Math-for-industry
Education & Research Hub

ISSN2188-1200

SICE-JSAE-AIMaP Tutorial

Advanced Automotive Control and Mathematics

Editors: Taketoshi Kawabe, Yoshihiro Mizoguchi,
Junichi Kako, Masakazu Mukai, Yuji Yasui

九州大学マス・フォア・インダストリ研究所

MI Lecture Note Vol.84 : Kyushu University

SICE-JSAE-AIMaP Tutorial

Advanced Automotive Control and Mathematics

September 8th, 2021

Online, SICE Annual Conference 2021

■**Organizer:**

Taketoshi Kawabe (Kyusyu University, SICE-JSAE Automotive Control and Modeling)

Yoshihiro Mizoguchi (Kyusyu University, AIMaP)

Co-organizer:

Junichi Kako (Toyota Motor Cooperation)

Masakazu Mukai (Kogakuin University)

Yuji Yasui (Honda R&D Co., Ltd.)

About MI Lecture Note Series

The Math-for-Industry (MI) Lecture Note Series is the successor to the COE Lecture Notes, which were published for the 21st COE Program “Development of Dynamic Mathematics with High Functionality,” sponsored by Japan’s Ministry of Education, Culture, Sports, Science and Technology (MEXT) from 2003 to 2007. The MI Lecture Note Series has published the notes of lectures organized under the following two programs: “Training Program for Ph.D. and New Master’s Degree in Mathematics as Required by Industry,” adopted as a Support Program for Improving Graduate School Education by MEXT from 2007 to 2009; and “Education-and-Research Hub for Mathematics-for-Industry,” adopted as a Global COE Program by MEXT from 2008 to 2012.

In accordance with the establishment of the Institute of Mathematics for Industry (IMI) in April 2011 and the authorization of IMI’s Joint Research Center for Advanced and Fundamental Mathematics-for-Industry as a MEXT Joint Usage / Research Center in April 2013, hereafter the MI Lecture Notes Series will publish lecture notes and proceedings by worldwide researchers of MI to contribute to the development of MI.

October 2018
Osamu Saeki
Director
Institute of Mathematics for Industry

Advanced Automotive Control and Mathematics

MI Lecture Note Vol.84, Institute of Mathematics for Industry, Kyushu University
ISSN 2188-1200
Date of issue: December 27, 2021
Editors: Taketoshi Kawabe, Yoshihiro Mizoguchi, Junichi Kako, Masakazu Mukai, Yuji Yasui
Publisher:
Institute of Mathematics for Industry, Kyushu University
Graduate School of Mathematics, Kyushu University
Motooka 744, Nishi-ku, Fukuoka, 819-0395, JAPAN
Tel +81-(0)92-802-4402, Fax +81-(0)92-802-4405
URL https://www.imi kyushu-u.ac.jp/

Contents

SICE-JSAE-AIMaP Advanced Automotive Control and Mathematics I

Efficient Lightweight Solvers for Real-Time Embedded Nonlinear MPC ······················ **1**
Themelis, Andreas (Kyusyu University)

Application of Energy Optimal Control to Hybrid Electric Vehicle ······························ **25**
Uchida, Hiroshi (Fukuyama University)

Research of the Optimization Methodology for Advanced Powertrain Control 2 ············ **43**
Hayasaka, Masato (Toyota Motor Corporation)

SICE-JSAE-AIMaP Advanced Automotive Control and Mathematics II

Expectations for Mathematical Science Researchers in the Field of Autonomous Driving Technology ······························ **59**
Hagiwara, Ichiro (Meiji University)

Topological Methods for Causal Inference from Time-Series Data ······························ **85**
Kaji, Shizuo (Kyusyu University)

Automated Driving and Driving Assistance Systems Using Cooperative Intelligence ······ **95**
Yasui, Yuji (Honda R&D Co., Ltd.)

Special session:

SICE-JSAE-AIMaP Advanced Automotive Control and Mathematics

Organizer:
Taketochi Kawabe (Kyusyu University, SICE-JSAE Automotive Control and Modeling)
Yoshihiro Mizoguchi (Kyusyu University, AIMaP)
Co-organizer
Junichi Kako (Toyota Motor Cooperation)
Masakazu Mukai (Kogakuin University)
Yuji Yasui (Honda R&D Co., Ltd.)

Background

The automotive industry is currently undergoing a once in a century transformation aimed at reducing traffic accidents, realizing carbon neutral society and providing new values of movement. Because the automotive technologies have to be significantly evolved in order to meet new requirements such as carbon neutral, CASE, etc., the expectation for AI (artificial intelligence), machine learning, advanced control, optimization and mathematics has been increased year by year. Consequently, SICE-JSAE Automotive Control and Modeling Technical Committee established by SICE (The Society of Instrument and Control Engineer) and JSAE (Society of Automotive Engineers of Japan, Inc.) has conducted various activities with AIMaP (Advanced Innovation powered by Mathematics Platform). In this special session, the collaboration of latest automotive technology and mathematics will be introduced and discussed.

Program

SICE-JSAE-AIMaP Advanced Automotive Control and Mathematics I

Chair: Taketochi Kawabe (Kyusyu University)
8th September 2021 (Wednesday)

14:15-14:45 (30min)	"Efficient Lightweight Solvers for Real-Time Embedded Nonlinear MPC" Andreas Themelis (Kyusyu University)
14:45-15:15 (30min)	"Application of Energy Optimal Control to Hybrid Electric Vehicle" Hiroshi Uchida (Fukuyama University)
15:15-15:45 (30min)	"Research of the Optimization Methodology for Advanced Powertrain Control 2" Masato Hayasaka (Toyota Motor Corporation)

SICE-JSAE-AIMaP Advanced Automotive Control and Mathematics II

Chair: Yoshihiro Mizoguchi (Kyusyu University)
8th September 2021 (Wednesday)

16:00-16:30 (30min)	"Expectations for Mathematical Science Researchers in the Field of Autonomous Driving Technology" Ichiro Hagiwara (Meiji University)
16:30-17:00 (30min)	"Topological Methods for Causal Inference from Time-Series Data" Shizuo Kaji (Kyusyu University)
17:00-17:30 (30min)	"Automated Driving and Driving Assistance Systems Using Cooperative Intelligence" Yuji Yasui (Honda R&D Co., Ltd.)

Speaker and Presentation Information

Andreas Themelis

Associate Professor
Kyusyu University

Presentation title:
Efficient Lightweight Solvers for Real-Time Embedded Nonlinear MPC

Abstract:
Model predictive control (MPC) has become a popular strategy to implement feedback control loops for a variety of systems. Since most systems are nonlinear by nature, nonlinear MPC offers a more accurate modeling, but it leads to nonconvex and much more complicated problems that need to be solved at every sampling step. In "embedded" applications such as autonomous driving, the resulting problems easily become of large scale and the sampling time can be as low as few milliseconds, thus imposing an imperative demand for algorithmic speed and efficiency. In this talk we show how the scalability properties of "proximal algorithms" can conveniently be employed to design certifiable, fast, and lightweight algorithms perfectly suited for embedded applications.

Hiroshi Uchida

Professor
Fukuyama University

Presentation title:
Application of Energy Optimal Control to Hybrid Electric Vehicle

Abstract:
The stability of energy optimal control (EOC) when the hybrid electric vehicle performs self-driving with speed control is verified by simulation and numerical calculation. As for the effects of the dead time of acceleration and velocity feedback, simulation that add dead time stepwise to both feedback results in negligible control error even if the dead time up to 40 ms (equivalent to 4 cycles of control loop) is given. As for the convergence of control, since the necessary and sufficient condition for the solution of the Euler-Poisson equation gives the minimum value of the evaluation function is that the second variation of the evaluation function is positive, the second variation of the evaluation function for both control law of engine and motor are calculated. As a result, it was shown that the engine control law is positive over the whole operation area on the torque-angular velocity plane, and it is the optimum solution over the whole operation area. Regarding the motor control law, although a part of the locus passes through the negative region, most of it exist within the positive region, indicating that the motor operated generally under the condition that the evaluation function had a minimum value.

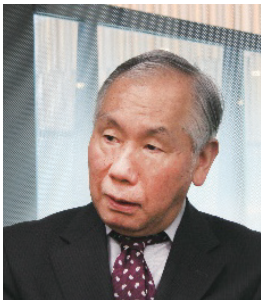
Masato Hayasaka

Project Manager
Toyota Motor Cooperation

Presentation title:
Research of the Optimization Methodology for Advanced Powertrain Control 2

Abstract:
Vehicle powertrain systems tend to be electrified in order to achieve carbon neutral. And development and the choice of various systems are necessary to meet different economic environment and energy policy, industrial policy and the needs of the customer every country, area. For advanced powertrain planning, we have to choose the best powertrain system from among many candidates expected by evaluating their potential correctly. Recent years, it has been clarified that some optimization methodologies are effective for potential evaluation of powertrain systems by obtaining optimal input time trajectory without any tailored controller. In this study, existing optimization methodology is improved to have wider coverage of powertrain system configurations. With proposed optimization methodology, the best input time trace is successfully achieved for fuel economy optimization of parallel-hybrid system.

Ichiro Hagiwara



Distinguished professor emeritus
Meiji University

Presentation title:
Expectations for Mathematical Science Researchers in the Field of Autonomous Driving Technology

Abstract:
Cars are now so popular that even from the general public, you can see what is being actively researched and developed now in the automobile industry. That is exactly the self-driving. Recently level 3 car was released where the system is responsible for driving until the system becomes difficult to drive. And at that time , the system entrusts driving to the driver. In this case the driver must keep getting more nervous than driving himself although it should be automatic driving for relaxation which gives the birth to the public opinion that level 3 is difficult in the first place. To realize the level 3 that should be, the system must explain the driver what and how the driver should do which necessitates to be installed causal machine learning system on the self-driving car.

It is expected to achieve level 4 quickly for regional revitalization where the self-driving is limited location and time. For level 4, remote monitoring system is used in such way that it can be easily monitored multiple self-driving cars by one person because one of the biggest expectations for the self-driving is to carry out transportation infrastructure with a small number of people. It also necessitates the causal machine learning and high-speed, high-precision image processing technology for this remote monitoring system.

As far as level 5, there is a problem "Safe but not relief" which requests technologies to achieve a higher level of ride quality. Among these technologies, it can be listed up origami engineering for realizing vibration isolation in the frequency band that affects ride comfort and generalization of energy control method for real time optimal control method. And also it is expected relief can be unraveled from deep mathematical point of view. The lecture will focus on the above.

Shizuo Kaji

Professor
Institute of Mathematics for Industry, Kyushu University

Presentation title:
Topological Methods for Causal Inference from Time-Series Data

Abstract:
The real world consists of a lot of inter-related systems that evolve over time. Understanding and modelling such relations is the main topic of (data) science. Given two systems, detecting causality between them is an important task. It is particularly difficult when we cannot intervene in the systems but can only observe their behaviours. In this talk, we give an overview of causal inference from observed data. There are mainly two approaches in causal inference that differ in the fundamental assumption for the system; deterministic or probabilistic. We mainly focus on the former case. We discuss the idea of topological methods, including the widely-used "convergent cross mapping (CCM)" and its variants, that do not require the underlying model identification and are applicable to complex non-linear systems.

Yuji Yasui

Executive Chief Engineer Honda R&D Co. Ltd
Innovative Research Excellence

Presentation title:
Automated Driving and Driving Assistance Systems Using Cooperative Intelligence

Abstract:
Honda has developed advanced driving assistance and automated driving vehicles (hereafter, AD/ADAS vehicle) in order to realize
"collision-free society" and to provide many people "joy and freedom of mobility. The vehicles with automation functions were released as driving support system for highway, and its autonomous level has been updated from Level2 to Level3 step by step while keeping wide operational design domain (ODD). However, there was huge technical gaps between Level2 and Level3. Honda released the AD/ADAS vehicle equipped with Level3 function for traffic jam situation on highway. In this session, how Honda broke the technical gaps through will be introduced. The level3 automated driving cannot be realized by only the application of AI (Artificial Intelligence), machine learning, and advanced control technologies.
Moreover, there are many tough subjects when the driving areas for automation functions are expanded from highway to city roads. The AD/ADAS vehicle has to coexist with other traffic participants in city areas. Of course, it has to avoid collision with them, even if it goes through congestion areas. However, if it indicates only passive movements following other traffic participants, it falls into dead-rock situation (continuous stop situation). The AD/ADAS vehicle has to become significantly smarter than current one in order to avoid the dead-rock situation by indicating cooperative behavior like a human. Honda expects AI and machine learning technology in order to realize the cooperative behavior of automated vehicles, and calls them "Cooperative intelligence: CI." Honda's latest research results for CI also will be introduced.

EFFICIENT LIGHTWEIGHT SOLVERS FOR REAL-TIME EMBEDDED NONLINEAR MPC

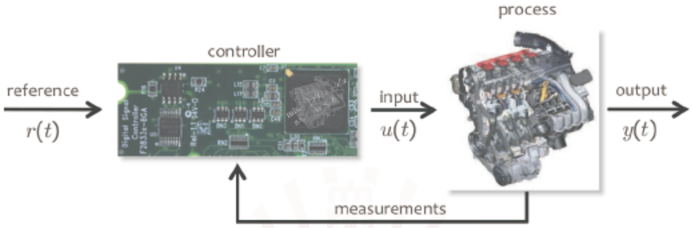
Andreas Themelis
Kyushu University
andreas.themelis@ees.kyushu-u.ac.jp

Control & decision making

1. Control & decision making
   - Model predictive control
   - Why is it hard
2. Problem setting & toolbox
   - Functions, variables, constraints
   - Simplest formulation
   - Speeding up (textbook attempts)
3. Novel speedup
   - Fast directions
   - Globalization
4. Embeddable ms-fast NMPC solvers
   - Handling state constraints
   - Experiments
5. Conclusions

Control & decision making

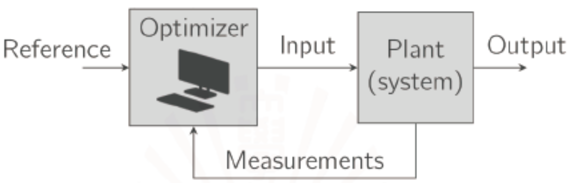
Model predictive control



A **controller** is a decision-making mechanism
who decides & actuates **inputs** to steer the plant/process/system

Control & decision making

Model predictive control



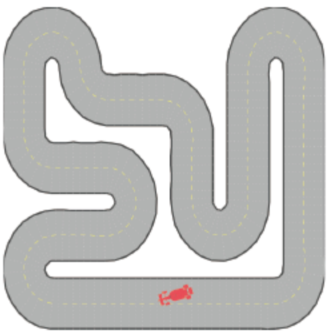
Conceptual example

Goal

- Fastest trajectory

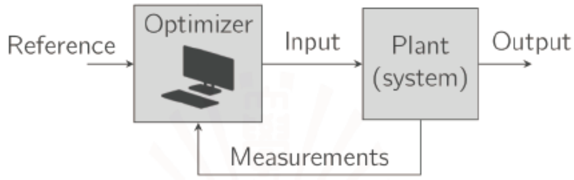
Constraints

- Stay on road
- Avoid other vehicles
- Physical limits (speed,. . .)



Control & decision making

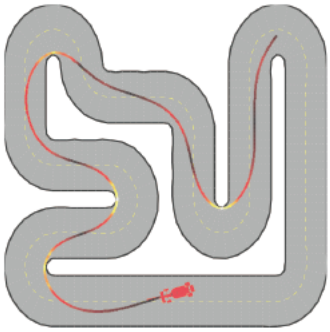
Model predictive control



Conceptual example

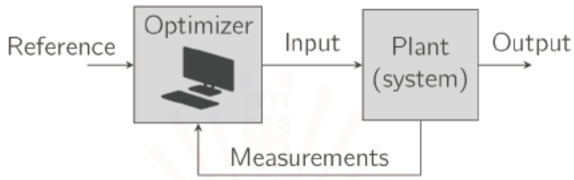
Problem

minimize	circuit time
considering	car dynamics road conditions
while	avoiding other cars staying on road



Control & decision making

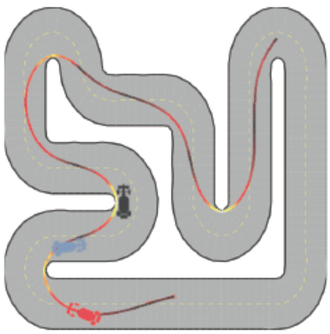
Model predictive control



Conceptual example

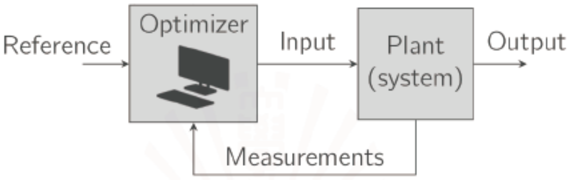
Input actuation

- Apply optimal input
- Check car/environment again!



Control & decision making

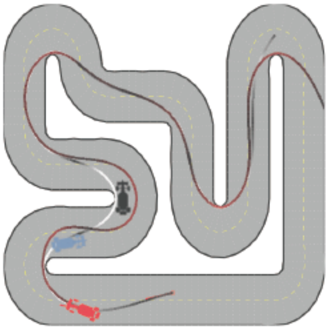
Model predictive control



Conceptual example

Feedback

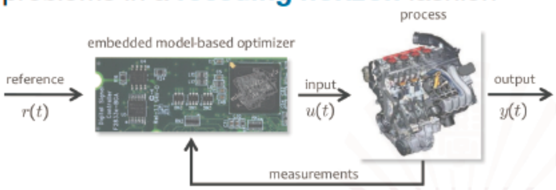
- **Discard** the long-term prediction
- **Restart** based on new info/measurements

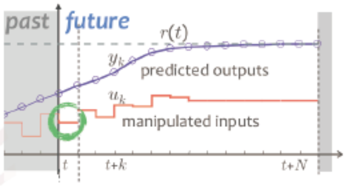


Control & decision making

Model predictive control

Use a dynamical model of the process to predict its evolution and choose control actions by **recursively solving** finite discrete-time optimal control problems in a **receding horizon** fashion





Optimal control problem

$$\underset{u}{\text{minimize}} \sum_{t=0}^{N-1} \ell_t(x_t(u),u_t) + \ell_N(x_N(u))$$

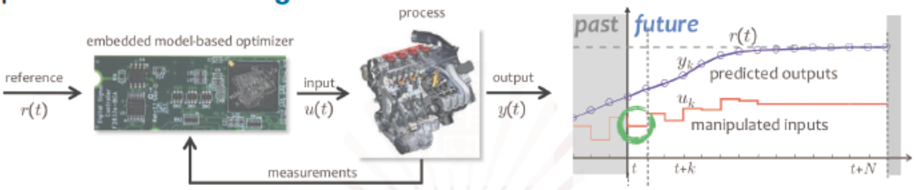
$$\text{s.t.} \quad u_t \in \mathcal{U}_t$$

$$x_t(u) \in X_t$$

Control & decision making

Model predictive control

Use a dynamical model of the process to predict its evolution and choose control actions by **recursively solving** finite discrete-time optimal control problems in a **receding horizon** fashion



Optimal control problem

$$\underset{u}{\text{minimize}} \sum_{t=0}^{N-1} \ell_t(x_t(u),u_t) + \ell_N(x_N(u))$$

$$\text{s.t.} \quad u_t \in \mathcal{U}_t$$

$$x_t(u) \in X_t$$

Optimization

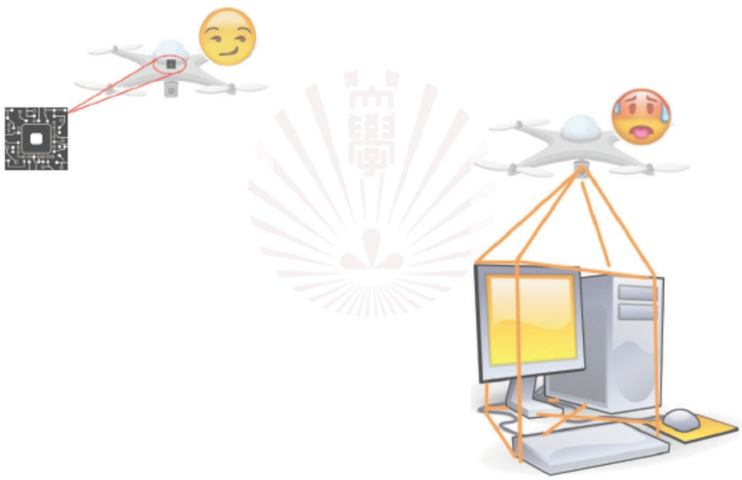
Technology of devising effective *decisions*:

- choose **variables**
- from within an **allowed set**
- that minimize a **cost**

Control & decision making

Why is it hard — Embeddability

Some applications **can't afford** high computing

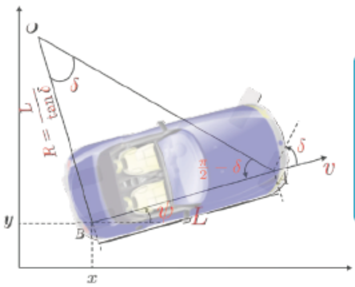


Algorithms must be **embeddable** on low-power chipsets

Control & decision making

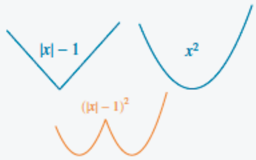
Why is it hard — Nonsmoothness + nonconvexity

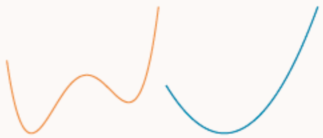
Nonlinear dynamics $\Rightarrow$ nonconvex problem



✓ convex $\circ$ **linear** = convex

✗ convex $\circ$ **nonlinear** = **?**





- Local VS global minima
  ...let's live with it
- Much theory **not applicable**
  - ✗ Duality
  - ✗ Monotone operators
  - ✗ Fejér monotonicity
  - ...

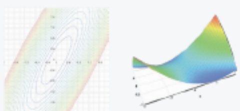
Control & decision making

Why is it hard — Summary

***ms*-fast**

< sampling time

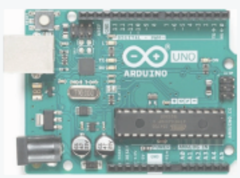
Reliable

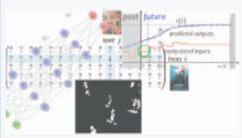


VS {ill-scaling, nonsmoothness, nonconvexity}

Embeddable

simple operations only





Multi-purpose(?)

Problem setting & toolbox

1. Control & decision making
   - Model predictive control
   - Why is it hard
2. Problem setting & toolbox
   - Functions, variables, constraints
   - Simplest formulation
   - Speeding up (textbook attempts)
3. Novel speedup
   - Fast directions
   - Globalization
4. Embeddable ms-fast NMPC solvers
   - Handling state constraints
   - Experiments
5. Conclusions

Problem setting & toolbox

Functions, variables, constraints

$$\operatorname*{minimize}_{u} \sum_{t=0}^{N-1} \ell_t(x_t(u), u_t) + \ell_N(x_N(u)) \quad \text{subject to} \begin{cases} u_t \in \mathcal{U}_t \\ x_t(u) \in X_t \end{cases}$$

Requirements

- ℓ_t, F_t smooth (e.g. C^2) — can be relaxed
- input constraints $\mathcal{U}_t$ *easy* to project onto (boxes, balls...) — no need for convex constraints, but typical in practice

Example: holonomic land vehicle with trailer

- steer to reference pos./orient. $(p_\star, \vartheta_\star)$
- avoid obstacles

Dynamics

$$\begin{cases} \dot{p}_x = u_x + L\dot{\vartheta}\sin\vartheta \\ \dot{p}_y = u_y - L\dot{\vartheta}\cos\vartheta \\ \dot{\vartheta} = \frac{1}{L}(u_y\cos\vartheta - u_x\sin\vartheta) \end{cases}$$

(input) velocity u
(state) trailer pos. p & head. angle ϑ
(const) trailer arm length L

MPC problem Given current pos. p_0 & head. angle θ_0,

$$\operatorname*{minimize}_{\substack{u_{t-1},(p_t,\vartheta_t)\\ t=1\ldots N}} \sum_{t=1}^{N} \tfrac{C_x}{2}\|p_t - p_\star\|^2 + \tfrac{C_x}{2}\|\vartheta_t - \vartheta_\star\|^2 + \tfrac{C_u}{2}\|u_{t-1}\|^2$$

$$\text{subject to} \begin{cases} (p_{t+1}, \theta_{t+1}) = F_t(p_t, \vartheta_t, u_t) & F_t \text{ discretized dynamics (e.g. RK4)} \\ u_{\min} \le u_t \le u_{\max} \\ p_t \notin \text{obstacles area} \end{cases}$$

Problem setting & toolbox

Functions, variables, constraints

$$\underset{u}{\text{minimize}} \sum_{t=0}^{N-1} \ell_t(x_t(u), u_t) + \ell_N(x_N(u)) \quad \text{subject to} \begin{cases} u_t \in \mathcal{U}_t \\ x_t(u) \in \mathcal{X}_t \end{cases}$$

Requirements

- ℓ_t, F_t smooth (e.g. C^2) — can be relaxed
- input constraints $\mathcal{U}_t$ *easy* to project onto (boxes, balls...) — no need for convex constraints, but typical in practice

States x_t recursively defined as

$$\begin{cases} x_0 \text{ fixed} \\ x_{t+1} = F_t(x_t, u_t) \end{cases}$$

$x_t(u) \in \mathcal{X}_t$ **difficult** to handle

$\Downarrow$

For now, **discard state constraints**
(we'll fix this later)

Problem becomes

$$\underset{u}{\text{minimize}} \ \underbrace{f(u; x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \leq u_t \leq u_{\text{MAX}})}$$

Problem setting & toolbox

Simplest formulation

$$\underset{u}{\text{minimize}} \ \underbrace{f(u; x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \leq u_t \leq u_{\text{MAX}})}$$

(Projected) gradient method (Cauchy, 1847)

$$\text{iterate} \quad u \leftarrow \underbrace{\Pi_{\mathcal{U}}}_{\text{projection on constraints}} \overbrace{(u - \gamma \nabla f(u))}^{\text{gradient descent step}}$$

Augustin-Louis Cauchy (1789-1857)

A. Cauchy, *Méthode générale pour la résolution des systemes d'équations simultanées*, Comp. Rend. Sci. Paris **25**:536–538, **1847**

Problem setting & toolbox

Simplest formulation

$$\underset{u}{\text{minimize}}\ \underbrace{f(u; x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \leq u_t \leq u_{\text{MAX}})}$$

(Projected) gradient method (Cauchy, 1847)

$$\text{iterate} \quad u \leftarrow \underbrace{\Pi_{\mathcal{U}}}_{\text{projection on constraints}} \overbrace{(u - \gamma \nabla f(u))}^{\text{gradient descent step}}$$

Augustin-Louis Cauchy (1789-1857)

Arguably the **simplest** possible method

☺ **Embeddable**
☺ **Minimal assumptions**

Unreliable for real-time applications

☹ **Slow**
☹ Sensitive to conditioning
Ex. $f(x, y) = x^2 + cy^2$
can take >1million iter. to reach accuracy $\varepsilon = 0.1$

A. Cauchy, *Méthode générale pour la résolution des systemes d'équations simultanées*, Comp. Rend. Sci. Paris **25**:536–538, **1847**

Problem setting & toolbox

Speeding up (textbook attempt)

If we didn't have constraints **AT ALL** (not even on the inputs!)

$$\underset{u \in \mathbb{R}^n}{\text{minimize}}\, f(u) \qquad f \text{ (twice) smooth}$$

$$u^+ = u - \tau B^{-1} \nabla f(u)$$

- Pure Newton: $B = \nabla^2 f(u)$
- Quasi-Newton: $B \approx \nabla^2 f(u)$ with linear algebra

Isaac Newton (1642-1727)

Key idea:

- $\nabla f(u^{(k)}) - \nabla f(u^{(k-1)}) \approx \nabla^2 f(u^{(k)})\,(u^{(k)} - u^{(k-1)})$

I. Newton, *Philosophiae Naturalis Principia Mathematica*, London, **1687**

Problem setting & toolbox

Speeding up (textbook attempt)

If we didn't have constraints **AT ALL** (not even on the inputs!)

$$\underset{u \in \mathbb{R}^n}{\text{minimize}}\, f(u) \qquad f \text{ (twice) smooth}$$

$$u^+ = u - \tau B^{-1} \nabla f(u)$$

Isaac Newton
(1642-1727)

- Pure Newton: $B = \nabla^2 f(u)$
- Quasi-Newton: $B \approx \nabla^2 f(u)$ with linear algebra

Key idea:

- $\underbrace{\nabla f(u^{(k)}) - \nabla f(u^{(k-1)})}_{y^{(k)}} \approx \underbrace{\nabla^2 f(u^{(k)})}_{\approx B_{k+1}} \underbrace{(u^{(k)} - u^{(k-1)})}_{s^{(k)}}$
- Update estimate $B_k \mapsto B_{k+1}$ by enforcing the *"secant condition"* $B_{k+1} s^{(k)} = y^{(k)}$

I. Newton, *Philosophiae Naturalis Principia Mathematica*, London, 1687

Problem setting & toolbox

Speeding up (textbook attempt)

If we didn't have constraints **AT ALL** (not even on the inputs!)

$$\underset{u \in \mathbb{R}^n}{\text{minimize}}\, f(u) \qquad f \text{ (twice) smooth}$$

$$u^+ = u - \tau B^{-1} \nabla f(u)$$

Isaac Newton
(1642-1727)

- Pure Newton: $B = \nabla^2 f(u)$
- Quasi-Newton: $B \approx \nabla^2 f(u)$ with linear algebra

Key idea:

- $\underbrace{\nabla f(u^{(k)}) - \nabla f(u^{(k-1)})}_{y^{(k)}} \approx \underbrace{\nabla^2 f(u^{(k)})}_{\approx B_{k+1}} \underbrace{(u^{(k)} - u^{(k-1)})}_{s^{(k)}}$
- Update estimate $B_k \mapsto B_{k+1}$ by enforcing the *"secant condition"* $B_{k+1} s^{(k)} = y^{(k)}$

- Converges with suitable **linesearch** (to tune τ)

I. Newton, *Philosophiae Naturalis Principia Mathematica*, London, 1687

Problem setting & toolbox

Speeding up — Summary

Projected grandient method

☺ Cheap(est)
☺ Constraints ✓
☹ Slow

Newton method

☺ Fast (very!)
☹ Expensive
☹ Constraints ✗

Quasi-Newton methods

☺ Fast
☺ Cheap
☹ Constraints ✗

Problem setting & toolbox

Speeding up — Summary

Projected grandient method

☺ Cheap(est)
☺ Constraints ✓
☹ Slow

Newton method

☺ Fast (very!)
☹ Expensive
☹ Constraints ✗

Quasi-Newton methods

☺ Fast
☺ Cheap
☹ Constraints ✗

Linesearch methods

$$u^+ = u + \tau d$$

all require f smooth

⇒ (no constraints)

Novel speedup

1 Control & decision making
   Model predictive control
   Why is it hard

2 Problem setting & toolbox
   Functions, variables, constraints
   Simplest formulation
   Speeding up (textbook attempts)

3 **Novel speedup**
   Fast directions
   Globalization

4 Embeddable *ms*-fast NMPC solvers
   Handling state constraints
   Experiments

5 Conclusions

Novel speedup

Fast directions — for nonsmooth problems

$$\operatorname*{minimize}_{u}\ \underbrace{f(u;x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \le u_t \le u_{\text{MAX}})}$$

Optimality conditions

$$u \text{ local minimum} \quad \Rightarrow \quad u - \Pi_{\mathcal{U}}(u - \gamma \nabla f(u)) = 0$$

- If $\mathcal{U} = \mathbb{R}^n$ (unconstrained), reduces to $\nabla f(u) = 0$

Novel speedup

Fast directions — for nonsmooth problems

$$\underset{u}{\text{minimize}}\ \underbrace{f(u;x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \le u_t \le u_{\text{MAX}})}$$

Optimality conditions

$$u \text{ local minimum} \quad \Rightarrow \quad \overbrace{u - \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))}^{\mathcal{R}(u)} = 0$$

- **Idea** for "fast" directions

Quasi-Newton on $\mathcal{R}$

$$\underbrace{\mathcal{R}(u^{(k)}) - \mathcal{R}(u^{(k-1)})}_{y^{(k)}} = B_{k+1} \underbrace{(u^{(k)} - u^{(k-1)})}_{s^{(k)}}$$

Novel speedup

Fast directions — for nonsmooth problems

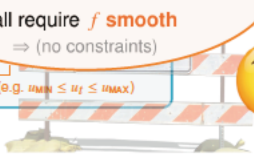
Linesearch methods

$$u^+ = u + \tau d$$

all require f smooth

$\Rightarrow$ (no constraints)

$$\underset{u}{\text{minimize}}\ \underbrace{f(u;x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \le u_t \le u_{\text{MAX}})}$$



Can't use **(classical)** linesearch

The **true** cost $u \mapsto \begin{cases} f(u) & \text{if } u \in \mathcal{U} \\ \infty & \text{if } u \notin \mathcal{U} \end{cases}$ is nonsmooth

Novel speedup

Globalization — A novel nonsmooth LS

$$\underset{u}{\text{minimize}}\ \underbrace{f(u; x_0)}_{\text{smooth}} \quad \text{subject to}\ u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \leq u_t \leq u_{\text{MAX}})}$$

New tool: for $\gamma > 0$, define

$$\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$$

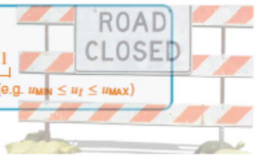
and

$$\varphi_\gamma(u) = f(u) + \langle \nabla f(u), \bar{u} - u \rangle + \tfrac{1}{2\gamma}\|\bar{u} - u\|^2$$

Novel speedup

Globalization — A novel nonsmooth LS

$$\underset{u}{\text{minimize}}\ \underbrace{f(u; x_0)}_{\text{smooth}} \quad \text{subject to}\ u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \leq u_t \leq u_{\text{MAX}})}$$



New tool: for $\gamma > 0$, define

$$\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$$

and

$$\varphi_\gamma(u) = f(u) + \langle \nabla f(u), \bar{u} - u \rangle + \tfrac{1}{2\gamma}\|\bar{u} - u\|^2$$

Key facts

0. u loc. min. $\Rightarrow \overbrace{u - \bar{u}}^{\mathcal{R}(u)} = 0$
1. $u \mapsto \varphi_\gamma(u)$ is **continuous**
2. $\gamma > 0$ can be chosen such that

$$\varphi_\gamma(\bar{u}) \leq \varphi_\gamma(u) - c\|\bar{u} - u\|^2$$

for some $c > 0$

Novel speedup

Globalization — A novel nonsmooth LS

$$\underset{u}{\text{minimize}}\ \underbrace{f(u;x_0)}_{\text{smooth}} \quad \text{subject to } u \in \underbrace{\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}}_{\text{easy to project onto (e.g. } u_{\text{MIN}} \le u_t \le u_{\text{MAX}})}$$

New tool: for $\gamma > 0$, define

$$\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$$

and

$$\varphi_\gamma(u) = f(u) + \langle \nabla f(u), \bar{u} - u \rangle + \tfrac{1}{2\gamma}\|\bar{u} - u\|^2$$

Key facts

0. u loc. min. $\Rightarrow$ $\overbrace{u - \bar{u}}^{\mathcal{R}(u)} = 0$
1. $u \mapsto \varphi_\gamma(u)$ is **continuous**
2. $\gamma > 0$ can be chosen such that

$$\varphi_\gamma(\bar{u}) \le \varphi_\gamma(u) - c\|\bar{u} - u\|^2$$

for some $c > 0$

Novel linesearch

constraints ✗ $\cancel{u^+ = u + \tau d}$

constraints ✓ $u^+ = (1-\tau)\bar{u} + \tau(u + d)$

reducing τ until

$$\varphi_\gamma(u^+) < \varphi_\gamma(u) - \tfrac{c}{2}\|\bar{u} - u\|^2 \qquad (\star)$$

1 + 2 $\Rightarrow$ $(\star)$ passed for small enough τ

Embeddable *ms*-fast NMPC solvers

1. Control & decision making
   - Model predictive control
   - Why is it hard
2. Problem setting & toolbox
   - Functions, variables, constraints
   - Simplest formulation
   - Speeding up (textbook attempts)
3. Novel speedup
   - Fast directions
   - Globalization
4. **Embeddable *ms*-fast NMPC solvers**
   - **Handling state constraints**
   - **Experiments**
5. Conclusions

Embeddable *ms*-fast NMPC solvers

(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u)=\ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \leq \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1-\tau)\bar{u} + \tau(u + d)$ reducing τ until $\varphi_\gamma(u^+) \leq \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$

Embeddable *ms*-fast NMPC solvers

(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u)=\ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \leq \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1-\tau)\bar{u} + \tau(u + d)$ reducing τ until $\varphi_\gamma(u^+) \leq \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$

CasADi Automatic Differentiation tool

Optimized C code for (backward) AD

J. Andersson, *A general-purpose software framework for dynamic optimization.* KU Leuven, **2013**

Embeddable *ms*-fast NMPC solvers

(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u)=\ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \le \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1 - \tau)\bar{u} + \tau(u + d)$
   reducing τ until
   $\varphi_\gamma(u^+) \le \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$

$\mathcal{U}_0 \times \cdots \times \mathcal{U}_{N-1}$ is separable

$$\bar{u}_t = \Pi_{\mathcal{U}_t}(u_t - \gamma \nabla_{u_t} f(u))$$

N projections in parallel

Embeddable *ms*-fast NMPC solvers

(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u)=\ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \le \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1 - \tau)\bar{u} + \tau(u + d)$
   reducing τ until
   $\varphi_\gamma(u^+) \le \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$

Optimality conditions $\mathcal{R}(u) = 0$ where

$$\mathcal{R}(u) := u - \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$$

Idea: quasi-Newton method on $\mathcal{R}$

e.g., **L-BFGS** (only scalar products)

Embeddable *ms*-fast NMPC solvers

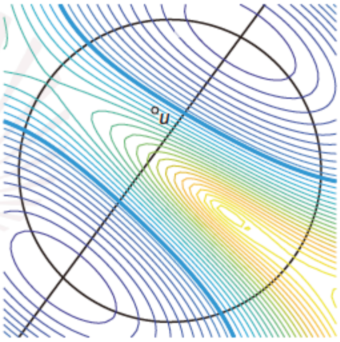
(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u) = \ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \leq \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1 - \tau)\bar{u} + \tau(u + d)$
   reducing τ until
   $\varphi_\gamma(u^+) \leq \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$



Illustrative (toy) example

$f(u) = \frac{1}{2}\operatorname{dist}^2(u, l)$

l is a line intersecting a circumference $\mathcal{U}$

Embeddable *ms*-fast NMPC solvers

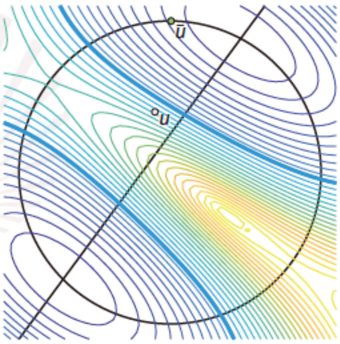
(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u) = \ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \leq \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1 - \tau)\bar{u} + \tau(u + d)$
   reducing τ until
   $\varphi_\gamma(u^+) \leq \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$



Illustrative (toy) example

$f(u) = \frac{1}{2}\operatorname{dist}^2(u, l)$

l is a line intersecting a circumference $\mathcal{U}$

Embeddable *ms*-fast NMPC solvers

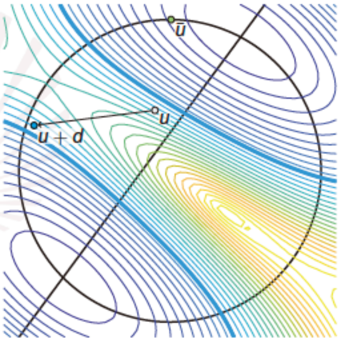
(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t,u_t) + \ell_N(x_N)}^{f(u)=\ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \le \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma\nabla f(u))$
3. Choose a direction d
4. $u^+ = (1-\tau)\bar{u} + \tau(u+d)$
   reducing τ until
   $\varphi_\gamma(u^+) \le \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$



Illustrative (toy) example

$f(u) = \frac{1}{2}\operatorname{dist}^2(u, l)$

l is a line intersecting a circumference $\mathcal{U}$

Embeddable *ms*-fast NMPC solvers

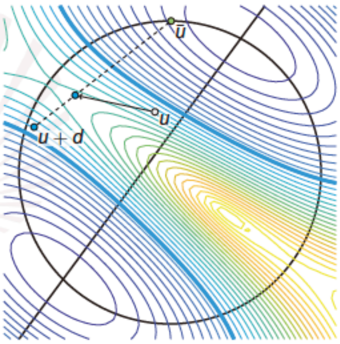
(still free states...)

$$\underset{u}{\text{minimize}} \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t,u_t) + \ell_N(x_N)}^{f(u)=\ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \le \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma\nabla f(u))$
3. Choose a direction d
4. $u^+ = (1-\tau)\bar{u} + \tau(u+d)$
   reducing τ until
   $\varphi_\gamma(u^+) \le \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$



Illustrative (toy) example

$f(u) = \frac{1}{2}\operatorname{dist}^2(u, l)$

l is a line intersecting a circumference $\mathcal{U}$

Embeddable *ms*-fast NMPC solvers

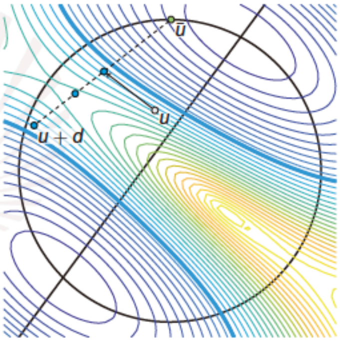
(still free states...)

$$\underset{u}{\text{minimize}} \; \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u) = \ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \le \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1 - \tau)\bar{u} + \tau(u + d)$
   reducing τ until
   $\varphi_\gamma(u^+) \le \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$



Illustrative (toy) example
$f(u) = \frac{1}{2}\operatorname{dist}^2(u, l)$
l is a line intersecting a circumference $\mathcal{U}$

Embeddable *ms*-fast NMPC solvers

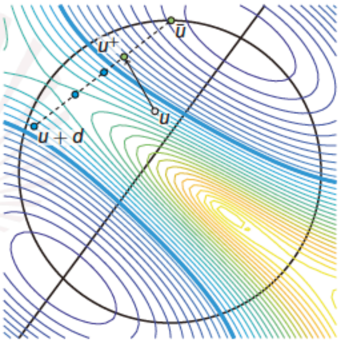
(still free states...)

$$\underset{u}{\text{minimize}} \; \overbrace{\sum_{t=0}^{N-1} \ell_t(x_t, u_t) + \ell_N(x_N)}^{f(u) = \ell(x(u),u))} \quad \text{subject to } u_t \in \mathcal{U}_t$$

Require suitable γ, c, initial $u = (u_0 \dots u_{N-1})$

Iterate until $\|u - \bar{u}\| \le \varepsilon_{\mathtt{tol}}$

1. Compute $\nabla f(u)$
2. $\bar{u} = \Pi_{\mathcal{U}}(u - \gamma \nabla f(u))$
3. Choose a direction d
4. $u^+ = (1 - \tau)\bar{u} + \tau(u + d)$
   reducing τ until
   $\varphi_\gamma(u^+) \le \varphi_\gamma(u) - \frac{c}{2}\|u - \bar{u}\|^2$

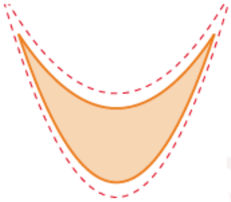


Illustrative (toy) example
$f(u) = \frac{1}{2}\operatorname{dist}^2(u, l)$
l is a line intersecting a circumference $\mathcal{U}$

Embeddable ms-fast NMPC solvers

Handling state constraints

- novel obstacle avoidance constraints encoding
- uses a single equality constraint!



$$x \notin O := \{z \mid h_i(x) > 0,\ i = 1, \ldots, m\}$$

$$\Updownarrow$$

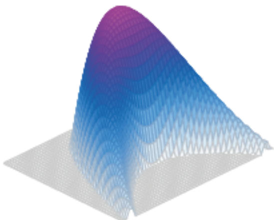
$$\exists i \quad \text{such that} \quad h_i(x) \le 0$$

$$\Updownarrow$$

$$\exists i \quad \text{such that} \quad \max\{h_i(x), 0\} = 0$$

$$\Updownarrow$$

$$\psi(x) := \prod_{i=1}^{m} \max\{h_i(x), 0\} = 0$$

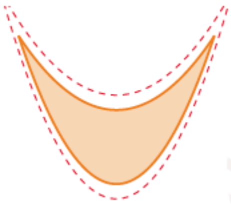


$\psi(x) = \prod_{i=1}^{m} \max\{h_i(x), 0\}$

Embeddable ms-fast NMPC solvers

Handling state constraints

- novel obstacle avoidance constraints encoding
- uses a single equality constraint!



$$x \notin O := \{z \mid h_i(x) > 0,\ i = 1, \ldots, m\}$$

$$\Updownarrow$$

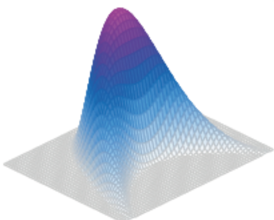
$$\exists i \quad \text{such that} \quad h_i(x) \le 0$$

$$\Updownarrow$$

$$\exists i \quad \text{such that} \quad \max\{h_i(x), 0\} = 0$$

$$\Updownarrow$$

$$\psi(x) := \prod_{i=1}^{m} \max\{h_i(x), 0\} = 0$$



$\psi^2(x) = \prod_{i=1}^{m} \max\{h_i(x), 0\}^2$

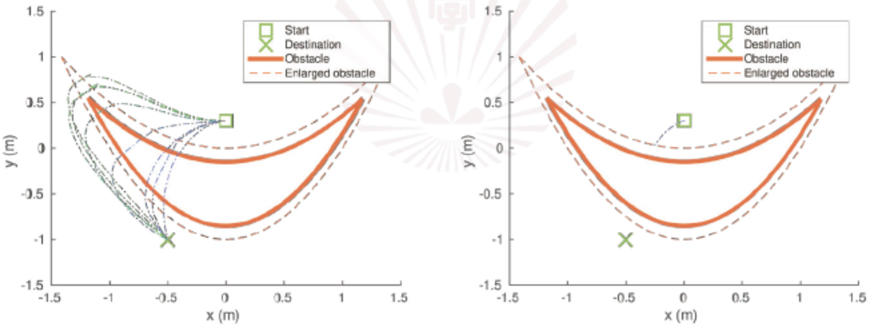
Quadratic penalty

Soften obstacle avoidance by adding $\mu\psi^2$ to cost function ($\mu > 0$)

Embeddable *ms*-fast NMPC solvers

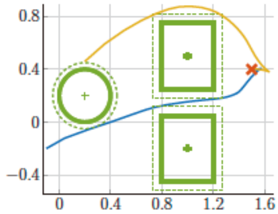
Handling state constraints

- **Quadratic penalty method:** gradually increase penalty μ
- Solve subproblems, **warm starting** with previous solution
- **Helps avoiding local minima** (getting stuck to obstacles)



Embeddable *ms*-fast NMPC solvers

Comparisons — Obstacle avoidance



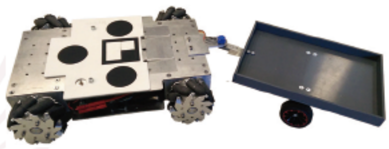
Goals

- steer vehicle to reference pos./orient. $(p_\star, \vartheta_\star)$
- avoid obstacles

Nonlinear system

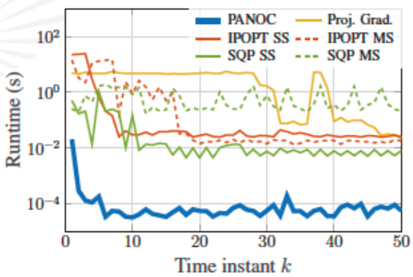
$$\begin{cases} \dot{p}_x = u_x + L\dot{\vartheta}\sin\vartheta \\ \dot{p}_y = u_y - L\dot{\vartheta}\cos\vartheta \\ \dot{\vartheta} = \frac{1}{L}(u_y\cos\vartheta - u_x\sin\vartheta) \end{cases}$$

u: velocity
p: position
ϑ: head. angle



Implementation

- discretized with RK4
- horizon $N = 50$
- 10Hz NMPC control rate
- $\|u\|_\infty \leq 0.8 m/s$
- soft-constrained enlarged obstacles with adaptive penalty



Conclusions

1. Control & decision making
   - Model predictive control
   - Why is it hard
2. Problem setting & toolbox
   - Functions, variables, constraints
   - Simplest formulation
   - Speeding up (textbook attempts)
3. Novel speedup
   - Fast directions
   - Globalization
4. Embeddable *ms*-fast NMPC solvers
   - Handling state constraints
   - Experiments
5. **Conclusions**

Conclusions

We ended up explaining how **PANOC** algorithm works

- Efficient, **QP-free**, NMPC line-search algorithm
- Give it a try
  - Embeddable NMPC `C` code generator (`Matlab` & `Python` interfaces)
    `https://github.com/kul-optec/nmpc-codegen`
  - Standalone `Julia` version
    `https://github.com/kul-optec/PANOC.jl`
- More than NMPC:
  engine of **generic optimization solvers**
  - OpEn (embedded Optimization Engine)
    `https://alphaville.github.io/optimization-engine/`
  - ALM solver
    `https://github.com/tttapa/PANOC-ALM`

Lorenzo **Stella**
Amazon Berlin

Pantelis **Sopasakis**
QU Belfast

Panos **Patrinos**
KU Leuven

More info on PANOC (shameful self-advertisement)

- AT, L. Stella and P. Patrinos, *Forward-backward envelope for the sum of two nonconvex functions: Further properties and nonmonotone linesearch algorithms*, SIAM J Opt **28**(3):2274-2303, **2018**
- L. Stella, AT, P. Sopasakis and P. Patrinos, *A simple and efficient algorithm for nonlinear model predictive control*, In: *IEEE 56th CDC*, **2017**
- A. Sathya, P. Sopasakis, R. Van Parys, AT, G. Pipeleers and P. Patrinos, *Embedded nonlinear model predictive control for obstacle avoidance using PANOC*, In: *IEEE ECC*, Jun **2018**

Conclusions

We ended up explaining how **PANOC** algorithm works

Lorenzo **Stella**
Amazon Berlin

- Efficient, **QP-free**, NMPC line-search algorithm
- Give it a try
  - Embeddable NMPC `C` code generator (`Matlab` & `Python` interfaces)
    `https://github.com/kul-optec/nmpc-codegen`
  - Standalone `Julia` version
    `https://github.com/kul-optec/PANOC.jl`

Pantelis **Sopasakis**
QU Belfast

- More than NMPC:
  engine of **generic optimization solvers**
  - OpEn (embedded Optimization Engine)
    `https://alphaville.github.io/optimization-engine/`
  - ALM solver
    `https://github.com/tttapa/PANOC-ALM`

Panos **Patrinos**
KU Leuven

More info on PANOC (shameful self-advertisement)

- AT, L. Stella and P. Patrinos, *Forward-backward envelope for the sum of two nonconvex functions: Further properties and nonmonotone linesearch algorithms*, SIAM J Opt **28**(3):2274-2303, **2018**
- L. Stella, AT, P. Sopasakis and P. Patrinos, *A simple and efficient algorithm for nonlinear model predictive control*, In: *IEEE 56th CDC*, **2017**
- A. Sathya, P. Sopasakis, R. Van Parys, AT, G. Pipeleers and P. Patrinos, *Embedded nonlinear model predictive control for obstacle avoidance using PANOC*, In: *IEEE ECC*, Jun **2018**

Application of Energy Optimal Control to Hybrid Electric Vehicle

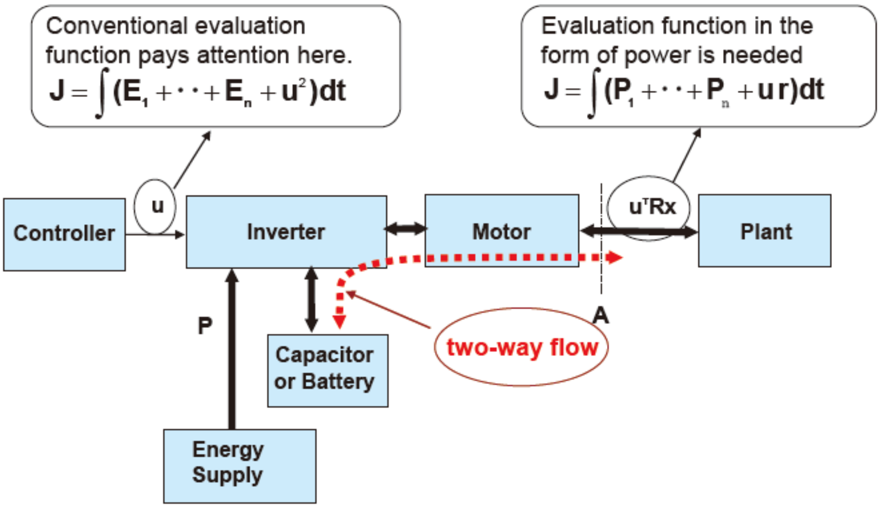
Hiroshi Uchida

Dept. of Mechanical System Engineering,
Fukuyama University

Background

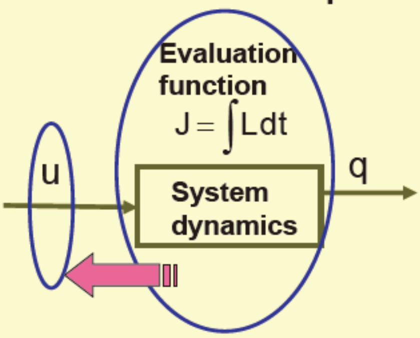
- When considering the application of optimal control for the purpose of improving the energy efficiency of HEV, there are the following problems.
  1. In normal optimal control with the evaluation function of a quadratic form, it is difficult to obtain an analytic solution to achieve the control purpose of minimizing energy consumption and maximizing the amount of regenerative energy (equivalent to negative energy consumption) at the same time.
  2. On the other hand, when nonlinear control is performed while solving differential equations of Lagrange multiplier in real time, it is difficult to implement this with inexpensive computers for automotive use because the efficiency functions of the engine and motor are nonlinear.

Energy Optimal Control



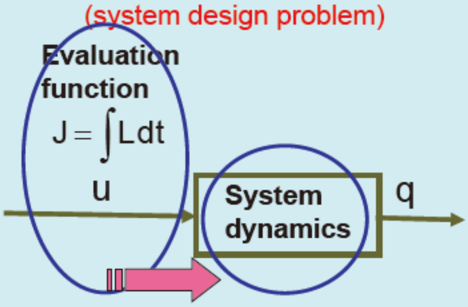
Energy Optimal Control

Conventional control problem



Acquire the control law to minimize the evaluation function J when J and the system dynamics are given.

New control problem
(system design problem)



Acquire the system dynamics to minimize the evaluation function J when J and control law u are given.

The system dynamics is easily obtained using the calculus of variations

$$\int \frac{\partial L}{\partial q}dt-\frac{\partial L}{\partial \dot{q}}+\frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right)=0$$

Energy Optimal Control

Use the system dynamics $B(\ddot{q},\dot{q},q)=u$
as the control law $u=B(\ddot{q},\dot{q},q)$

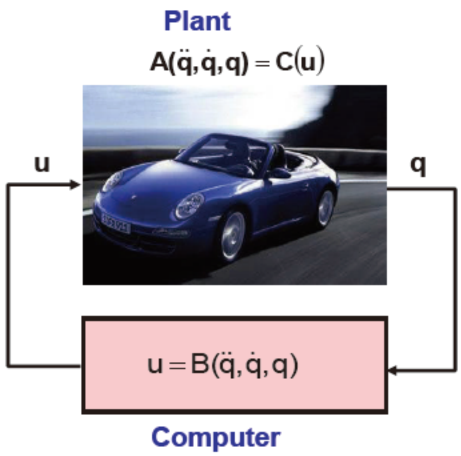
Combine the plant dynamics A and
the control law B in a closed-loop

u and **q** will be the solution of
simultaneous differential equation

$$A(\ddot{q},\dot{q},q)=C(u)$$

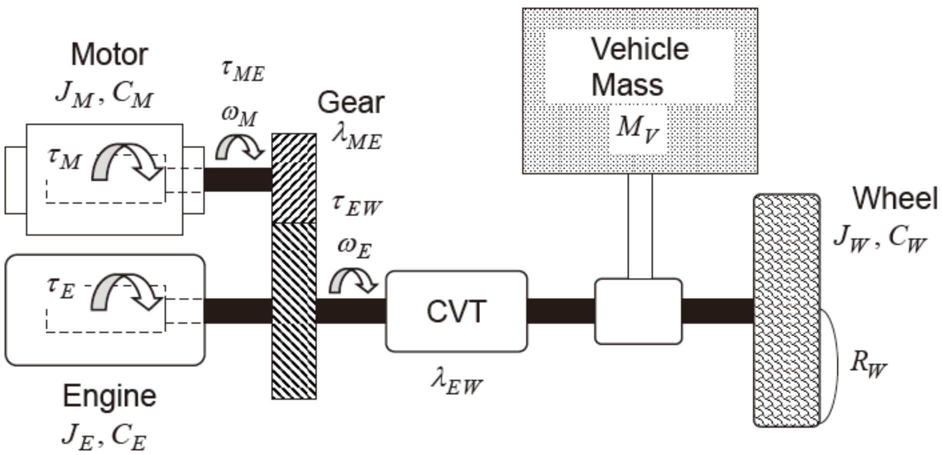
$$u=B(\ddot{q},\dot{q},q)$$

(q: generalized coordinate)



The plant behaves as if its dynamics is
that of the optimal system

HEV model



Schematic Model of Parallel HEV

HEV model

- Dynamics of wheel, engine, and motor

$$\lambda_{EW}\tau_{EW} = J_W{}'\dot{\omega}_W + C_W\omega_W + R_W(f_A + f_R)$$
$$\tau_E - \tau_{EW} + \lambda_{ME}\tau_{ME} = J_E\dot{\omega}_E + C_E\omega_E$$
$$\tau_M - \tau_{ME} = J_M\dot{\omega}_M + C_M\omega_M$$

- Combined form of the dynamics

$$\boldsymbol{\tau} + \begin{bmatrix} \lambda_{ME} \\ -1 \end{bmatrix}\tau_{ME} = \mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{C}\boldsymbol{\omega} + \begin{bmatrix} L_{ES} \\ 0 \end{bmatrix}$$

- Dissipated power is defined as

$$P = -\boldsymbol{\omega}^{\mathrm{T}} J\dot{\boldsymbol{\omega}} - \boldsymbol{\omega}^{\mathrm{T}} C\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{T}} \begin{bmatrix} \lambda_{ME} \\ -1 \end{bmatrix}\tau_{ME}$$

6

Evaluation Function

- Integrand of Evaluation Function

$$L = \kappa P + g + r\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\tau}$$

- Performance Index

$$g = r_1(\hat{\omega}_E - \omega_E)(\hat{\omega}_M - \omega_M) + r_2\dot{\omega}_E(\hat{\omega}_M - \omega_M)$$
$$+ r_2(\hat{\omega}_E - \omega_E)\dot{\omega}_M + r_4 q + r_5 p$$

$$q = \frac{\tau_E\omega_E}{f_E} \qquad p = \frac{\tau_M\omega_M}{f_M}$$

  – Product of the angular velocity error (and angular acceleration) of engine and motor is used as the performance index term for speed tracking
  – q and p are the energy consumption of engine and motor, respectively. f_E, f_M are the efficiency functions.

7

Derivation of Control Law

- Control law of engine and motor is derived by applying Euler Poisson's equation to L.

$$\tau_E = \frac{\kappa\left(2C_{ES} - \dot{J}_{ES}\right)\omega_E + r_1(\widehat{\omega}_M - \omega_M) + r_2\widehat{\dot{\omega}}_M}{r + \frac{r_4}{f_E}\left(1 - \frac{\omega_E}{f_E}\frac{\partial f_E}{\partial \omega_E}\right)}$$

$$\tau_M = \frac{2\kappa C_M\omega_M + r_1(\widehat{\omega}_E - \omega_E) + r_2\widehat{\dot{\omega}}_E}{r + \frac{r_5}{f_M}\left(1 - \frac{\omega_M}{f_M}\frac{\partial f_M}{\partial \omega_M}\right)}$$

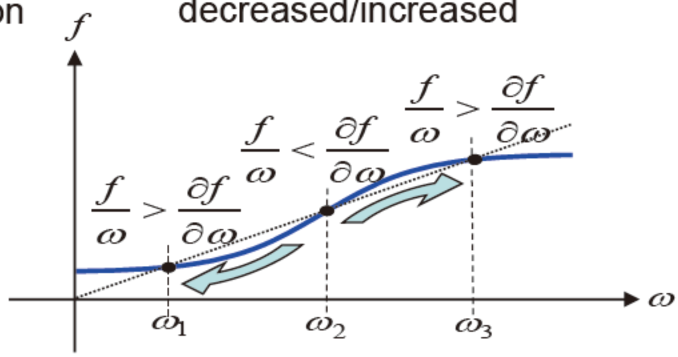
Speed control law that has both velocity feedback and acceleration feedforward is obtained.

Control Law - Function of Denominator

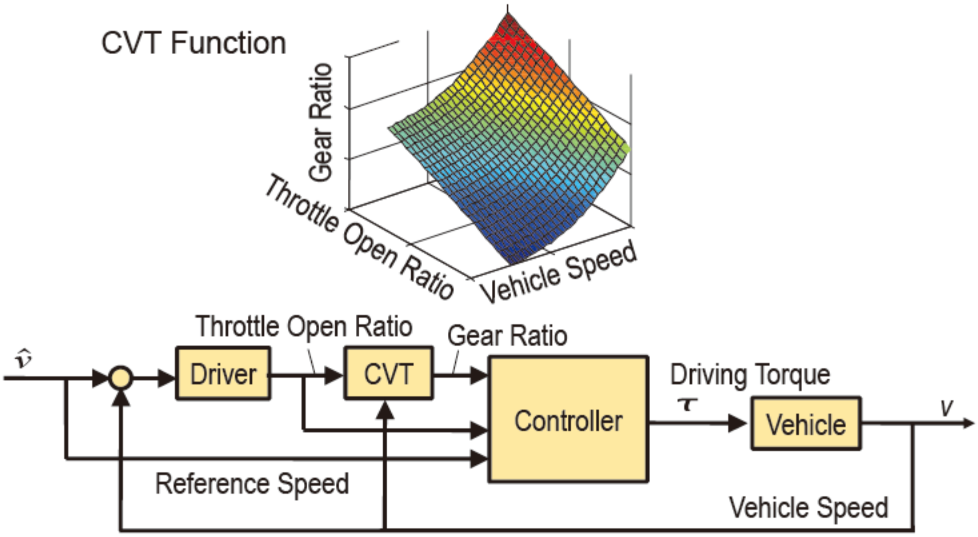
$$\frac{r}{\boxed{f}}\boxed{\left(1 - \frac{\omega}{f}\frac{\partial f}{\partial \omega}\right)}$$

Torque is increased under the high energy efficiency condition

When the efficiency function is concave/convex, torque is decreased/increased



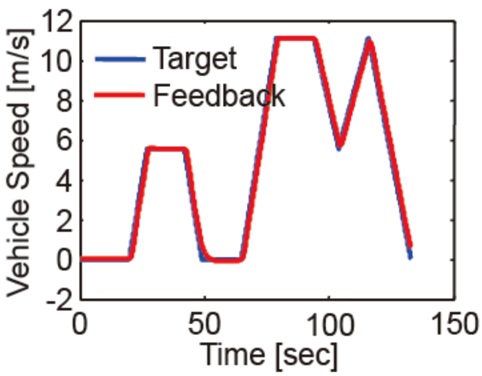
Simulation



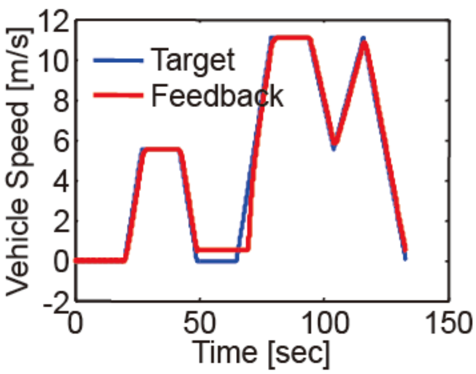
Simulation

- For comparison: Rule-based control (RBC)
  1. During acceleration: Both the engine and the motor generate driving torque. The torque distribution is determined based on battery SOC (state of charge).
  2. In cruising speed: Either the engine or the motor allots 100% torque, and the other side is assumed to be 0. When SOC is below a certain threshold, the engine is selected and when SOC exceeds the threshold, the motor will take the place.
  3. During deceleration: The motor allots 100% of the braking torque. The regenerative electric power is obtained from the motor.

Speed Tracking Performance



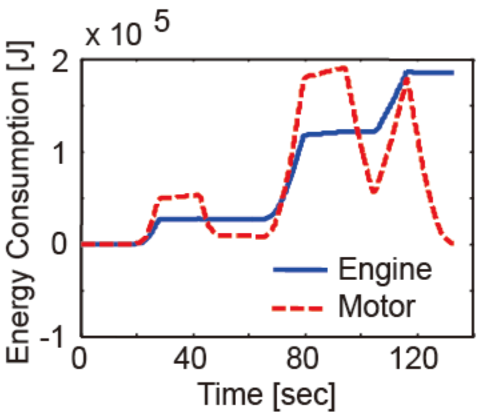
RBC



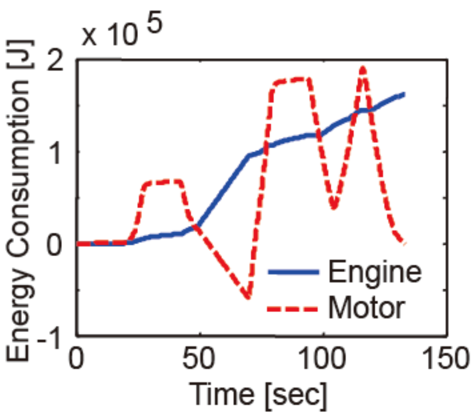
EOC

12

Energy Consumption



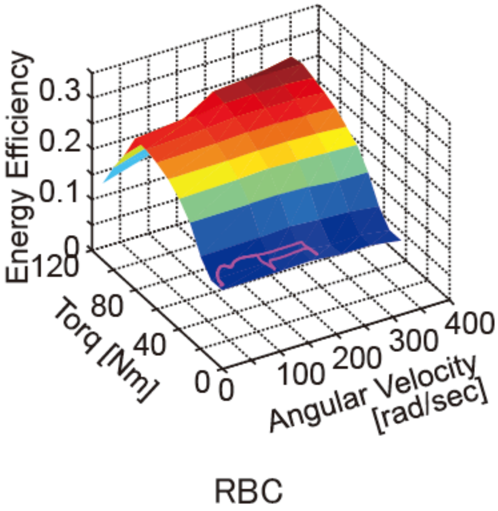
RBC

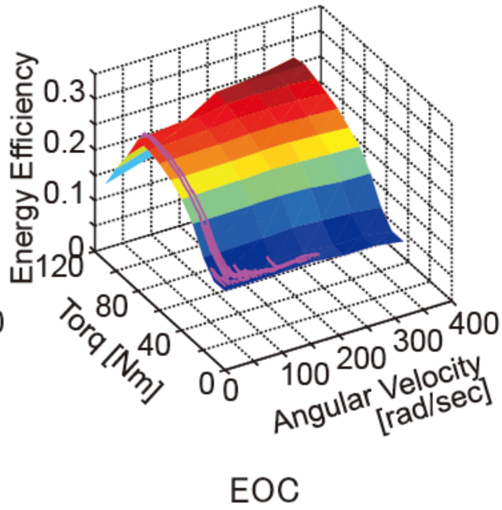


EOC

13

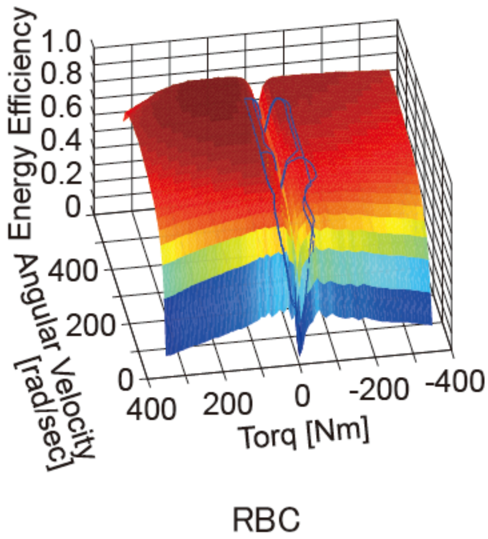
Energy Efficiency of Engine

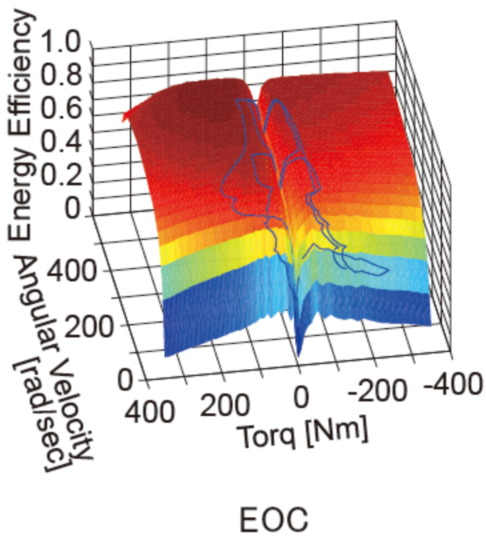




14

Energy Efficiency of Motor





15

Conclusions

- The energy optimal control (EOC) has been applied to the energy flow control of hybrid electric vehicle (HEV).
- Since the differential equation did not have to be solved in real time in EOC, its control law is very simple and easy to implement to onboard computers.
- In the 10-mode driving simulation, the emergent control pattern in that the engine torque increase during idling and the regenerative energy by a motor is positively accumulated has been appeared. As a result, the fuel cost of EOC is 15% lower than the rule-based control (RBC).

Effects of Dead Time of Acceleration Feedback

Effects of Dead Time of Acc. Feedback

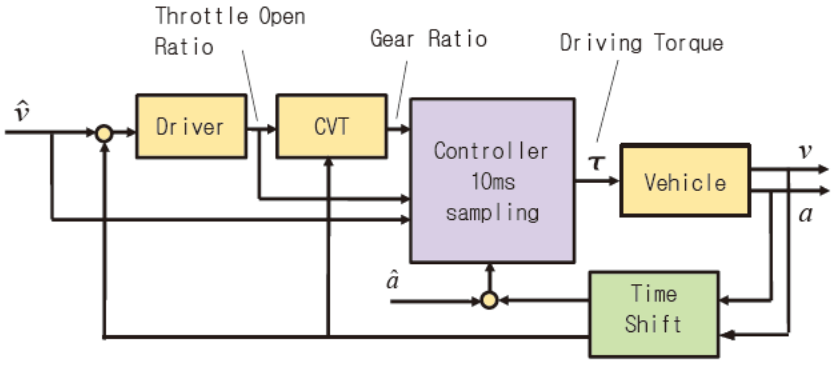
- In a real control system, acceleration feedback without delay and dead time is not feasible.
- In order to determine the implementation condition under which acceleration feedback works well, the range of the dead time when the control operates stably was decided by simulation.

$$\tau_E = \frac{\kappa\left(2C_{ES} - \dot{J}_{ES}\right)\omega_E + r_1(\widehat{\omega}_M - \omega_M) + r_2\left(\widehat{\dot{\omega}}_M - \dot{\omega}_M\right) + r_3\dot{\omega}_M - \kappa\lambda_{ME}\tau_{ME}}{\kappa + r - \frac{r_4}{f_E}\left(1 - \frac{\omega_E}{f_E}\frac{\partial f_E}{\partial \omega_E}\right)} \quad (28)$$

18

Dead Time Simulation

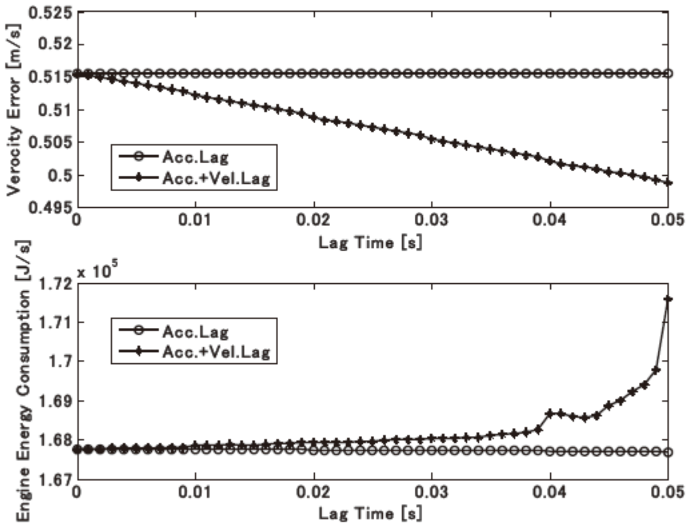
- The sampling time of the simulation was set to 1ms and that of the control loop was set to 10ms, and the simulation was carried out to increase the dead time of acceleration feedback by each 1ms.



Simulation: 1ms sampling

19

Results of Dead Time Simulation

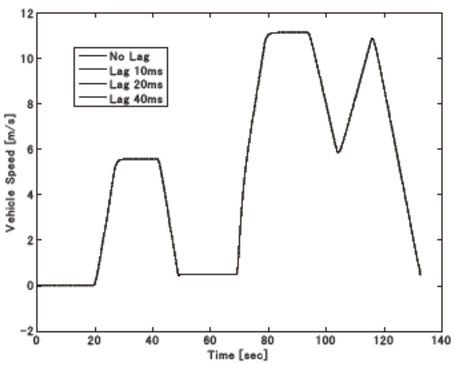


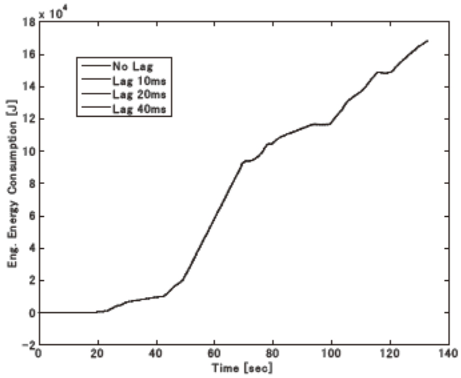
Results of Dead Time Simulation

Acceleration feedback with dead time

Vehicle speed

Engine energy consumption



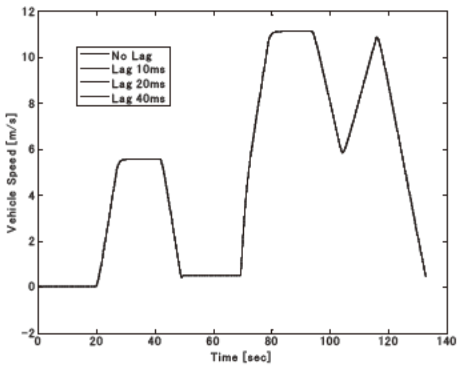


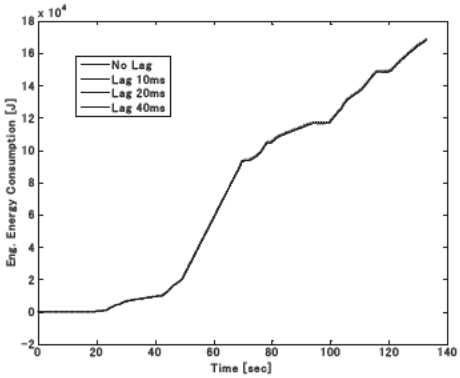
Results of Dead Time Simulation

Acceleration and velocity feedback with dead time

Vehicle speed

Engine energy consumption





22

Effects of Dead Time of Acc. Feedback

- To consider the reason why the effect of dead time of the acceleration is small, the control law of the engine shown in Eq. (28) are transformed into the following form.

$$\tau_E = \frac{\kappa\left(2C_{ES} - j_{ES}\right)\omega_E + r_1(\widehat{\omega}_M - \omega_M) + r_2\widehat{\omega}_M - (r_2 - r_3)\dot{\omega}_M}{\kappa + r - \frac{r_4}{f_E}\left(1 - \frac{\omega_E}{f_E}\frac{\partial f_E}{\partial \omega_E}\right)} \qquad (29)$$

- Looking at the 4th term of the numerator, the acceleration feedback term is multiplied by the difference of r_2 and r_3.

23

Effects of Dead Time of Acc. Feedback

- Therefore, if the values of r_2 and r_3 are close, the acceleration feedback gain decreases, and a large influence does not appear even if dead time is given.
- In addition, since r_2 is independently multiplied to the acceleration feedforward term (3^{rd} term of numerator), and even if the values of r_2 and r_3 are close, the gain does not decrease.

24

Optimality of Control Law

25

Uniqueness of Control Law

- Since the Euler-Poisson equation is a stationary condition of the evaluation function, the control law satisfies the necessary condition of stationary.
- When this is incorporated into a closed loop, the simultaneous solution of the vehicle dynamics and control law is uniquely determined by Pontryagin's theorem that the solution is unique if the control law and its partial derivative are continuous.
- Since it is obvious that the partial derivatives of τ_E and τ_M related to ω_E and ω_M are continuous, τ_E and τ_M will be uniquely determined.

26

Optimality of Control Law

- The sufficient condition that the solution of the Euler-Poisson equation gives the minimal value of the evaluation function is that the 2nd variational of the evaluation function is positive.
- That is, the 2nd variational of the evaluation function J by the function $q(t)$

$$\begin{aligned}\frac{d^2J}{d\varepsilon^2} &= \frac{d^2}{d\varepsilon^2}\int_a^b L(t,q,\dot{q},\ddot{q})dt \\ &= \int_a^b \left(\frac{\partial^2 L}{\partial q^2}\eta(t)^2 + \frac{\partial^2 L}{\partial \dot{q}^2}\dot{\eta}(t)^2 + \frac{\partial^2 L}{\partial \ddot{q}^2}\ddot{\eta}(t)^2\right)dt\end{aligned} \quad (30)$$

must be positive.

27

Optimality of Control Law

- Therefore, for any error function $\eta(t)$, at least one of

$$\frac{\partial^2 L}{\partial q^2}, \quad \frac{\partial^2 L}{\partial \dot{q}^2}, \quad \frac{\partial^2 L}{\partial \ddot{q}^2}$$

must be positive and the rest must be non-negative.

Optimality of Control Law

- In the control law of engine and motor, since the equivalent of q(t) is ω_E and ω_M , the coefficients in the previous page become these equations.

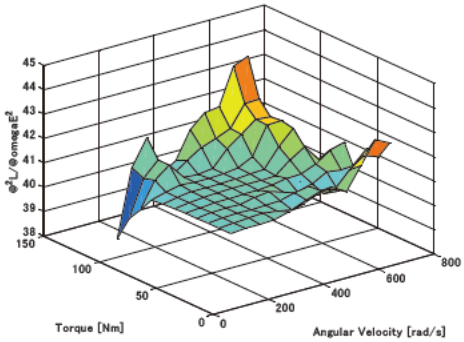
$$\frac{\partial^2 L}{\partial {\omega_E}^2} = 2\kappa C_{ES} + r_4 \left(2\tau_E \frac{\partial \varphi_E}{\partial \omega_E} + \tau_E \omega_E \frac{\partial^2 \varphi_E}{\partial {\omega_E}^2} \right) \tag{31}$$

$$\frac{\partial^2 L}{\partial {\omega_M}^2} = 2\kappa C_M + r_5 \left(2\tau_M \frac{\partial \varphi_M}{\partial \omega_M} + \tau_M \omega_M \frac{\partial^2 \varphi_M}{\partial {\omega_M}^2} \right) \tag{32}$$

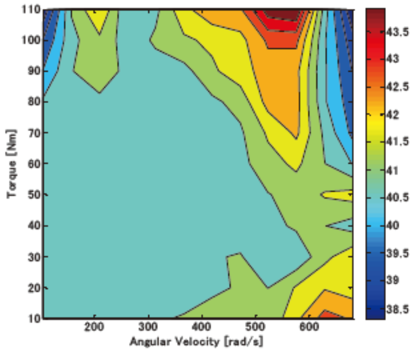
$$\frac{\partial^2 L}{\partial {\dot{\omega}_E}^2} = 0 \qquad \frac{\partial^2 L}{\partial {\dot{\omega}_M}^2} = 0 \tag{33}$$

2nd-order partial derivative by ω_E

3D View

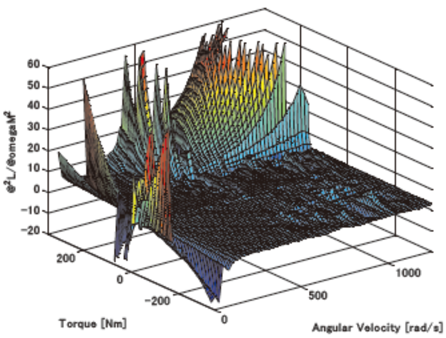


Contour Map

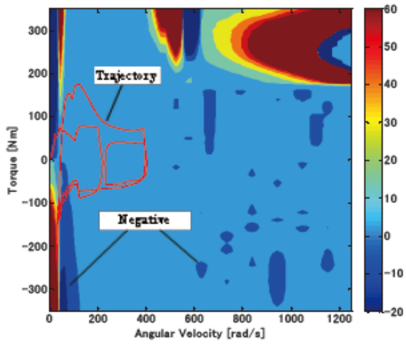


2nd-order partial derivative by ω_M

3D View



Contour Map



Conclusion

- The stability of energy optimal control (EOC) when hybrid electric vehicle performs autonomous driving was verified by simulation.
- As for the dead time of acceleration feedback and velocity feedback, there was almost no control error even if the dead time was given up to 40ms (equivalent to 4 cycles of control loop).
- As for the convergence of the control, it was shown that the 2nd variational of the evaluation function for the engine control law is positive in the entire operating range of torque-angular velocity, and the control law is the optimal solution. In the motor control law, it was shown that most of the motion trajectory exist in the positive region, and the control law is almost optimal.

32

Research of the Optimization Methodology for Advanced Powertrain Control 2

Masato Hayasaka

Hideo Asai, Junichi Kako
Toyota Motor Corporation

TOYOTA

Contents

2

1 Introduction

2 Issue with Previous Research

3 New optimization process

4 Validation

5 Conclusion

Contents

1 Introduction

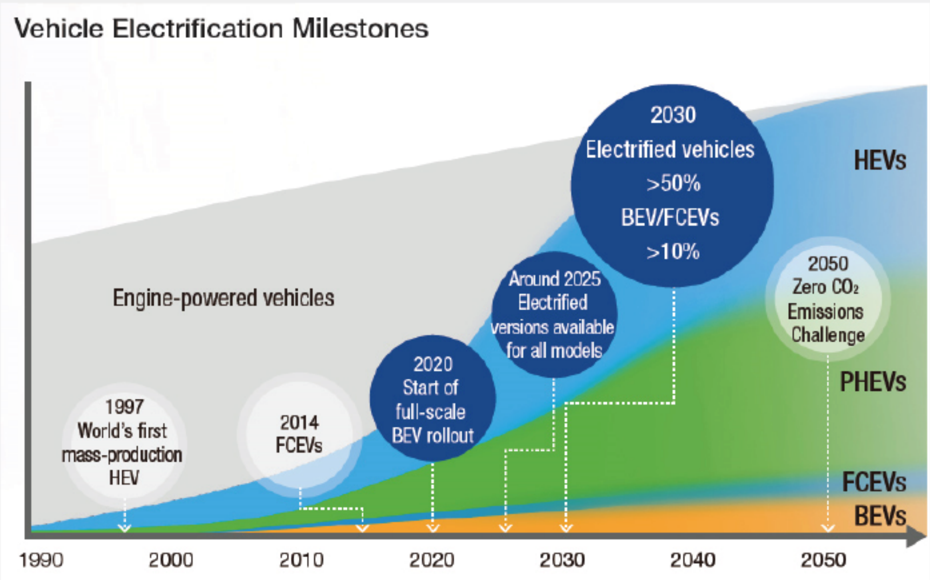
2 Issue with Previous Research

3 New optimization process

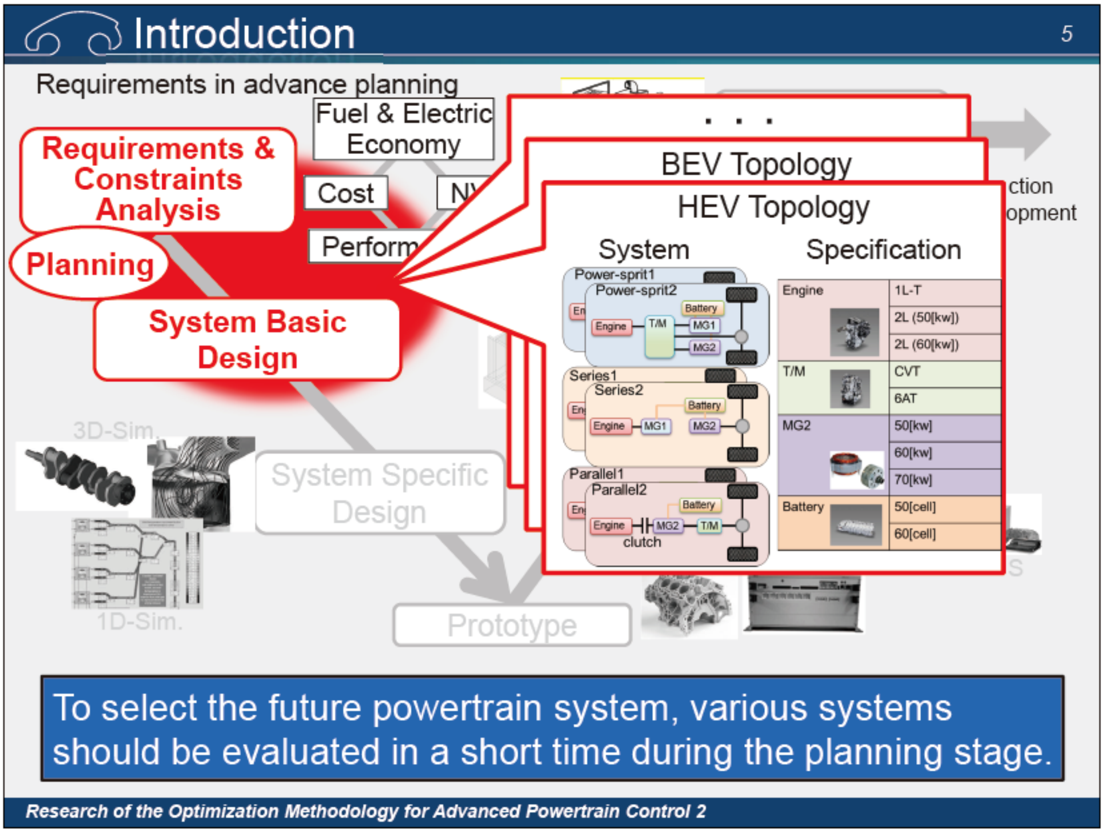
4 Validation

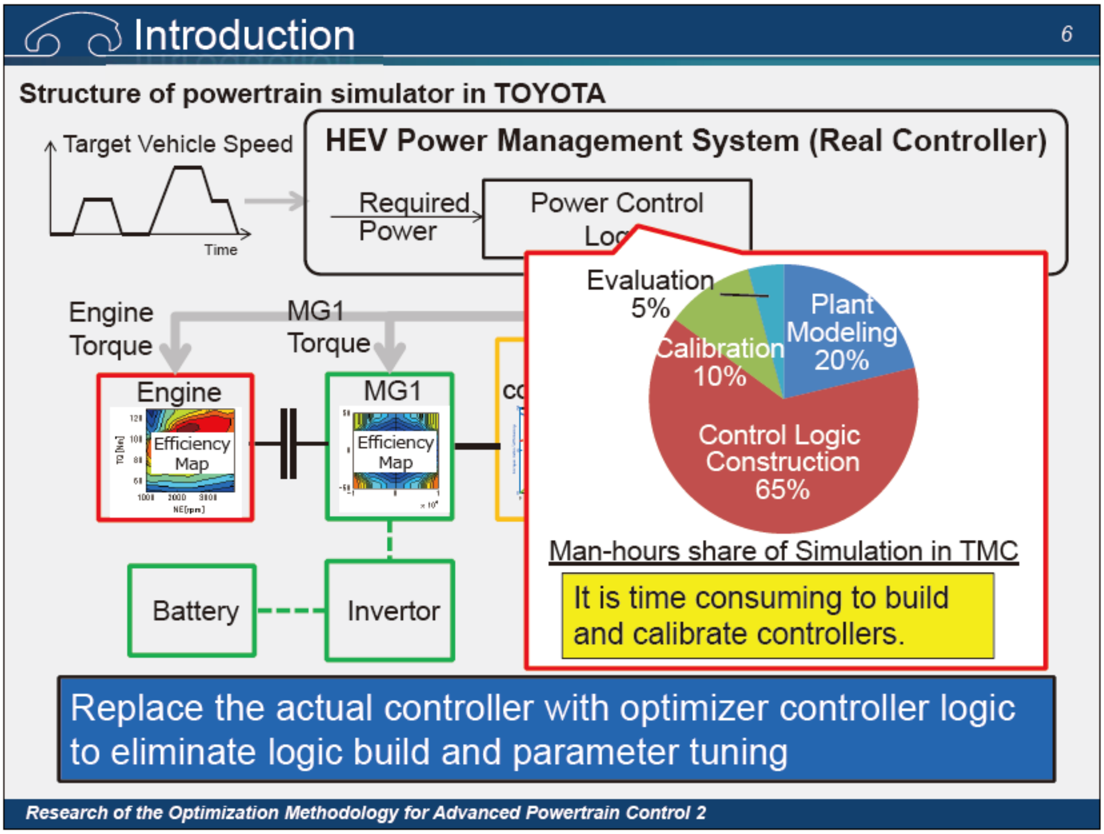
5 Conclusion

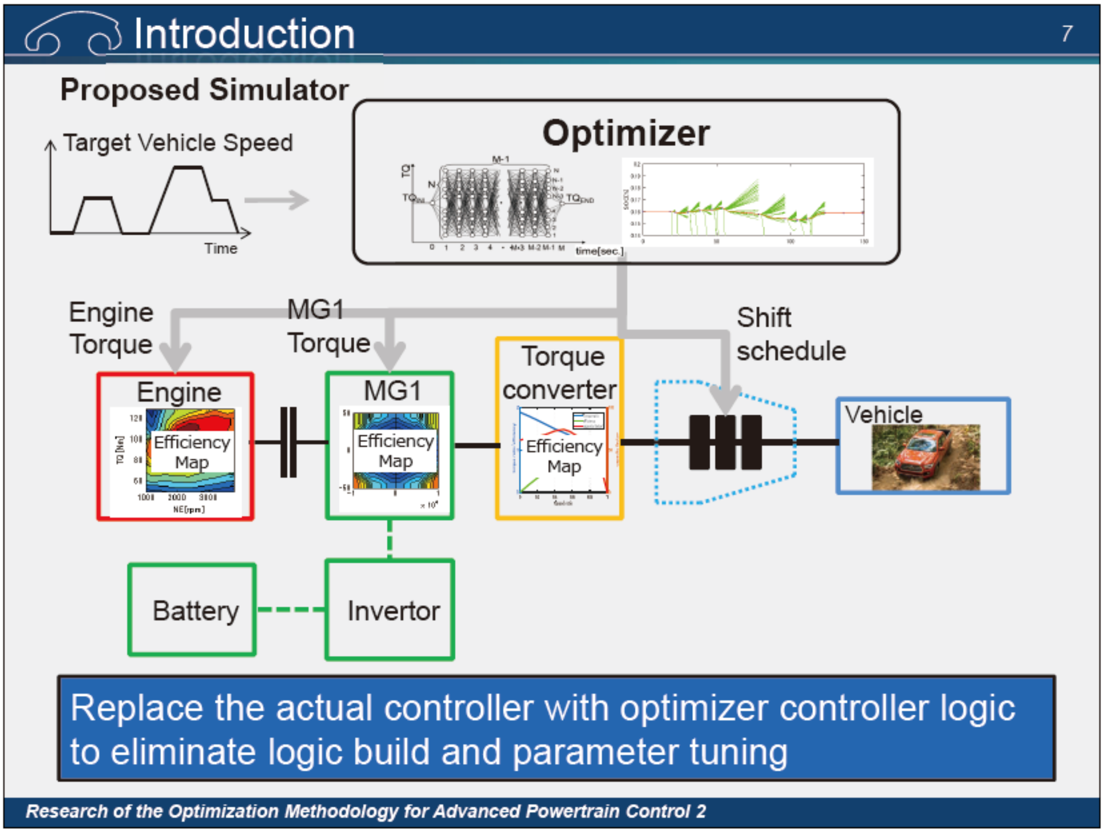
Introduction



The needs for complex and various electric vehicles are rapidly expanding.


Introduction
5
Requirements in advance planning
Requirements & Constraints Analysis
Planning
System Basic Design
Fuel & Electric Economy
Cost
Perform
NV
. . .
BEV Topology
HEV Topology
System
Specification
Power-sprit1
Power-sprit2
Engine
T/M
MG1
MG2
Battery
Series1
Series2
Parallel1
Parallel2
clutch
1L-T
2L (50[kw])
2L (60[kw])
CVT
6AT
50[kw]
60[kw]
70[kw]
50[cell]
60[cell]
3D-Sim.
1D-Sim.
System Specific Design
Prototype
ction
opment
To select the future powertrain system, various systems should be evaluated in a short time during the planning stage.
Research of the Optimization Methodology for Advanced Powertrain Control 2


Introduction
6
Structure of powertrain simulator in TOYOTA
Target Vehicle Speed
Time
HEV Power Management System (Real Controller)
Required Power
Power Control Log
Engine Torque
MG1 Torque
Engine
Efficiency Map
MG1
Efficiency Map
Battery
Invertor
Evaluation 5%
Plant Modeling 20%
Calibration 10%
Control Logic Construction 65%
Man-hours share of Simulation in TMC
It is time consuming to build and calibrate controllers.
Replace the actual controller with optimizer controller logic to eliminate logic build and parameter tuning
Research of the Optimization Methodology for Advanced Powertrain Control 2



Contents

1 Introduction

2 Issue with Previous Research

3 New optimization process

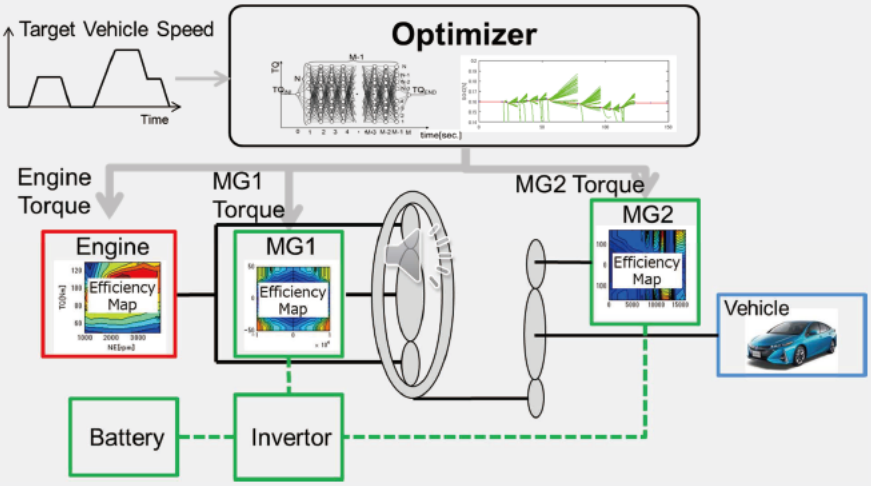
4 Validation

5 Conclusion

Issue with previous methods

Existing Optimization Methodology for Power-Split HEV

IFAC E-CosM(2018) p207-212
Research of the Optimization Methodology for the Advanced Powertrain Control

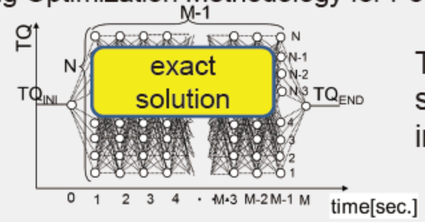


Reduce computational cost by discretization.
Realized acceptable calculation time.(about 30[min.]@LA#4)

Issue with previous methods

Existing Optimization Methodology for Power-Split HEV

IFAC E-CosM(2018) p207-212
Research of the Optimization Methodology for the Advanced Powertrain Control



The time consuming exact solution was achieved in the following way.

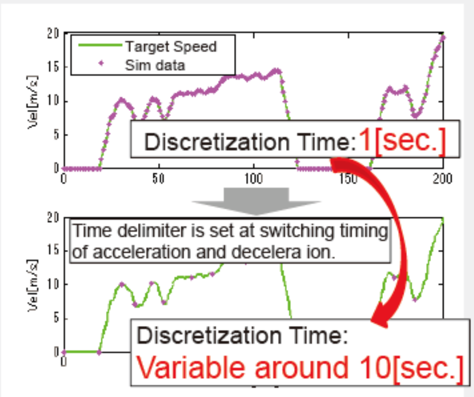
Engine and motor torques candidate number
Around 1500（Torque）

1500 modes

Table. Operation modes

*	Vehicle Operation Mode	Component Operation Mode	Constraints: Vehicle Speed	Engine Torque	Engine Speed	MG1 Speed	MG2 Speed
1	STOP	All Stop		- (=0)	- (=0)	=0	=0
2	BEV	MG1 or MG2				F	= 0
3						= 0	F
...	...	...			...	...	...
25	HEV	Engine + MG1 + MG2			F	F	= const. (5 levels)
26					F	F	
27					F	F	
28					F	F	
					F	F	

28 modes



Reduce computational cost by discretization.
Realized acceptable calculation time.(about 30[min.]@LA#4)

Issue with previous methods

Issue with existing methods at parallel HEV

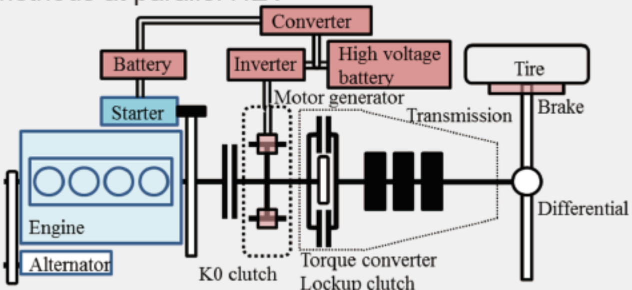
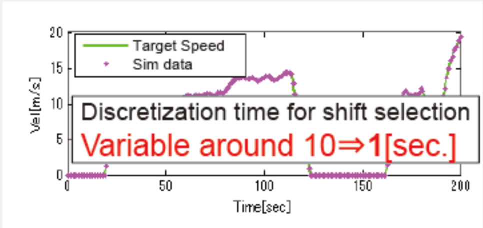


Fig. Target system

Table. Operation modes

Vehicle Operation Mode	Component Operation Mode	Constraints			
		Vehicle Speed	Engine Torque	MG Torque	Shift Select
STOP	All Stop		-	-	N
			β : (0.1∼1.2) (12levels)	F	

Power-Split:28 ⇒ **Parallel:320** modes



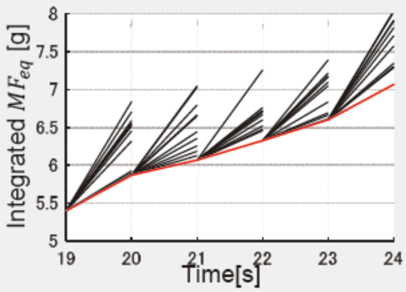
With overall optimization, the calculation time will be about 120 times longer(about 60[hr.]). Need to reduce computational costs.

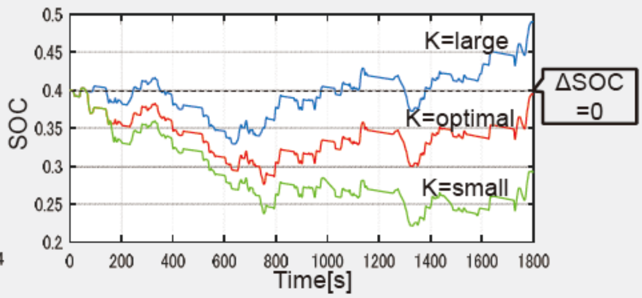
Instantaneous optimization method

ECMS (Equivalent fuel Consumption Minimization Strategy).

$$MF_{eq} = MF - K \times P_{ele}$$

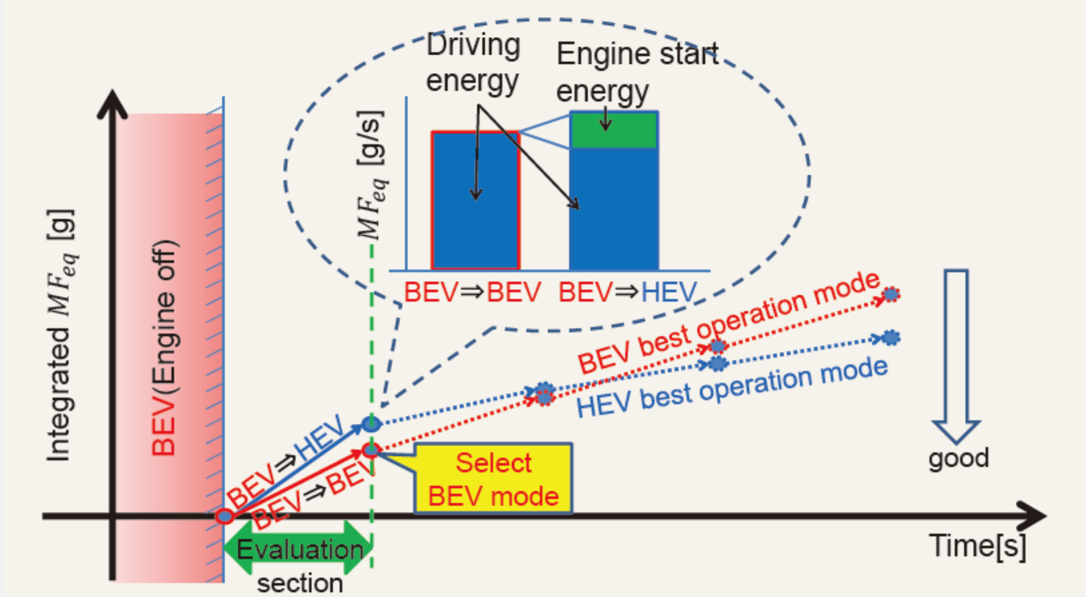
- MF_{eq}: Equivalent fuel consumption
- MF: Fuel consumption
- P_{ele}: Electrical energy
- K :Conversion ratio from electric energy to fuel heat energy





Sequentially select the mode that minimizes MF_{eq} for each time step.

Influence by evaluation section length



Mode transition including engine start will not be selected.
Therefore, we propose a new method to obtain more accurate engine start / stop timing.

Contents

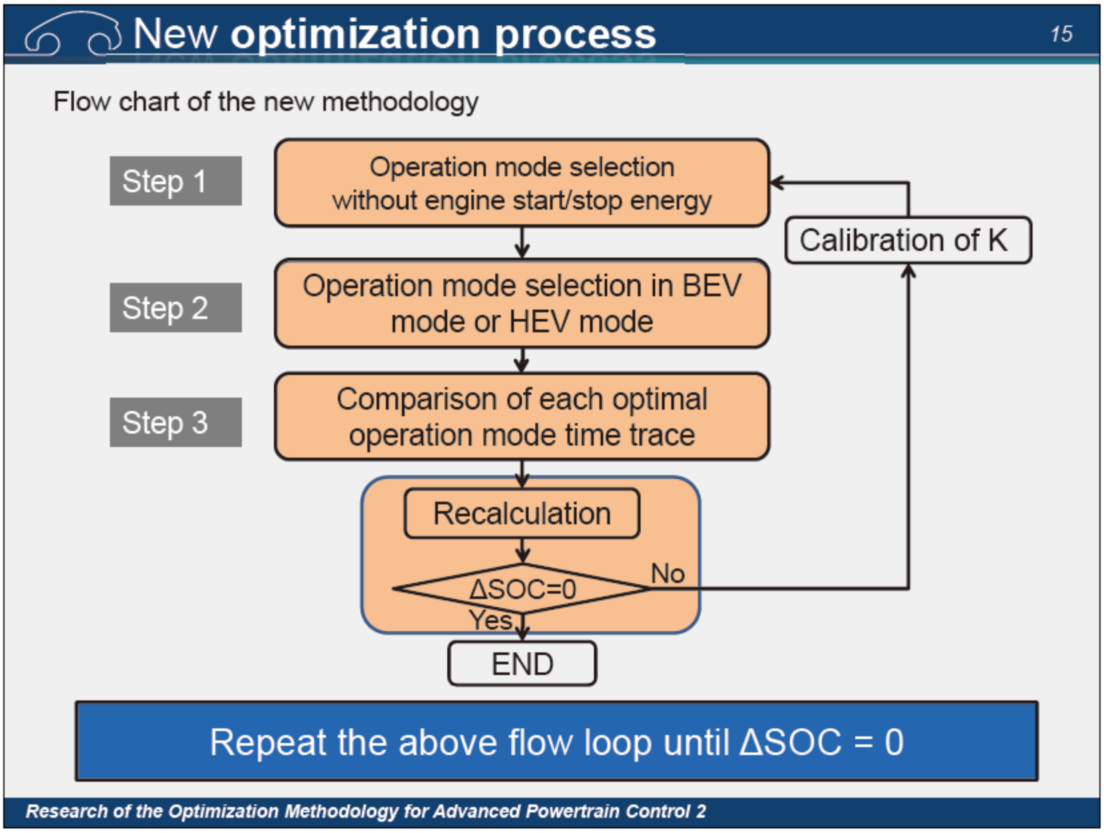
1 Introduction

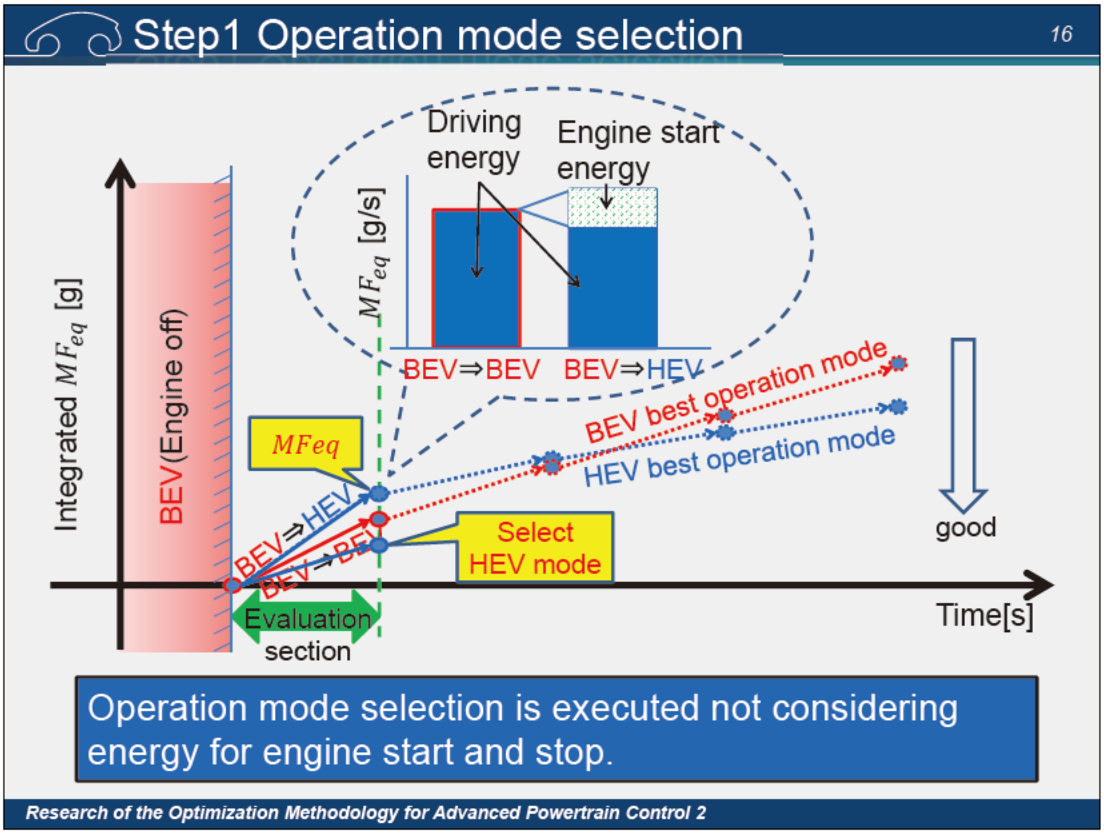
2 Issue with Previous Research

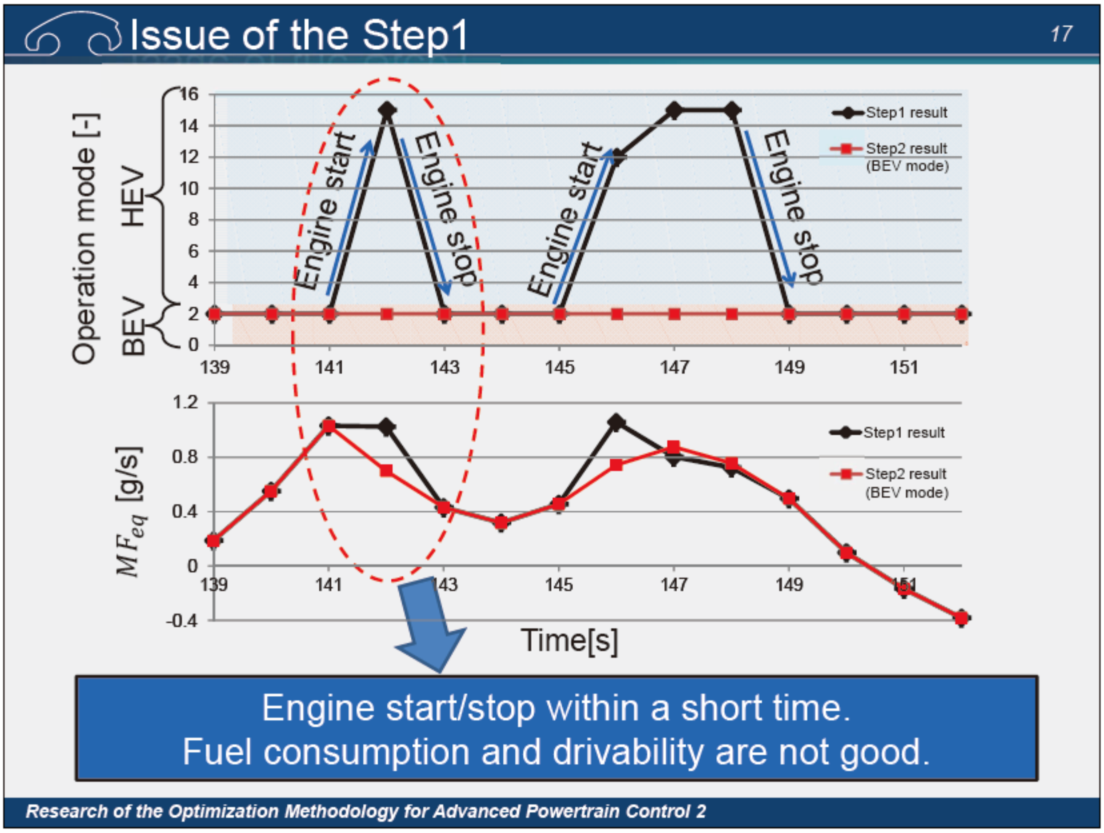
3 New optimization process

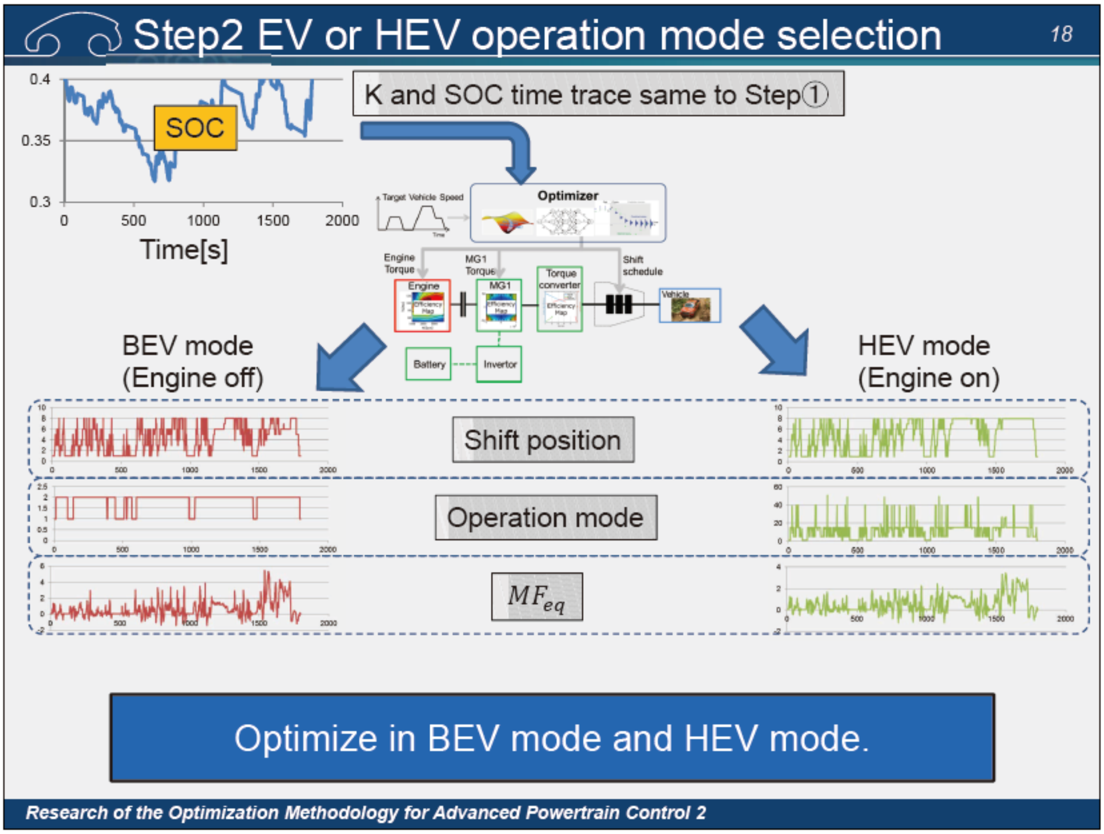
4 Validation

5 Conclusion

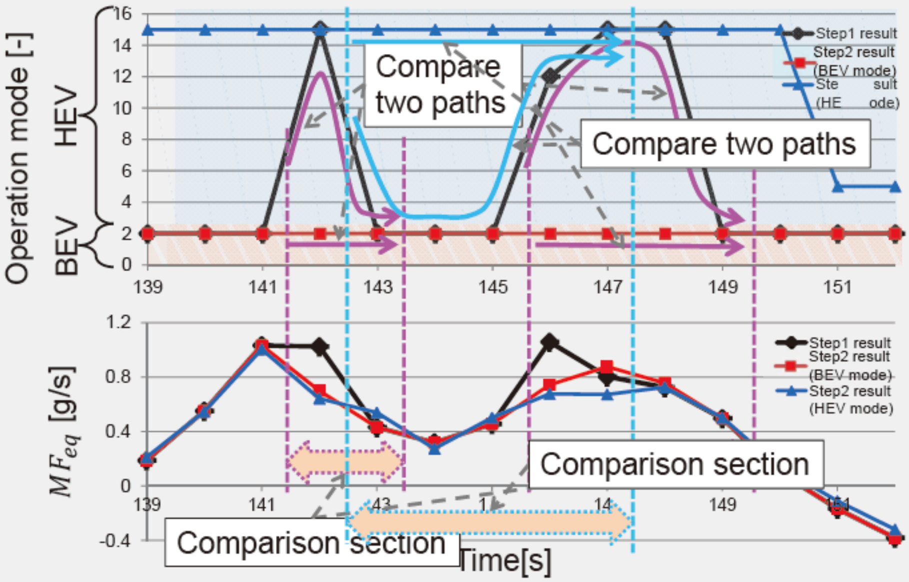

New optimization process
15
Flow chart of the new methodology
Step 1
Operation mode selection without engine start/stop energy
Step 2
Operation mode selection in BEV mode or HEV mode
Step 3
Comparison of each optimal operation mode time trace
Recalculation
ΔSOC=0
No
Yes
END
Calibration of K
Repeat the above flow loop until ΔSOC = 0
Research of the Optimization Methodology for Advanced Powertrain Control 2


Step1 Operation mode selection
16
Driving energy
Engine start energy
MF_{eq} [g/s]
BEV⇒BEV
BEV⇒HEV
Integrated MF_{eq} [g]
BEV(Engine off)
$MFeq$
BEV⇒HEV
BEV⇒BEV
BEV best operation mode
HEV best operation mode
Select HEV mode
good
Evaluation section
Time[s]
Operation mode selection is executed not considering energy for engine start and stop.
Research of the Optimization Methodology for Advanced Powertrain Control 2


Issue of the Step1
17
Operation mode [-]
HEV
BEV
Engine start
Engine stop
Engine start
Engine stop
Step1 result
Step2 result
(BEV mode)
MF_{eq} [g/s]
Step1 result
Step2 result
(BEV mode)
Time[s]
Engine start/stop within a short time.
Fuel consumption and drivability are not good.
Research of the Optimization Methodology for Advanced Powertrain Control 2

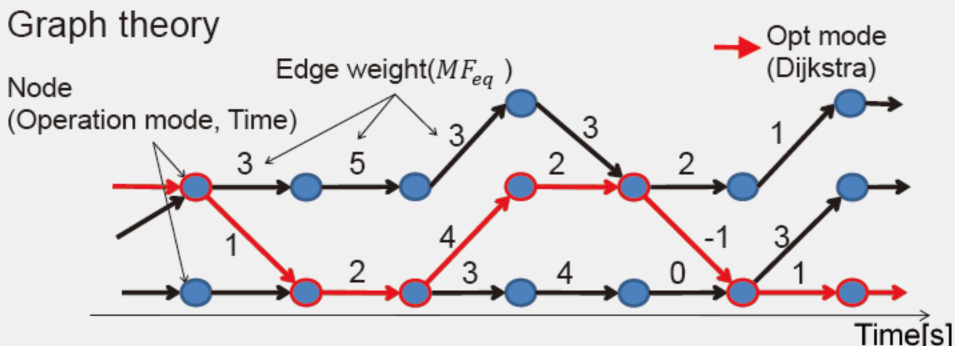

Step2 EV or HEV operation mode selection
18
SOC
Time[s]
K and SOC time trace same to Step①
Optimizer
BEV mode
(Engine off)
HEV mode
(Engine on)
Shift position
Operation mode
MF_{eq}
Optimize in BEV mode and HEV mode.
Research of the Optimization Methodology for Advanced Powertrain Control 2

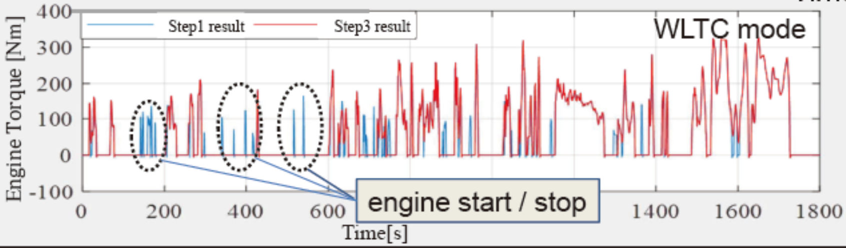
Step3 Comparison of each optimal operation mode path



The three operation mode time traces obtained in first and second optimization are compared.

Step③ Comparison of each optimal operation mode path





Reduced engine start / stop in short cycles.

Contents

1 Introduction

2 Issue with Previous Research

3 New Methodology

4 Validation

5 Conclusion

Validation

There are 2 cases to validate considering how to use.

Case 1

To improve real world fuel consumption with resent technology, for example ADAS.

->Target vehicle speed is pre-known

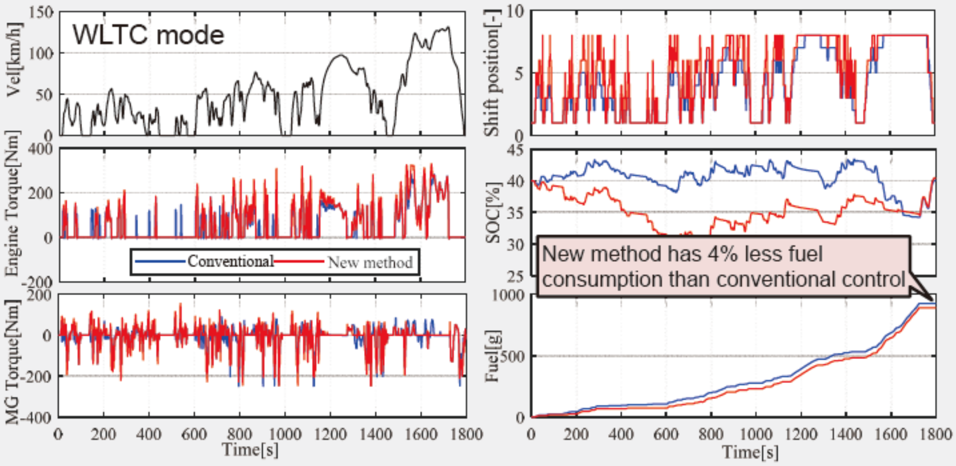
Case 2

To improve existing rule based controller with considering driver's acceleration demand.

->Target vehicle speed might be changed by driver

Validation (Case1)

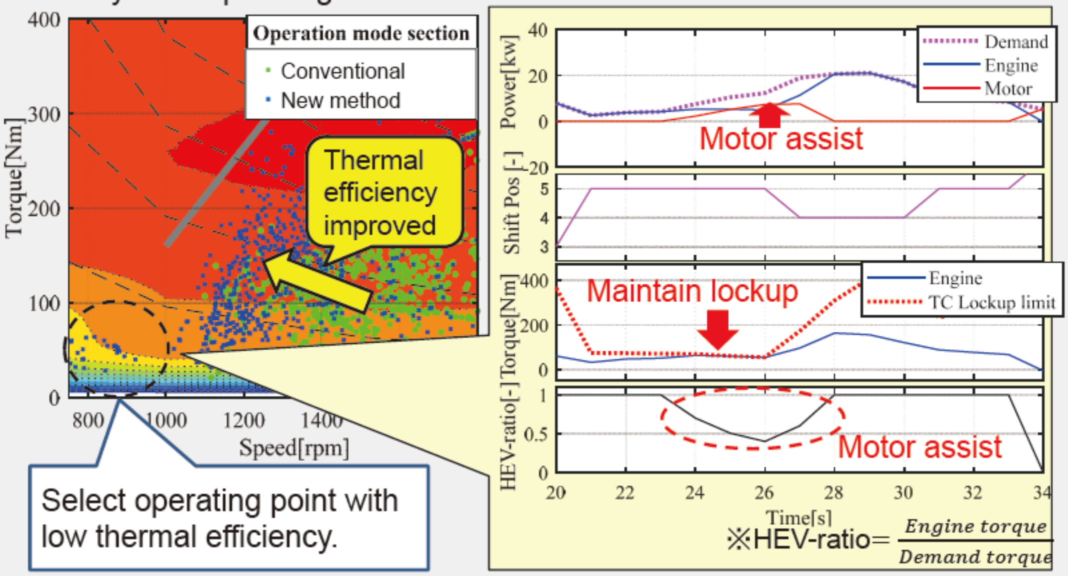
Fuel efficiency comparison of the optimization results with the results by the conventional control.



Fuel consumption was reduced by 4% compared to conventional control.
Acceptable calculation time(about 37 [min.] @WLTC) was achieved.

Validation (Case1)

Analysis of operating condition

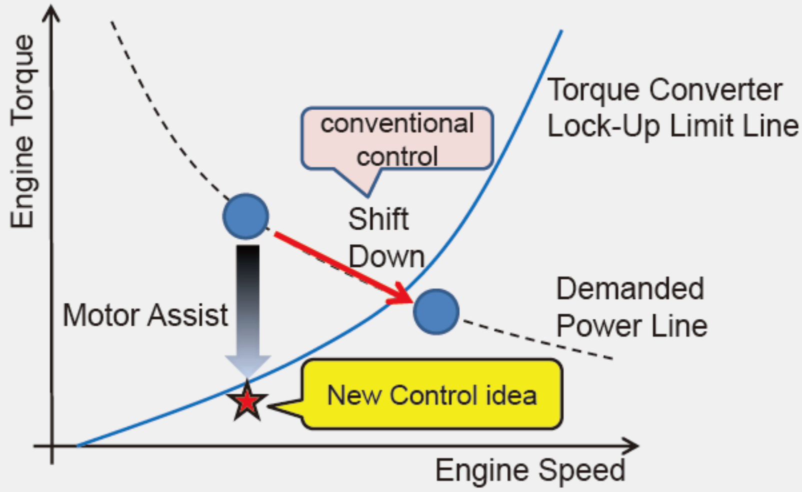


Maintain lockup without increasing engine torque.
Motor assists for the lack of torque.

Validation (Case1)

25

New control idea

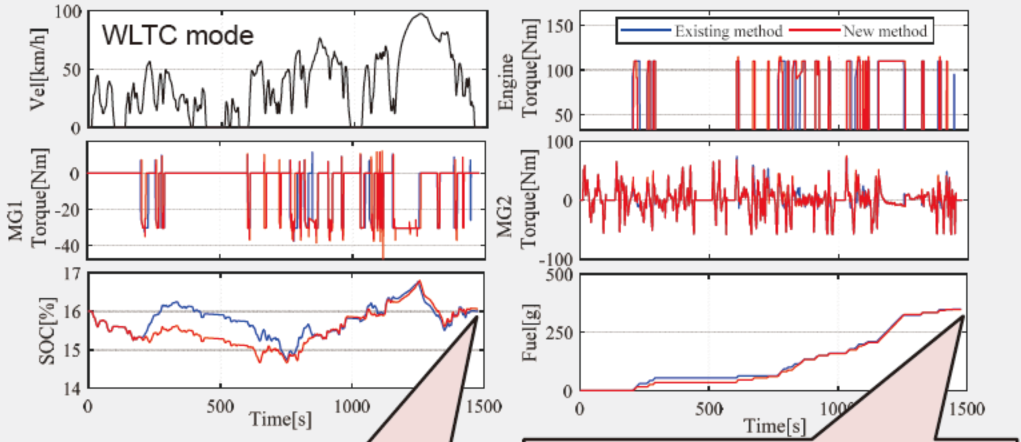


Optimized operation is different from conventional control concept.

Validation (Case1)

26

Fuel efficiency comparison of the optimization results when applied to Power-Split HEV with the results by existing method.



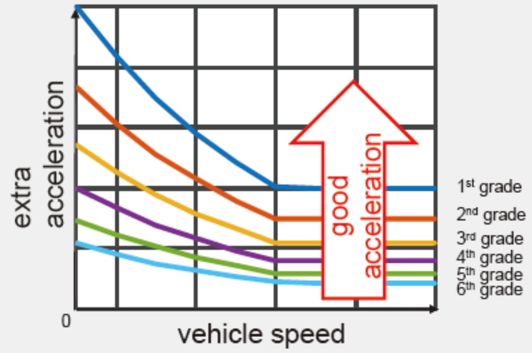
1. New method increases ΔSOC = 0.06%

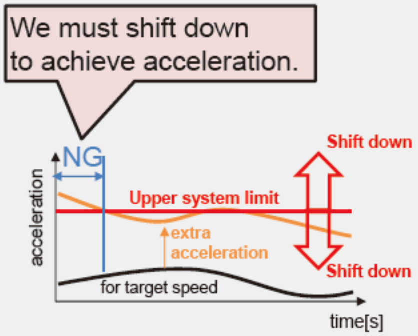
2. New method has 0.46% less fuel consumption than existing method.

Equivalent fuel consumption was reduced by 1% compared to existing method.

Constrain for acceleration grade

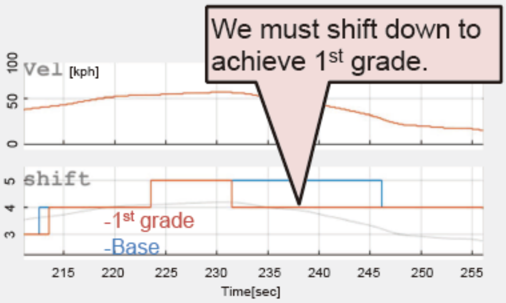
It is necessary to achieve acceleration which driver need.
Extra acceleration is additional torque for future driver's demand.
There are several grades for each vehicle character.

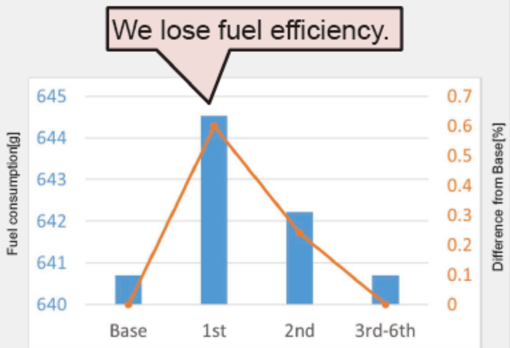




To achieve driver's acceleration demand, we must change shift position, and find best operating point.

Result with considering acceleration grade





If we want better acceleration grade, we should select lower shift position, and we lose fuel efficiency.

Conclusion

Optimization methodology for power management of electrified power train systems have been developed. It can obtain optimal input trace in acceptable time.

- The proposed method achieves better results than well-tuned conventional controllers.

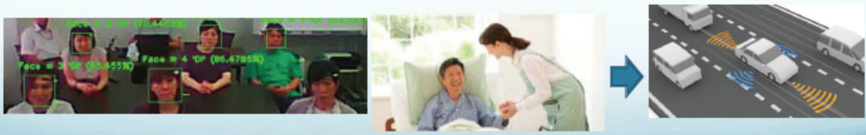
- By analyzing the input trace, new control idea was obtained.

- Even with different systems, we could derive the potential performance of fuel consumption.

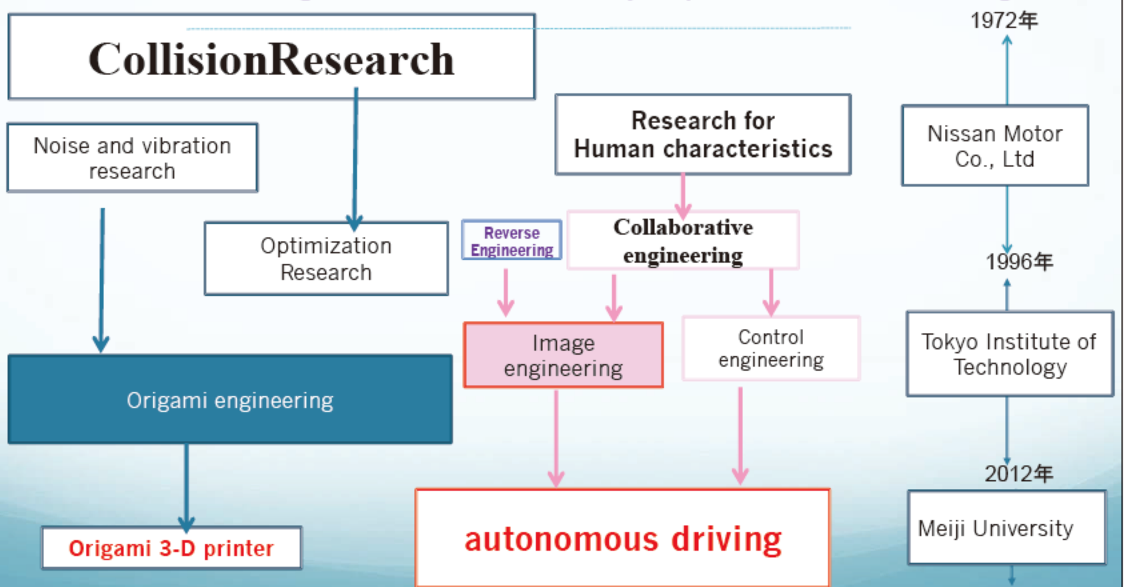
Expectations for mathematical science researchers in the field of autonomous driving technology

Distinguished Professor Emeritus
Ichiro Hagiwara

Advanced Study of Mathematical Sciences &
Autonomous Driving Social Research Institute
Meiji University



Self-introduction/working field is extensive, necessary to promote autonomous driving.



0	*All operation by driver* **Normal passenger car**
1	***Steering operation or Acceleration & deceleration is supported by system／【Driving support】***
2	***Steering operation and Acceleration & deceleration are supported by system／【Driving support】***
3	***full automatic operation in a specific location, in an emergency operated by driver ／【automatic operation 】***
4	***full automatic operation in a specific location ／【automatic operation 】***
5	***full automatic operation without location limitations ／【full***

Level 3

出典:OLVO web記事より

History of *Establishment of Meiji Institute for* Autonomous Driving (MIAD)

- 2018年2月20日　企画会議*Planning meetings*、懇親会*Get-together*
- 2018年3月28日　設立記者会見*Press conference for Establishment*
- 2018年4月28日　開所式*Opening Ceremony*

Content

1 Self-driving class

2 Introduction of Meiji University's Institute of Autonomous Driving(MIAD)

3 Mathematical Sciences for regional revitalization

4 High speed and high accuracy image processing and machine learning technologies

5 Causal neural network is efficient for cooperative control between system and driver in level 3, between system and remote operator in levels 4&5.

6 New topology optimization is effective for ride quality improvement and fear relief

7 Accident and near miss accident from drive recorder are led to diferential equation, modeling ,simulation and real time optimum control

8 Conclusion

Abstract(1/2)

Cars are now so popular that even from the general public, you can see what is being actively researched and developed now in the automobile industry.

That is exactly the self-driving. Recently level 3 car was released where the system is responsible for driving until the system becomes difficult to drive.

And at that time , the system entrusts driving to the driver. In this case the driver must keep getting more nervous than driving himself although it should be automatic driving for relaxation which gives the birth to the public opinion that level 3 is difficult in the first place.

To realize the level 3 that should be, the system must explain the driver what and how the driver should do which necessitates to be installed causal machine learning system on the self-driving car.

Abstract(2/2)

It is expected to achieve level 4 quickly for regional revitalization where the self-driving is limited location and time. For level 4, remote monitoring system is used in such way that it can be easily monitored multiple self-driving cars by one person because one of the biggest expectations for the self-driving is to carry out transportation with a small number of people.

It also necessitates the causal machine learning and high-speed, high-precision image processing technology for this remote monitoring system.
As far as level 5, there is a problem "Safe but not relief" which requests technologies to achieve a higher level of ride quality. Among these technologies, it can be listed up origami engineering for realizing vibration isolation in the frequency band that affects ride comfort and generalization of energy control method for real time optimal control method.

And also it is expected relief can be unraveled from deep mathematical point of view. The presentation will focus on the above.

Main activities Since the establishment of MIAD

❶ JSME Annual convention Special plan Workshop : Role of of Mathematical Sciences for Industry—Role of Mathematical Sciences for Self-driving : AIMaP Plan 11th September,2018
❷ TUM-UT German-Japan Workshop / Symposium: **Technische Universität München** AI-Accountability and Autonomous Car Driving/Invited speech: ***Autonomous Driving & AI*** *～ Take advantage of the human skills*
❸ Transdisciplinary Federation Science and Technology ConferenceOS **Mathematical Sciences of Automotive industry such as autonomous driving** : AIMaP Plan 9^{th} of October ,2020.
❹ JSME Annual convention Special plan Workshop : Current status and issues of Mathematical Sciences to realize highly automatic driving : AIMaP Plan: 7th September,2021
❺ **SICE-JSAE-AIMaP Advanced Automotive Control** : 8th September,2021
❻ JSST Autonomous driving special feature plan
❼ MIMS Joint research meeting/ Current status and issues of Mathematical Sciences to realize highly automatic driving Conducted once a year since 2018

Safety and security are the most important for self driving

.

Safety is the most important. But even we know it's safe, we don't feel secure. To ensure safety, self-position estimation and obstacle detection

To realize this, fusion of high-speed cameras and machine learning is effective.

To ensure a sence of secure, it is effective to move eigen frequencies within vibration frequency range that promotes fear outside the range and also it is effective to cut off the vibration within the frequency range.

To realize the former, topology optimization is effective and for the latter,origami structure is effective.

Safety and security are the most important for self driving

In level 3, system takes charge of driving operation. In the case of the difficulty for system to operate driving, it requests to intervene (RtI).

Can driver make decisions instantly and operate driving properly? To realize level 3, only then will it be possible that the driver is concentrated and the system understands why it is an emergency and explains the driver what should be done,

For this achievement, the machine learning must be not only high-speed and high accuracy but also be causal

Safety and security are the most important for self driving

In levels 4&5, remote operation management is mandatory to drive safely self driving car

To solve the shortage of drivers in rural areas , it is important that one remote operation manager can handle multiple cars aided by machine learning.

Here also the machine learning must be not only high-speed and high accuracy but also be causal like level 3.

As mentioned above, safety is of the utmost importance in the end. Therefore, we are proceeding with the following concept.

Concept of multiple barrier based on Swiss cheese model

Risk aversion responsibility sharing

Automotive hardware/software

Basic design barrier
- Redundant system

FMEA(Failure Mode and Effect Analysis)
FTA(Fault Tree Analysis)

Soft barrier
- AI Risk prediction avoidance

Social system

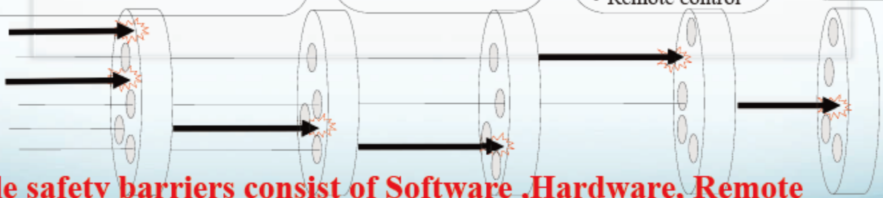
Social barrier
- Road infrastructure
- Traffic rules
- Laws and regulations、Insurance
- ODD limited range

Remote service

Remote monitoring
- AI
- Image confirmation
- Emergency stop
- Remote control

Body structure

Stiffness control
- Flexible crush absorbing structure



Multiple safety barriers consist of Software ,Hardware, Remote monitoring,Body structure and also Social system such as special traffic rule ,barriers.

Related properties and mathematics

1） Safety

Self-position identification
Obstacle detection

High-speed High precision
Image processing
Machine learning

2） Relief

Ride quality improvement
Fear relief

Real time optimal control
(Energy Optimal Control(EOC))
Vibration cut off,
Eigen frequencies movement

Prevention of motion sickness
Ride quality improvement

Vibration cutoff
Using Origami structure

Hagiwara,I.,Ishida,S.,Uchida,H., Patent application number2013-220548
,Patent publication number 2015-81655..

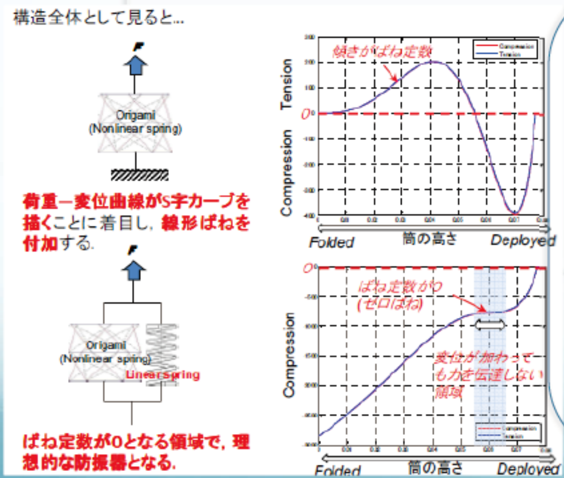
Sachiko Ishida, Hiroshi Uchida, Haruo Shimosaka and Ichiro Hagiwara,Design and Numerical Analysis of Vibration Isolators With Quasi-Zero-StiffnessCharacteristics Using Bistable Foldable Structures, J. Vib. Acoust 139(3),031015 (Apr 24, 2017) (8 pages)

解析モデル **Reversed Spiral Origami Structure(RSO)**

2 stable points exist at complete folded and complete deployed points.

Prevention of motion sickness
Ride quality improvement

Vibration cutoff
Using Origami structure



2 stable points exist at complete folded and complete deployed points. This brings minus stiffness part and so brings zero stiffness part with linear stiffness spring. In such way, we have vibration cut off region. This prevents motion sickness and improves ride quality.

出典:日産自動車(株)HP

level 5 passenger car

資料提供:JARI(日本自動車研究所)　内閣府SIP実証実験　福岡県みやま市出典 CityMobil 2 Project

truck platooning　small low speed bus

恐怖(fear)

Relieve fear

There are many demonstration experiments of autonomous driving car such as level 5 passenger car, truck platooning and small low speed bus. Among this, in level 5 car, at 80km / h, it must be safe, but not relieved and the passenger feels fear. How is safety different from relief? Maybe there is a frequency band that promotes fear.

Topology optimization is effective for controlling plural eigen frequencies

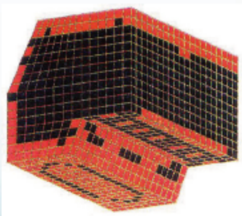
Homogenization [Bendsoe-Kikuchi:1[illegible]]

Density method [[illegible]-[illegible]:1[illegible]]

Energy density method [[illegible]-[illegible]:2021]

No.	Initial Values	Target Values	Result Values
1	6.2	15.0	15.0
2	8.2	20.0	19.1
3	11.7	25.0	25.0
4	14.3	30.0	29.7

Control eigenfrequencies



Optimal placement of frames

Initial values : Flame is placed all over on the Cabin panels

Target values: to control all eigen frequencies over 15 Hz

Optimization : Object function: Index of generalized eigenfrequencies

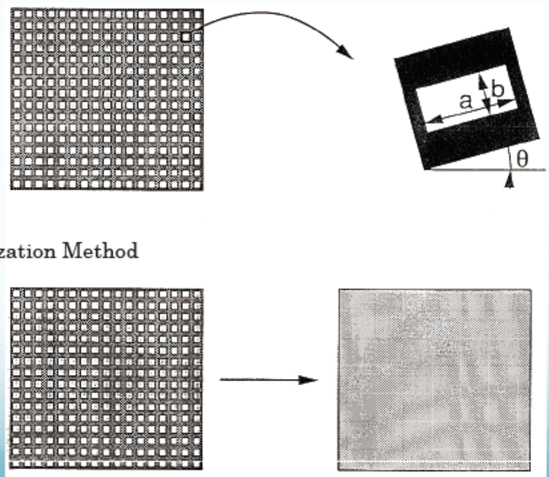
Constraint function:Half of weight

Result: Frame remains only where it is needed and result values

Optimization method using homogenization method (Bendsoe-Kikuchi 1988)

Optimization method

Design valuable
a, b, & θ

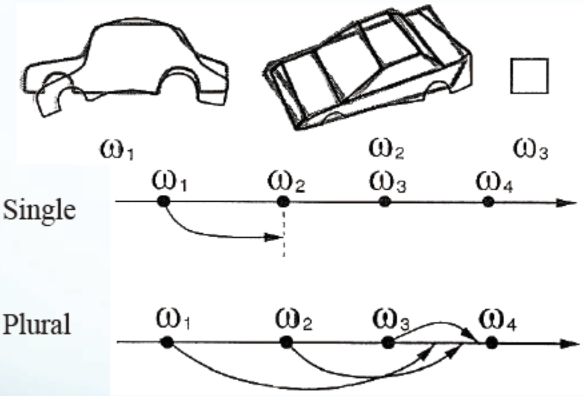


Homogenization Method

Homogeneous material

E^H, V^H

Simultaneous optimization for plural eigen frequencies



In topology optimization, the plural eigen frequencies change significantly, so that it is unstable. In this case, generalized eigen value index(weighted average of m eigenvalues) is effective.

$$\lambda^* = \begin{cases} \lambda_0^* + \left(\sum_{i=1}^{m} w_i (\lambda_{n_i} - \lambda_{0_i})^n \Big/ \sum_{i=1}^{m} w_i \right)^{\frac{1}{n}} & (n \neq 0 \text{ の整数}) \\ \lambda_0^* + \exp\left(\sum_{i=1}^{m} w_i \ln\left|\lambda_{n_i} - \lambda_{0_i}\right| \Big/ \sum_{i=1}^{m} w_i \right) & (n=0) \end{cases}$$

パラメーター： w_i , n , λ_{0_i} , λ_0^*

It is difficult to move only one and even the slightest frequency without topology optimization

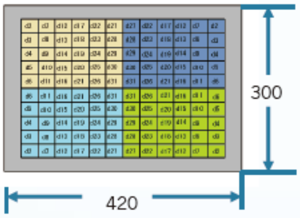
Weighted average of m eigenvalues”

History of topology optimization method

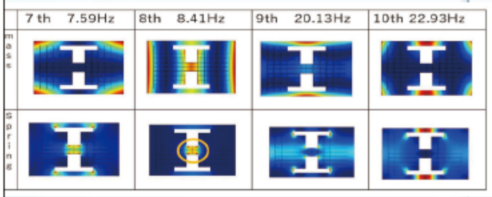
Year	Researchers	Contents
1988	Bendso-Kikuchi:*Comput. Meths Appl. Mech. Engrg.*, **71**, (1988).	!st development for static problem See through as difficult for vibration problem with used optimization method
1993	**Tenek- Hagiwara** :Comput. Methodsin Appl. Mech. Engrg. , Vol.109 (1993-10).	**application to vibration problem**
1993	Ma—Kikuchi—Hagiwara: Transactions of the JSME, Vol 59,No 562(1993	**Development of generalized eigenvalues to control plural eigen frequencies**
1994	**Tenek- Hagiwara** :Comput Methods in Appl Mech Engrg Vol 115(1994)Nos 1&2,	**development of density method**
1995	Ma—Kikuchi—Hagiwara: Transactions of the JSME, Vol 60,No 577 (1994-),	**Development of the most general optimization method**
2021	Sasaki—Hagiwara:Submitted to Transactions of the JSME	Eliminates the drawbacks of traditional topology optimization by 1stly using strain and kinetic eigenmodes

Simultaneous optimization for plural eigen frequencies

New Technology !(2021)



Eigen frequencies	7th	8th	9th	10th
Initial values	8.24Hz	9.37Hz	19.11Hz	19.29Hz
Target values	6.0Hz	6.5Hz	21.5Hz	22.0Hz
Result values	6.74Hz	7.62Hz	20.64Hz	22.29Hz
Design values with holes	6.81Hz	7.08Hz	17.87Hz	19.23Hz



Most previous studies ended with Result values. Actual design is to make a hole below a certain threshold of plate thickness distribution which gives not the objected one. Return to the origin of vibration such that to lower the eigen frequency, a hole is made in the spring part of the mode and to raised the one, a hole is made in the mass part of the mode. With this, the previous drawbacks are eliminated.

Remote operation monitoring・ management system concept for autonomous vehicle at low cost

A remote monitoring / support function that can respond to abnormal situations such as unusual behavior and outside ODD etc. is required to realize level 4 and 5 autonomous vehicles

One observer monitors and manages multiple autonomous vehicles aided by AI. This is our object.

Use data from street cameras, etc

Sends an abnormal signal in the event of outside ODD and an abnormality。 At the same time, the supportfunction is activated

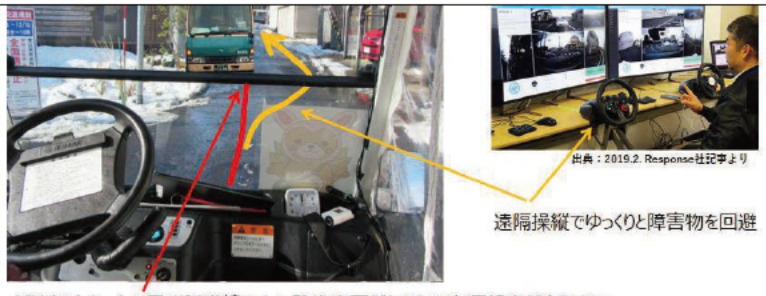
Remote monitoring・ control center

Configure remote monitor support function with less burden for remote observer by using AI etc. from camera images of various places

Get images from cameras in 4 directions

AI processing reduces the burden on the

Realization of operation management function of autonomous vehicle at low cost

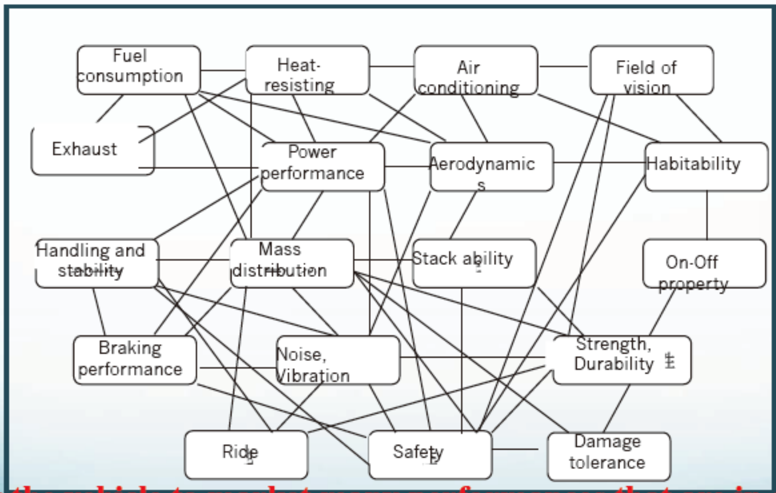


低速無人カートの電磁誘導線の上に駐車車両があるため自運転車が走れない。

It is important to determine the types of obstacles and whether they are moving away or approaching. To achieve that, high-speed, high-precision image processing technology and as far as machine learning, real-tyme, incremental learning and causality are necessary.

Let's see how machine learning has been used in automotive industry.

Relevance of performance



To bring the vehicle to market,many performances that are in a trade-off relationship with each other must be realized simultaneously.

Revolution of developing automobile

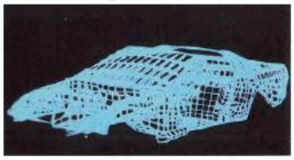
Clay model

Data model

Test car

Precise model

Experimenta l equipment



Computer simulation

Most of performances can be developed with CAD/CAM/CAE.Developing period：largely shorten(40～60 months→12～20months)

Approach from new Kansei engineering:

Advocating a concept to measure satisfaction of the driver by driver's facial expression(Hagiwara,SICE1996年)

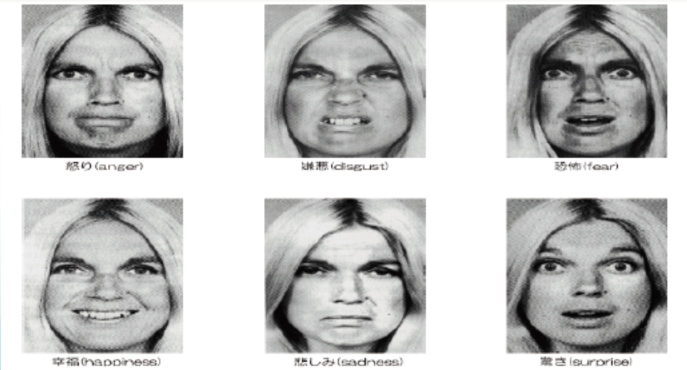
With advances in computational mechanics, most of vehicle performances can be designed in the initial stage of design. But performance related to human characteristics such as ride quality, steering stability and sound quality are difficult to be designed because their evaluations vary depending on race, age, and gender. Therefore, it is advocated to evaluate these characteristics with facial expressions because communication consists of language 7% and facial expression 55%.

Recognition of 6 basic facial expressions

6 basic facial expressions

Altough there are more than 400 words about facial expressions in the world ･

6 basic facial expression as anger ,disgust, fear, sadness, happiness, surprise is the same in the world regardless of race, culture, sex



Facial Action Unit(FAU) number

A total of 44 units have been identified and each numbered

AU No.	AU 名	AU No.	AU 名
1	眉の内側を上げる	20	唇両端を横に引く
2	眉の外側を上げる	23	唇を固く閉じる
4	眉を下げる	24	唇を押えつける
5	上瞼を上げる	25	顎を下げずに唇を開く
6	頬を持ち上げる	26	顎を下げて唇を開く
7	瞼を緊張させる	27	口を大きく開く
8	唇を互いに接近させる	28	唇を吸い込む
9	鼻にしわを寄せる	29	下顎を突き出す
10	上唇を上げる	30	顎を左右にずらす
11	鼻唇溝を深める	32	唇を噛む
12	唇両端を引き上げる	35	頬を吸い込む
13	唇を鋭く引き上げる(頬を膨らませる)	41	上瞼を（力なく）下げる
14	えくぼを作る	42	薄目
15	唇両端を下げる	43	閉眼
16	下唇を下げる	44	細眼にする
17	下顎（おとがい）を上げる	45	まばたく
18	唇をすぼめる	46	ウィンクする
			眼球の回転

*No.1:*Raise the inside of the eyebrows
No.2: Raise the outside of the eyebrows
No.4 Lower eyebrows
No.6 Lift cheeks
No.9 Wrinkle nose
No.14 Make dimples
No.16 Lower the lower lip
No.17 Raise the lower jaw
No18 Pursed lips
No.27 Open the mouth wide
No.28 Inhale lips
No.29 Stick out the lower jaw
No.32 Biting lips.*No.35* Inhale the cheeks
No.42 Thin,*No.43* Closed eyes
No.45 Blink *No.46 Wink*

6 basic facial expressions expressed by AU

Expression	Action Unites
Surprise	1+2+5+26
Fear	1+2+4+5+7+20+26
Disgust	4+9+17
Anger	4+5+7+10+26
Happiness	6+12(+26)
Sadness	1+4+15

The facial expression is a combination of several FACs. To recognize the combination, machine learning was used. In second AI generation, mostof machine learnings were back propagation neuralnetwork

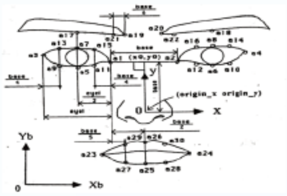
6 basic facial expressions by Hierarchical NN (Kobayashi etc)

◉ ***Procedure***

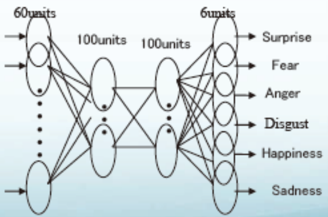
1. ***Designate 30 characteristics responding to AU and Calculate the amount of movement from a neutral facial expression***

2. ***Input the amount of movement to NN, output 6 basic facial expressions***

3. ***Learning by NN and recognition of facial expressions***

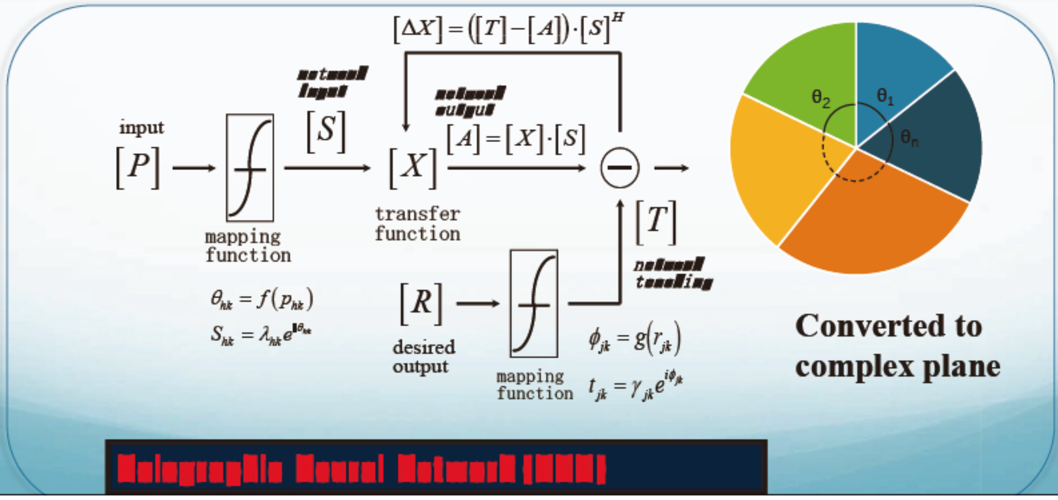


characteristics of face



NN for 6 basic facial expressions

HNN is an extended Laplace transform type neural network utilizing the fact that the input / output relationship can be expressed linearly by mapping input / output data to the complex plane.



Basic Theory of HNN。

$x = \{x_1, x_2, \ldots, x_k\}^{\mathrm{T}}$: stimulus data,

$y = \{y_1, y_2, \ldots, y_m\}^{\mathrm{T}}$: training data

n : learning number, X, Y : are input and output matrices

$$X = \begin{pmatrix} X_1^T \\ \vdots \\ X_n^T \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}$$

$$Y = \begin{pmatrix} Y_1^T \\ \vdots \\ Y_n^T \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{pmatrix} \qquad (1)$$

.

Each element of X and Y is converted to angles $\theta_{ai}(a=1,\cdots,n\,;i=1,\cdots,k)$ and $\phi_{aj}(a=1,\cdots,n\,;j=1,\cdots,m)$. (2)

Linear, Sigmoid and Arc Tangent functions for f_x and f_y.

the above angles are mapped on complex representation by using exponential function.

$$s_{ai}=\lambda_{ai}e^{\hat{\imath}\theta_{ai}} \qquad r_{aj}=\gamma_{aj}e^{\hat{\imath}\phi_{aj}} \qquad (3)$$

From the above, X and Y are converted to stimulus S and response R.

$$S=\begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1k} \\ s_{21} & s_{22} & \cdots & s_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nk} \end{pmatrix} \quad R=\begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{pmatrix} \qquad (4)$$

Formula for transfer function of HNN

$H=[h_1,\ldots,h_m]$, the transfer function of HNN is presented by minimizing the difference between teaching data R and $S\cdot H$ as follows;

$$H=(S^*S)^{-1}S^*R \qquad (5)$$

* : conjugate transpose . Output V can be predicted to be V :$V=U\cdot H$ for new input U.

From knowing the elements of transfer function like this, we can understand the causality.

$$H_0=S^*Y \qquad (6)$$

$$\Delta H_{k+1}=S^*(Y-SH_k) \qquad (7)$$

$$H_{k+1}=H_k+\Delta H_{k+1} \qquad (8)$$

$$J(H_{k+1})=(Y-S\,H_{k+1})^*(Y-S\,H_{k+1})\le \text{eps} \qquad (9)$$

History of HNN

1990	1997	2004	2009	2011

J.G.Sutherland : HNN paper

Hagiwara : apply HNN to Optimyzation and system identification : HNN extended by Hagiwara

Hagiwara,I. and Diago,L., : Apply HNN combined with Fuzzy inference class 2 to Iyashi and facial expression

J.G.Sutherland, The Holographic Model of Memory, Learning and Expression, International Journal of Neural System, 1990, Vol.1 No.3, 259

Q. Shi, I Hagiwara, S. Azetsu, T. Ichikawa, Holographic Neural NetworkApproximations for Acoustics Optimization, JSAE Review, Japan Society ofAutomobile Engineering, Vol.19 No.4 (1998-10), pp.361-363.

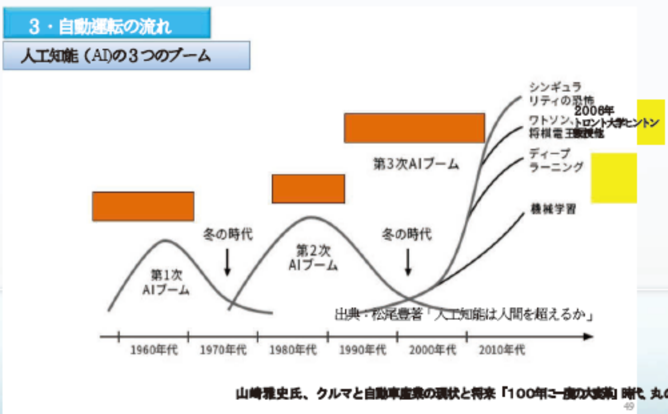
Diago, L., Kitaoka, T., Hagiwara, I. and Kambayashi, T., Neuro-Fuzzy Quantification of Personal Perceptions of Facial Images Based on a Limited Data Set, IEEE Transactions on Neural Networks, Vol. 22, No. 12 (2011), pp.2422-2434.

I was convinced that extended HNN and HNN combined with Fussy inference class 2 were better than Back propagation neural network which towedthe second AI boom

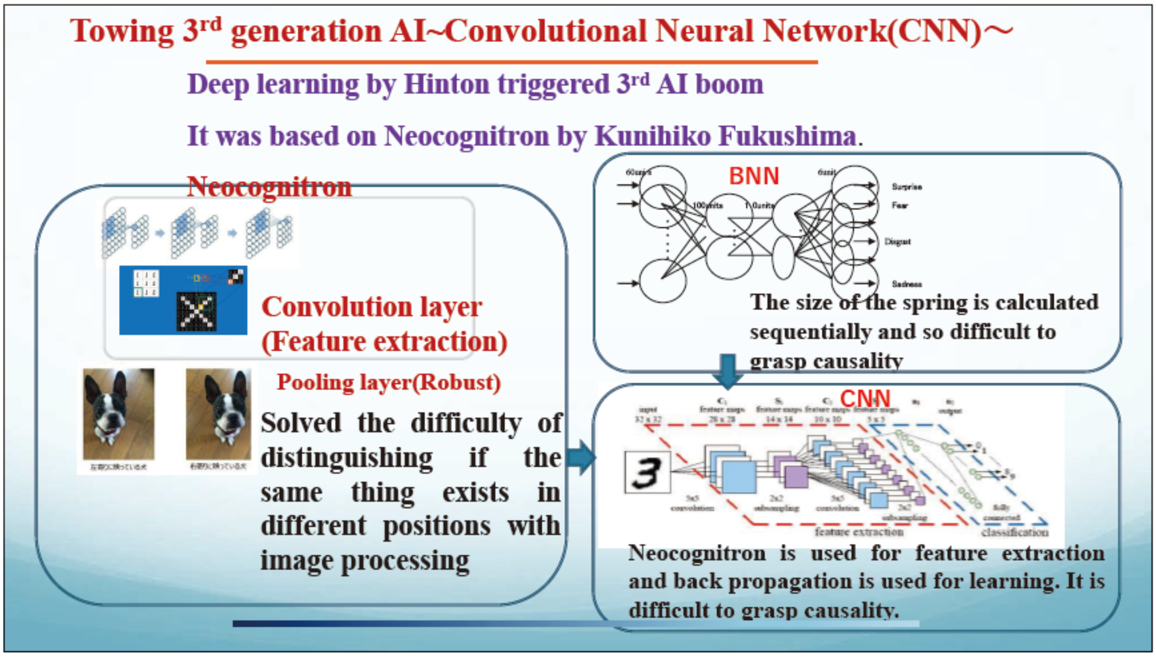
35

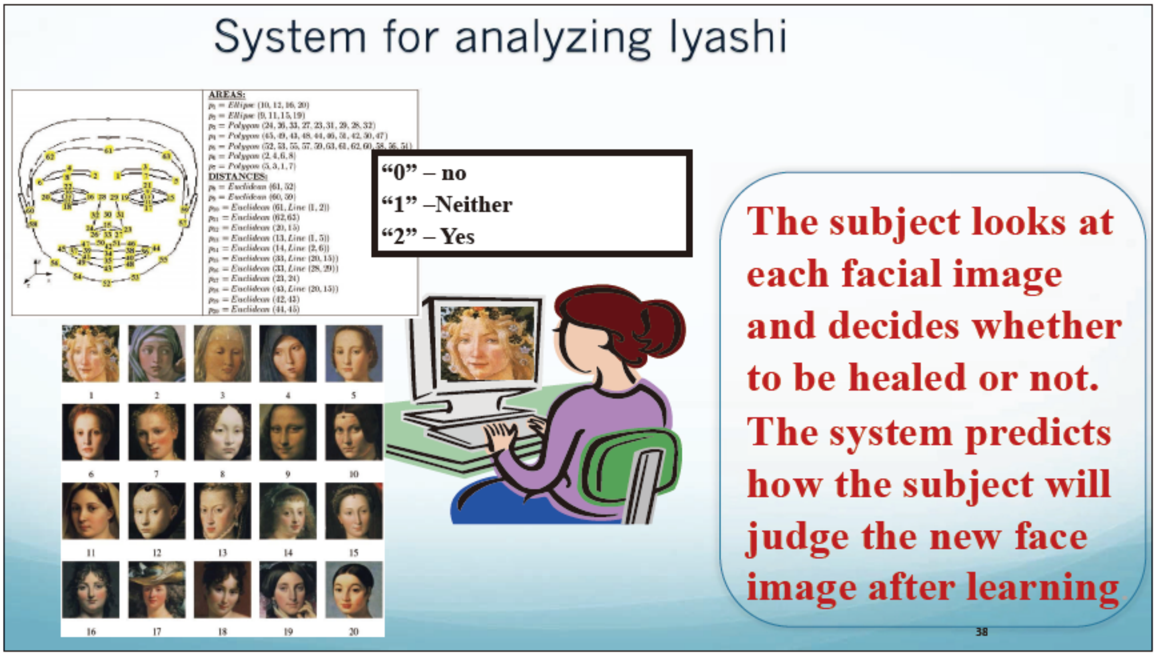
From 2nd generation AI to 3rd generation AI ━Causal AI left behind━

(Hagiwara, Sound quality collegium in JSAE,10th of May,2021.)



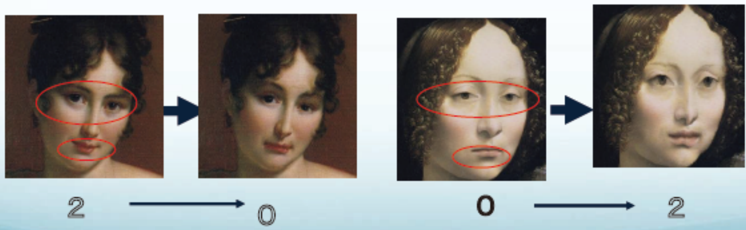
Search / reasoning (1st generation AI)⇒Knowledge(2nd generation AI)
⇒Machine learning･Expression(3rd generation AI)


Towing 3rd generation AI~Convolutional Neural Network(CNN)～
Deep learning by Hinton triggered 3rd AI boom
It was based on Neocognitron by Kunihiko Fukushima.
Neocognitron
Convolution layer
(Feature extraction)
Pooling layer(Robust)
Solved the difficulty of distinguishing if the same thing exists in different positions with image processing
BNN
Surprise
Fear
Disgust
Sadness
The size of the spring is calculated sequentially and so difficult to grasp causality
CNN
feature extraction
classification
Neocognitron is used for feature extraction and back propagation is used for learning. It is difficult to grasp causality.


System for analyzing Iyashi
AREAS:
DISTANCES:
"0" – no
"1" –Neither
"2" – Yes
The subject looks at each facial image and decides whether to be healed or not. The system predicts how the subject will judge the new face image after learning
38

Merits of HNN

- Not only the correct answer rate is high but also the system lets us know which of FACs the subject focuses on when making decisions
- From this information,by changing FACs, the subject's favorite facial expression can be created.
- This technology can be used for cooperated control of systems and humans in levels 3-5.

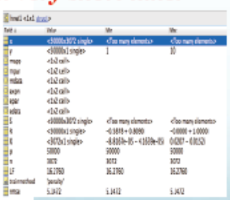


39

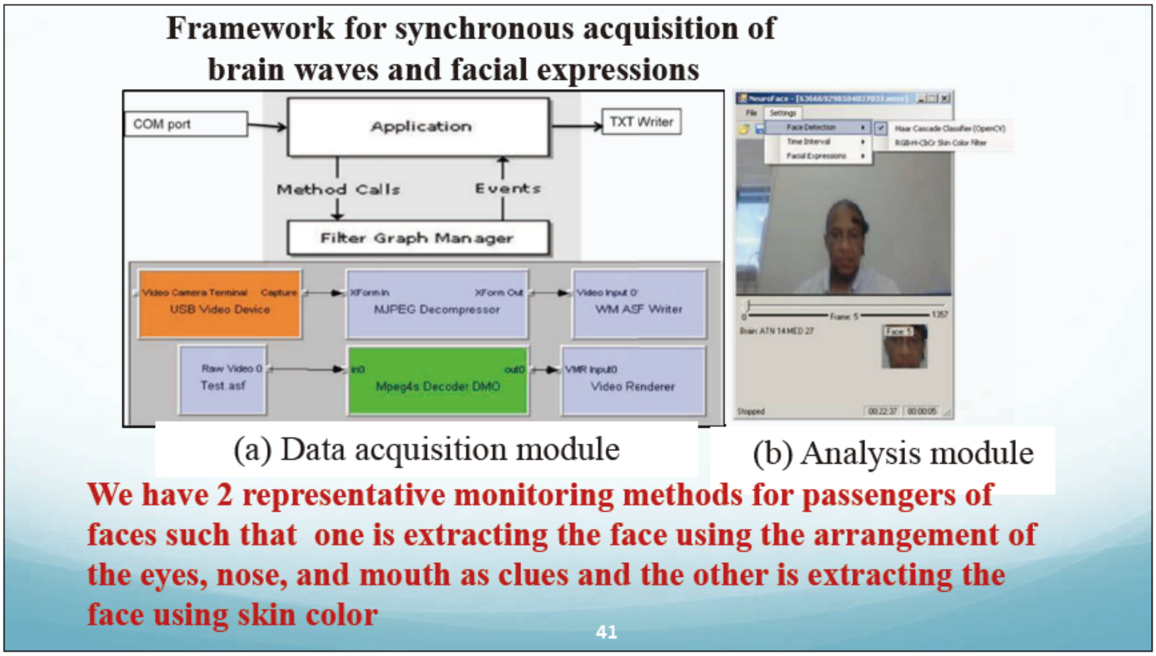
CNN vs HNN - ImageNet

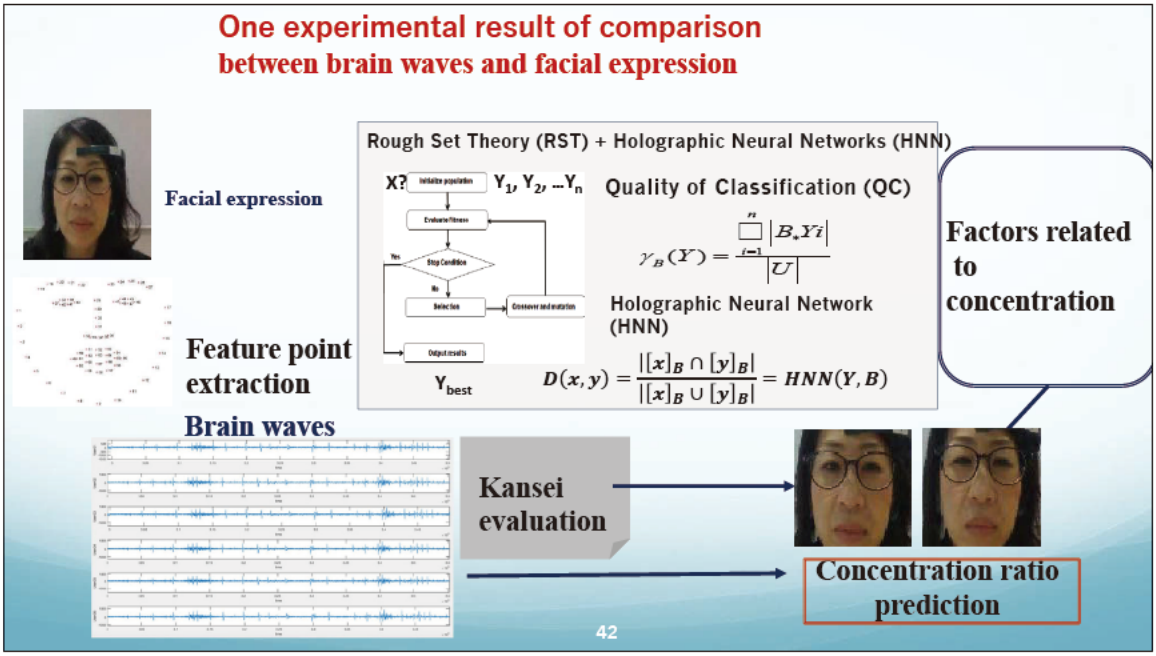
- *CNN(Convolutional Neural Network) has developed in various versions and played a leading role for deep learning. On the other hand ,our group is progressing an extended Laplace Transform (HNN). After learning with 10 types 60 thousand such as airplane,car,bird,catdeer,dog,fog,horse,ship,truck, Some types of CNN diverges、HNN had more than 90 percentage of correct answers.*

HNN Test_error = 9.147%
In very short time.

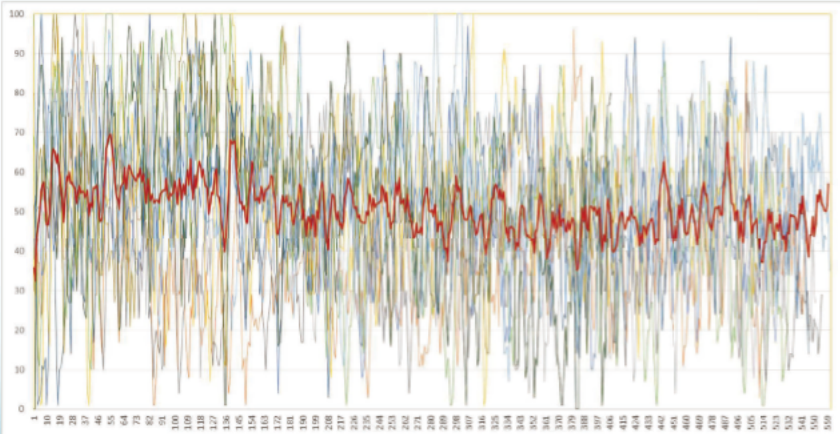


CNN has developed in various versions. Compared with these ,HNN is within 3rd place in accuracy and is overwhelmingly fast and top in calculation speed.


Framework for synchronous acquisition of
brain waves and facial expressions
COM port
Application
TXT Writer
Method Calls
Events
Filter Graph Manager
Video Camera Terminal Capture
USB Video Device
XForm In
XForm Out
MJPEG Decompressor
Video Input 0
WM ASF Writer
Raw Video 0
Test.asf
in0
out0
Mpeg4s Decoder DMO
VMR Input0
Video Renderer
File
Settings
Face Detection
Time Interval
Facial Expressions
Haar Cascade Classifier (OpenCV)
RGB-H-CbCr Skin Color Filter
Stopped
(a) Data acquisition module
(b) Analysis module
We have 2 representative monitoring methods for passengers of faces such that one is extracting the face using the arrangement of the eyes, nose, and mouth as clues and the other is extracting the face using skin color
41


One experimental result of comparison
between brain waves and facial expression
Facial expression
Feature point extraction
Brain waves
Rough Set Theory (RST) + Holographic Neural Networks (HNN)
X?
$Y_1, Y_2, \dots Y_n$
Stop Condition
Yes
No
Y_{best}
Quality of Classification (QC)
$\gamma_B(Y) = \frac{\sum_{i=1}^{n} |B_* Yi|}{|U|}$
Holographic Neural Network (HNN)
$D(x,y) = \frac{|[x]_B \cap [y]_B|}{|[x]_B \cup [y]_B|} = HNN(Y,B)$
Factors related to concentration
Kansei evaluation
Concentration ratio prediction
42

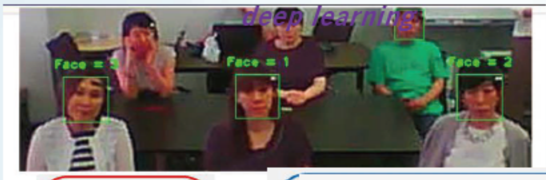
Experimental result　～ Changes in driver concentration assuming level 3 ～



43

Comparison between facial expressions extracted in real time and brain waves of plural persons

Measure Fatigue and Looseness through extracting negative facial expressions and brain waves by deep learning



6 Basic Facial Expressions

Development of breakthrough automatic driving technology by fusion of 3 splendid technologies such as HNN,EOC and Neural differential equation. EOC: by Fukishima-Uchida-Hagiwara

Giving autonomous driving car learning how to deal with accidents and near miss accident

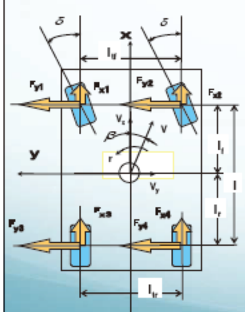
Modeling and Simulation are automatically generated by the approach of neural differential equation

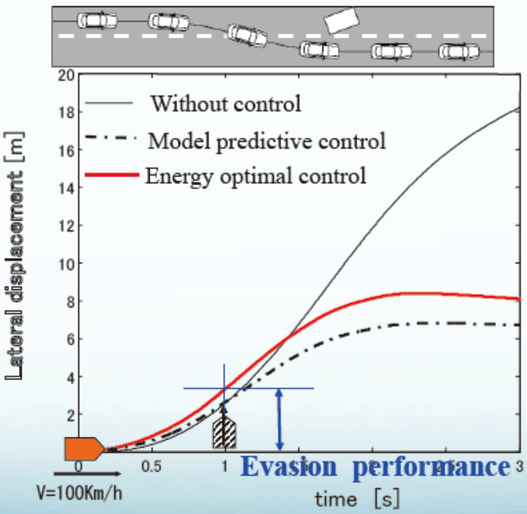
Making solutions with Energy optimal control (EOC). Change parameters and create a lot of data for deep learning.

Since Neural differential equation was proposed by Chen et al. in 2018, many learning methods have been discovered how to learn differential equation model which satisfies given data.

5

Construction of accident avoidance control model from drive recorder(DR)





Accident discovery that collided with a gutter from DR

It seems that it was caused by the sudden steering operation of the driver who found an obstacle ahead

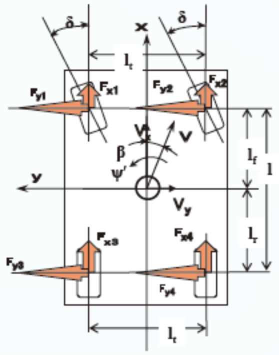
Suggestion of giving self-driving car learning measures to avoid accidents by generating accident model and solving with EOC.

It would be great if the differential equation, simulasiom model and the control method could be automatically generated from DR.

Now we are developing modeling and Simulation by ourselves.

Modeling of vehicle

The equation of motion in 3 degrees of freedom for vehicle is described as below



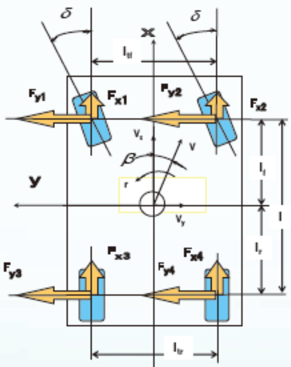
$$m\left(\ddot{x}-\dot{y}\dot{\phi}\right)=\sum_{i=1}^{4}F_{xi}+F_{wx}$$

$$m\left(\ddot{y}+\dot{x}\dot{\phi}\right)=\sum_{i=1}^{4}F_{yi}+F_{wy}$$

$$I\ddot{\phi}=l_f\sum_{i=1}^{2}F_{yi}-l_r\sum_{i=3}^{4}F_{yi}+\frac{d}{2}\left(-F_{x1}+F_{x2}-F_{x3}+F_{x4}\right)+M_w$$

β : angle of sideslip
δ : steering angle
F_{xi} : anteroposterior force
F_{yi} : side force
V : vehicle speed
ϕ ': yaw rate

Applying EOC



m : 車両質量
I_z : ヨー慣性モーメント

β: 横すべり角
δ: 前輪舵角
V: 車速
r: ヨーレート

運動方程式

$m(\upsilon_x-\upsilon_y r)=X$

$m(\upsilon_y+\upsilon_x r)=Y$

$I_z\dot{\mathrm{r}}=M$

X,Y,Mは各輪タイヤの前後力,横力,ヨーモーメント

各輪タイヤの横滑りによる散逸パワーの総和

$P=Y\upsilon_y+Mr$

制御入力パワー

$P_u=U(\upsilon_y+l_f r)$

仮想パワー

$g=R_1(r_d-r)M$

R_1,R_d,R_bは重み計数

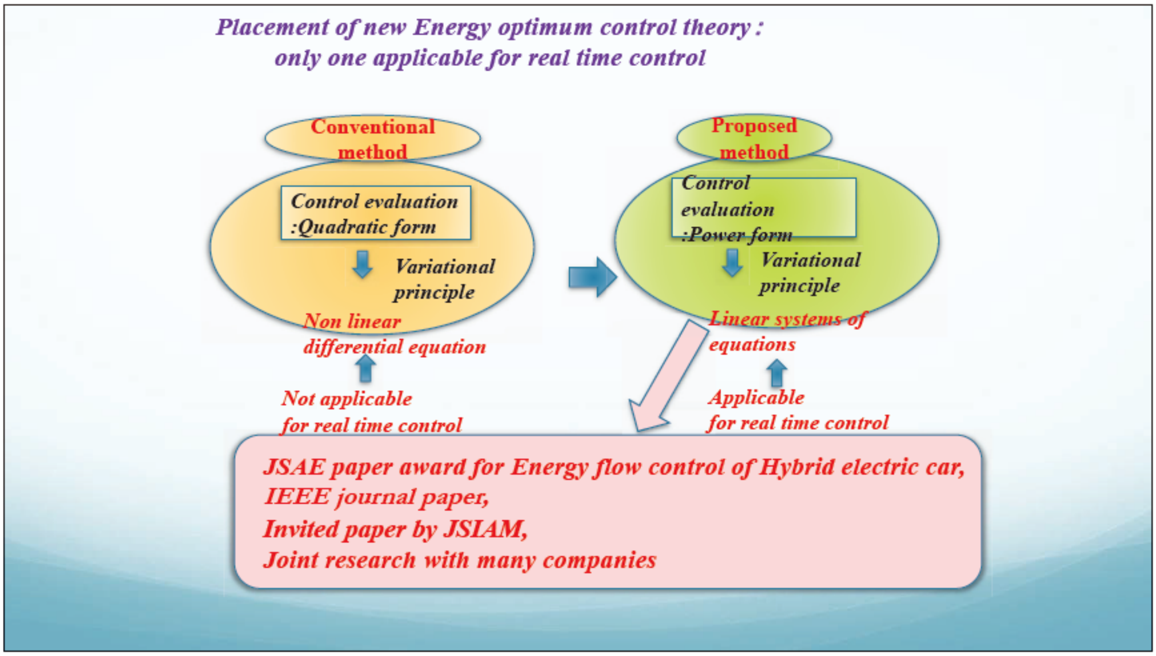
評価関数

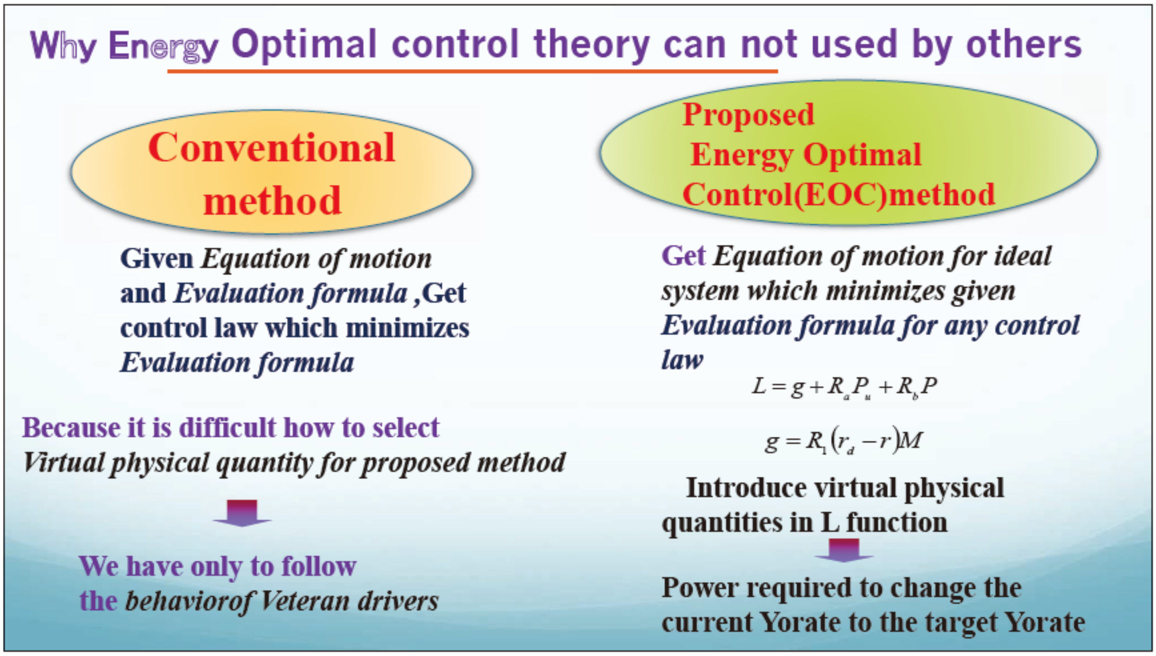
$J=\int L dt$

$L=g+R_a P_u+R_b P$

Lをrで偏微分して

$$U=\frac{R_1}{r_a l_f}\left\{M+(r-r_d)\frac{\partial M}{\partial r}\right\}-\frac{R_b}{R_a l_f}\left\{M+\left(\frac{\partial Y}{\partial r}\upsilon_y+\frac{\partial M}{\partial r}r\right)\right\}$$


Placement of new Energy optimum control theory :
only one applicable for real time control
Conventional method
Control evaluation :Quadratic form
Variational principle
Non linear differential equation
Not applicable for real time control
Proposed method
Control evaluation :Power form
Variational principle
Linear systems of equations
Applicable for real time control
JSAE paper award for Energy flow control of Hybrid electric car,
IEEE journal paper,
Invited paper by JSIAM,
Joint research with many companies


Why Energy Optimal control theory can not used by others
Conventional method
Given Equation of motion and Evaluation formula ,Get control law which minimizes Evaluation formula
Because it is difficult how to select
Virtual physical quantity for proposed method
We have only to follow
the behaviorof Veteran drivers
Proposed Energy Optimal Control(EOC)method
Get Equation of motion for ideal system which minimizes given Evaluation formula for any control law
$L = g + R_a P_u + R_b P$
$g = R_1 (r_d - r) M$
Introduce virtual physical quantities in L function
Power required to change the current Yorate to the target Yorate

2 Innovative Applications of Energy-Based Modeling and Simulation Based on Deep Learning (Neural differential Equation with data)

1st Item: Application to EOC

❶ In 2018, Neural differential equation was proposed based on given data with various learning methods by Chen et al.

❷ This means we can get system equation A. . On the other hand , with EOC system *equation B of motion is given for ideal system which minimizes given evaluation formula for any control*

❸ By comparison of A and B may lead to the virtual Drive system(**virtual physical quantities)**

2nd Item: Application to EOC

Generation of differential equation ,modeling and simulation based on accidents and near miss accidents on drive recorders.

Summary

1 The most important issue for self driving is to ensure safety.

2 To realize this, it is important high-speed, high-precision image processing technology and machine learning.

3 And also it is required higher level control technology.

4 Energy Optimal Control(EOC) law is effective which is only possible for real-time optimal control.

5 EOC must have virtual mechanism(virtual physical quantities) and for finding this it may be effective to use Neural differential equation by Chen et al.

6 The cooperative control for system & driver in level 3 and for system & remote operator for levels 4 &5 must have causal Holographic neural network

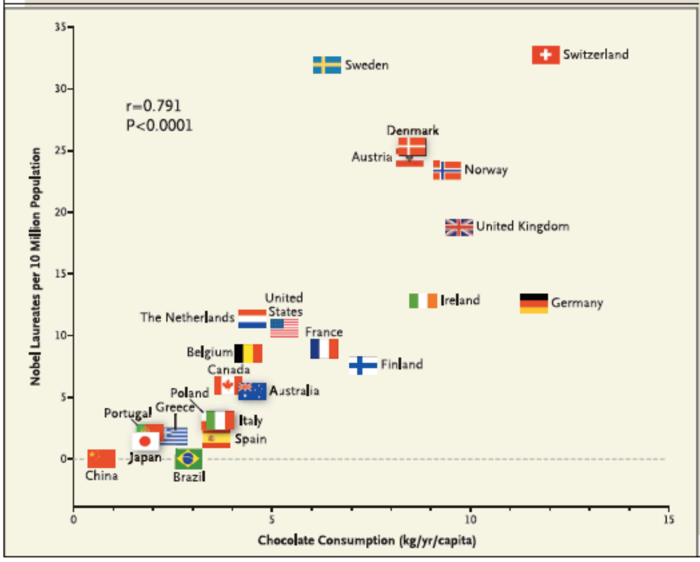
7 Vehicle structure must have property such that vibration cutoff in frequency range where ride quality and fear are much influenced and it is also effective to move eigen frequencies out of this frequency range.

Topological Methods for Causal Inference from Time-series Data

Shizuo KAJI (Institute of Mathematics for Industry Kyushu University)

Data Science
=
Description + Prediction
+ <u>Causal inference</u>

Eating chocolate produces Nobel prize winners(?)

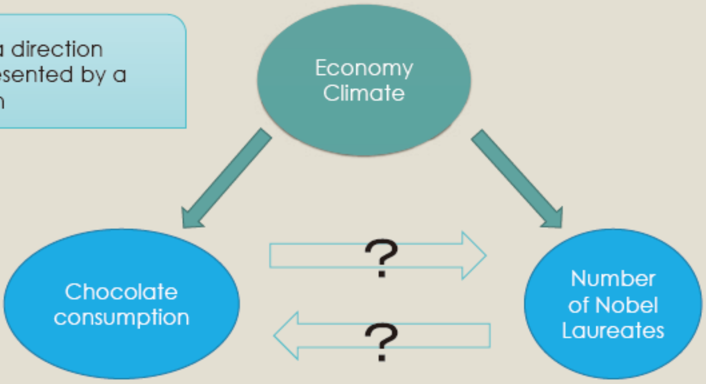


Causality is different from correlation

- Messerli, F. H. (2012) *Chocolate Consumption, Cognitive Function, and Nobel Laureates*, The New England Journal of Medicine 367
- Witty and amusing paper Highly recommended!

Causality in Network of variables

- Causality has a direction
- It can be represented by a directed graph



How to detect : Intervention (e.g., cut the supply of chocolate for 20 years!)
=> In many cases, it is not feasible
Goal today : Causal inference from observation (data)
⇒ Challenge: We cannot observe everything

Example: do they have any causal relation?

- Traffic congestion at different points
- Traffic accident vs Traffic flow
- Covid-19 vs people's activity
- CO_2 emission vs temperature anomalies

Basic causality test

Intervention (e.g., Randomised Control Trial)

Antibiotics → ? → Tuberculosis

Contrast with a placebo group

Medical Research Council (1948)
Streptomycin Treatment of Pulmonary Tuberculosis

Problem: in many practical situations, we are not allowed to intervene but only to observe.

Various causality inference methods are developed with different assumptions on the world

- Granger causality (GC)
- Structural equation models (SEM)
- Transfer Entropy (TE)
- Linear non-Gaussian acyclic model (LiNGAM)
- **Convergent Cross Mapping (CCM)**
  Sugihara et al. "Detecting causality in complex ecosystems" Science 338.6106 (2012), and its extensions

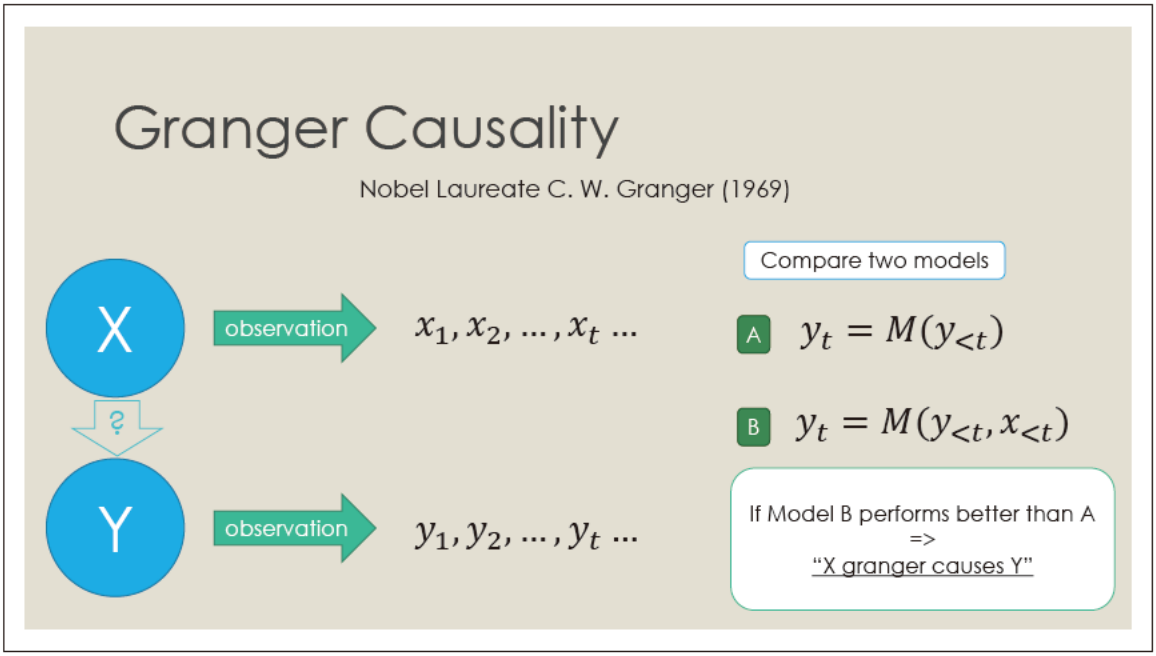
t = year	x = anchovy	y = np_sst
1929	-0.0075987	-0.347846
1930	-0.0096016	0.328734
1931	-0.0084442	1.61027
1932	-0.0083536	1.26534
1933	-0.0077497	0.0400459
...	...	...
2002	-0.741992	-1.18642
2003	-0.157825	-1.34262
2004	0.249897	-0.530206
2005	0.213121	-0.757965
2006	0.0730902	-0.583363

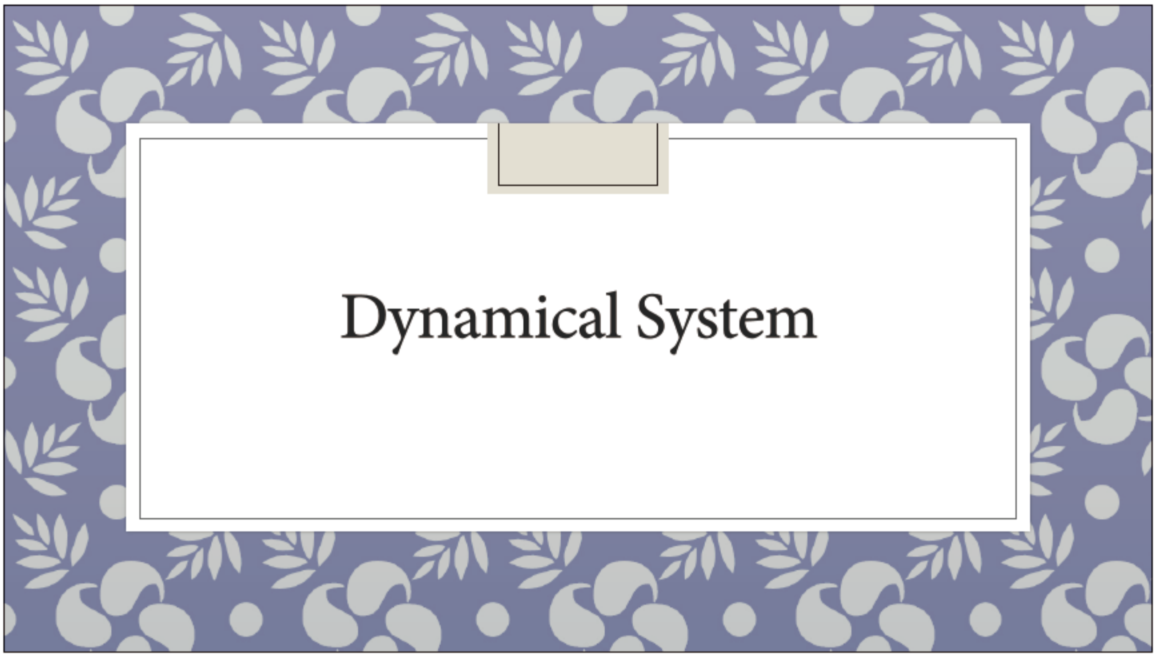
pyEDM built-in demo data

Time series

- Usually represented by a spreadsheet-lie table
- Each row corresponds a set of observation at a specific time
- Each column corresponds to a set of observations for a certain period

Example:
$x_{1930} = -0.0096016$
indicates the catch of anchovy in the year 1930 (relative to the mean)


Granger Causality
Nobel Laureate C. W. Granger (1969)
X
observation
$x_1, x_2, \dots, x_t \dots$
?
Y
observation
$y_1, y_2, \dots, y_t \dots$
Compare two models
A $y_t = M(y_{<t})$
B $y_t = M(y_{<t}, x_{<t})$
If Model B performs better than A
=>
"X granger causes Y"


Dynamical System

View of the world: dynamical system

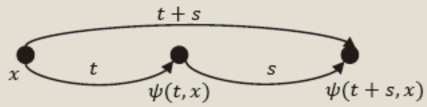
(assumption for today)

Defined by the triple (Time $T \subset Z$, State space X, Evolution rule ψ)

The system evolves in a deterministic way

$$t, s \in T, \quad x \in X, \quad \psi: T \times X \to X$$

- $\psi(0, x) = x$
- $\psi(s, \psi(t, x)) = \psi(t + s, x)$



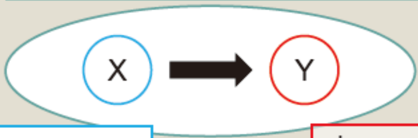
- (Continuous system governed by a differential equation: $\dot{x} = f(x)$)
- Discrete system governed by a difference equation: $x_{t+1} = f(x_t)$

Causality in dynamical systems

"X causes Y" = The evolution rule of Y is dependent on the state of X

total system $X \times Y$

$$\begin{cases} f: X \to X: & x_{t+1} = f(x_t) \\ g: X \times Y \to Y: & y_{t+1} = g(x_t, y_t) \end{cases}$$



upstream system X

$x_{t+1} = f(x_t)$

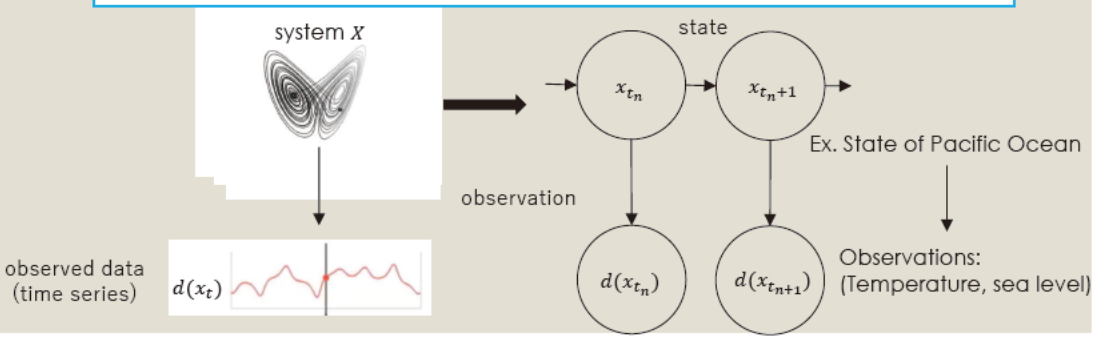
downstream system Y

$y_{t+1} = g(x_t, y_t)$

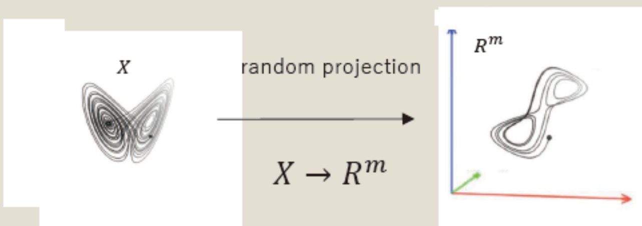
We have limited access to the system

- We have no way to gain complete knowledge of the state space
- Only partial information is available through observation

observation $d: X \rightarrow R^m$
(a map from the state space to a Euclidean space)



Reconstruction of the state space



- For a large m, a generic observation $d: X \rightarrow R^m$ is known to be an embedding
(more precisely, such d are dense in the space of smooth maps)

observation faithfully captures the state

But

- We cannot always make enough large observation.
(intuitively, m is the number of simultaneous observations; e.g., temperature, humidity,⋯)

Delay-coordinate embedding (Takens' embedding)

Instead of large simultaneous observation,
we can (surprisingly!) reconstruct the state space
from a series of **single observation**

Define $\boldsymbol{d}: X \to R^E$ by

$$\boldsymbol{d}(x) = (d(x_t), d(x_{t-1}), d(x_{t-2}), \ldots, d(x_{t-E+1}))$$

E: embedding dimension

in practice, E is a hyper-parameter

Theorem (Takens 1981)

Under a mild assumption on the periodic orbits of the system,
for a large E and a generic d,
the image of the delay-coordinate embedding approximates the attractor of the system

Note: Delay-coordinate embedding provides a powerful tool for time-series analysis (not only for causality inference but also e.g., prediction).

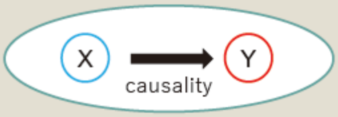
Delay-coordinate embedding for a coupled system

Observation from the caused system can recover the total system

Total system $X \times Y$

$$\begin{cases} x_{t+1} = f(x_t) \\ y_{t+1} = g(x_t, y_t) \end{cases}$$

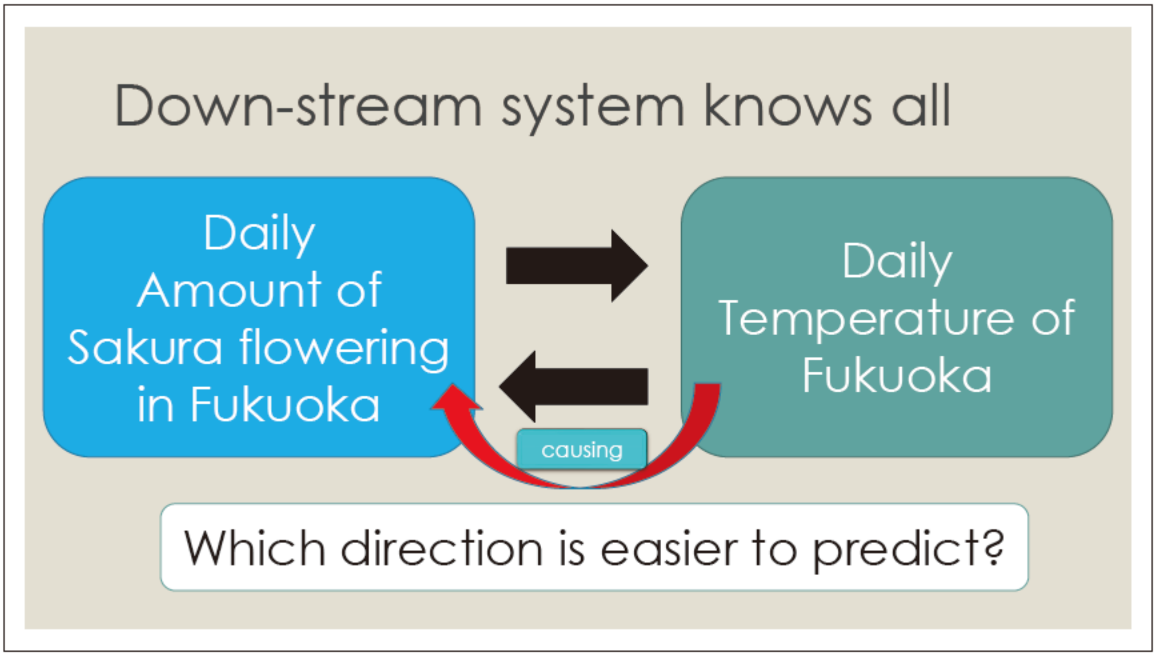
upstream X
$x_{t+1} = f(x_t)$
observation
$d_X(x_t)$

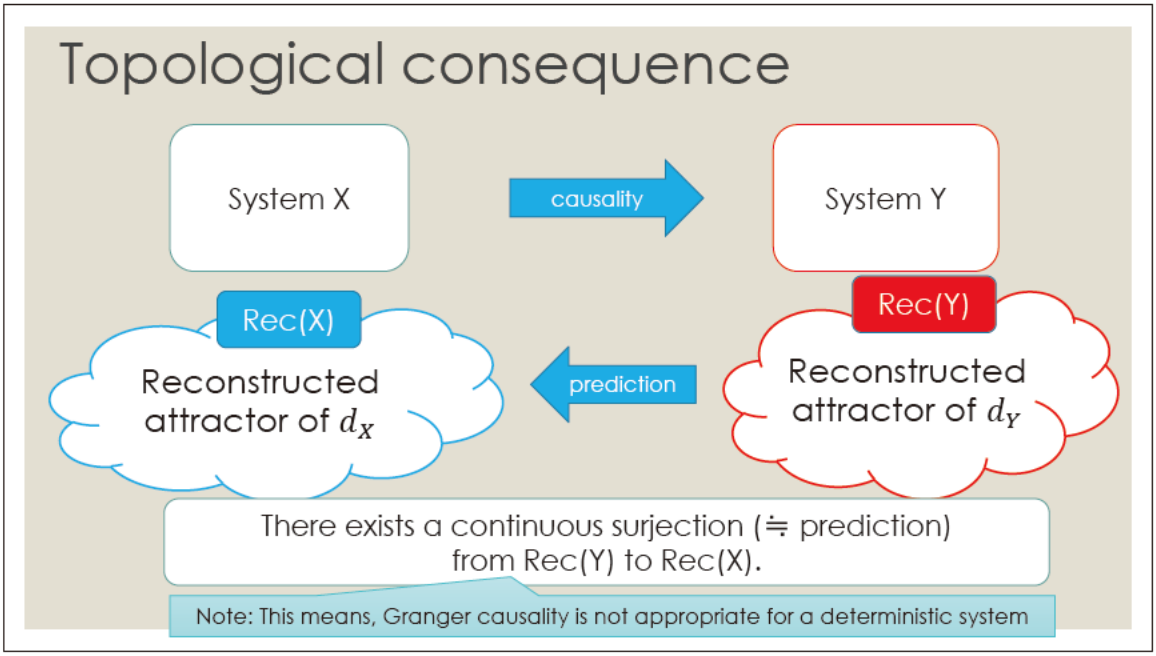


downstream Y
$y_{t+1} = g(x_t, y_t)$
observation
$d_Y(x_t)$

Theorem (Stark 1999)

Under a mild assumption on the periodic orbits of f,
the delay-coordinates of a generic observation $d_Y(x_t)$
approximates the attractor of the total system $X \times Y$


Down-stream system knows all
Daily Amount of Sakura flowering in Fukuoka
Daily Temperature of Fukuoka
causing
Which direction is easier to predict?


Topological consequence
System X
causality
System Y
Rec(X)
Reconstructed attractor of d_X
prediction
Rec(Y)
Reconstructed attractor of d_Y
There exists a continuous surjection (≒ prediction) from Rec(Y) to Rec(X).
Note: This means, Granger causality is not appropriate for a deterministic system

Convergent Cross Mapping (CCM)

utilises the idea in the previous slide for causality inference

pros
- A popular choice for causality check for deterministic systems
- Requires relatively small amount of data (e.g., 100 observations)
- Easy to use software packages
  - pyEDM: a Python package for CCM: https://github.com/SugiharaLab/pyEDM
  - rEDM: an R package for CCM: https://github.com/SugiharaLab/rEDM
  - mpEDM: massively parallel implementation: https://github.com/keichi/mpEDM
- Tutorial articles (in Japanese as well)

cons
- Not applicable to strongly correlated system
- Technical difficulties in hypothesis testing
  (but normally, one can just use the package)

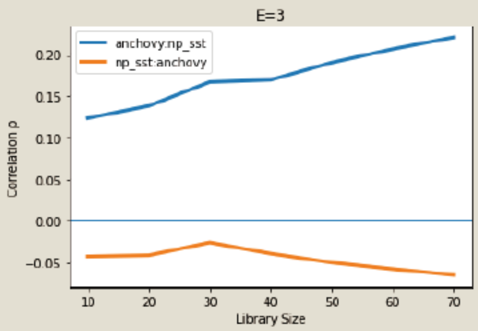
pyEDM Demo

pyEDM built-in demo data

year	anchovy	np_sst
1929	-0.0075987	-0.347846
1930	-0.0096016	0.328734
1931	-0.0084442	1.61027
1932	-0.0083536	1.26534
1933	-0.0077497	0.0400459
...	...	...
2002	-0.741992	-1.18642
2003	-0.157825	-1.34262
2004	0.249897	-0.530206
2005	0.213121	-0.757965
2006	0.0730902	-0.583363

78 rows, np_sst = sea surface temperature

a few dozen data is usually sufficient



The prediction anchovy -> np_sst gets better when Library Size is increased, whereas the opposite direction does not.

This indicates np_sst is a cause of anchovy.

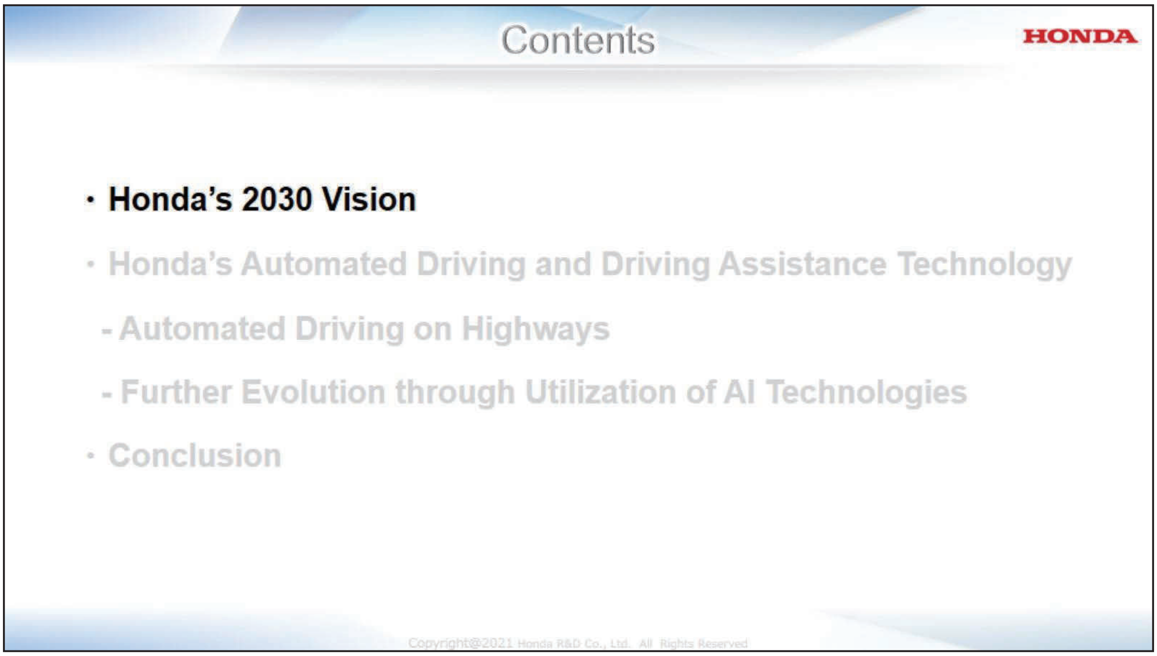
Automated Driving and Driving Assistance Systems Using Cooperative Intelligence

Honda R&D Co., Ltd
Innovative Research Excellence
Computer Science domain

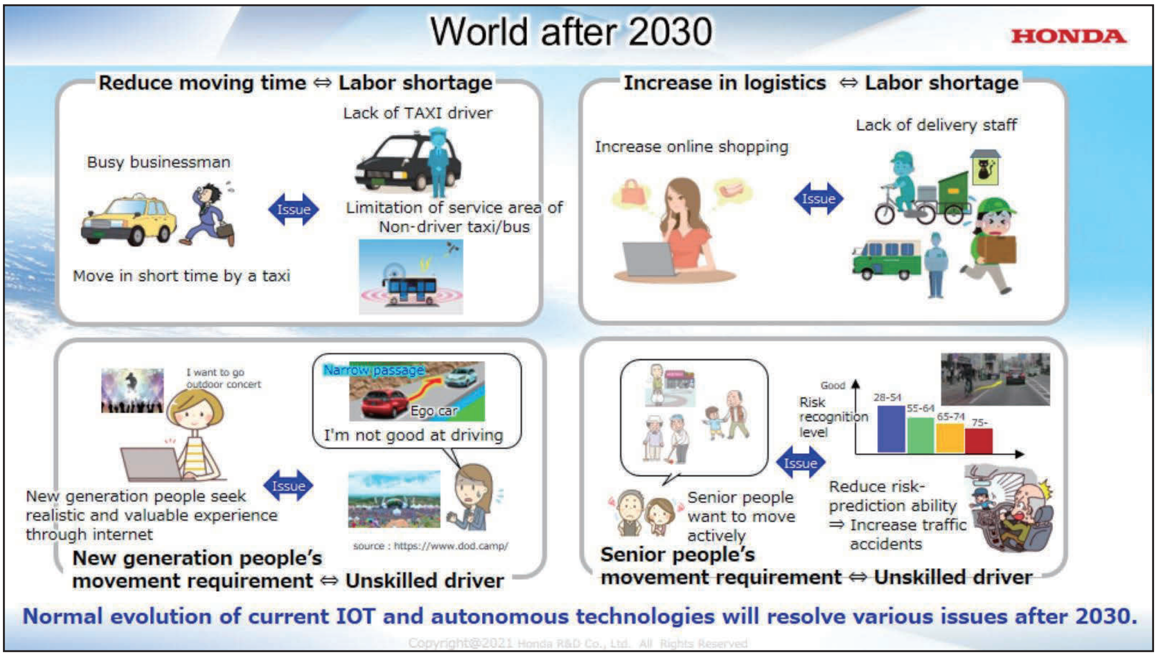
Executive Chief Engineer
Yuji Yasui

Contents

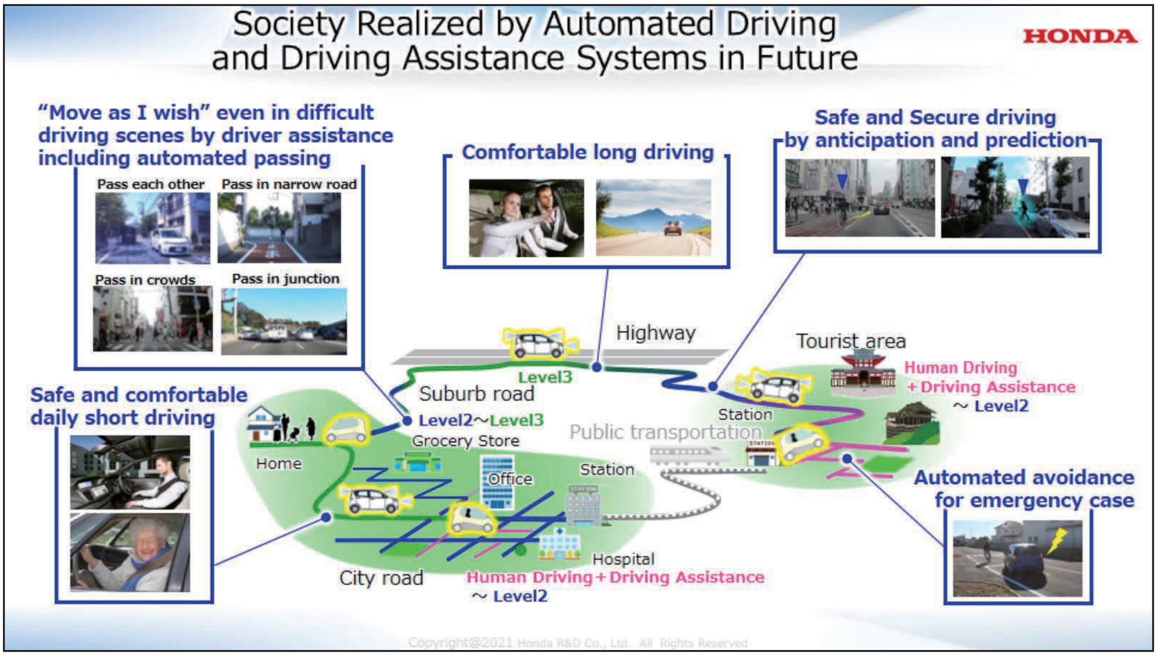
- Honda's 2030 Vision
- Honda's Automated Driving and Driving Assistance Technology
  - Automated Driving on Highways
  - Further Evolution through Utilization of AI Technologies
- Conclusion


Contents
HONDA
• Honda's 2030 Vision
• Honda's Automated Driving and Driving Assistance Technology
- Automated Driving on Highways
- Further Evolution through Utilization of AI Technologies
• Conclusion
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Honda's 2030 Vision
HONDA
Serve people worldwide with
the "joy of expanding their life's potential"
- Lead the advancement of mobility and enable people
everywhere in the world to improve their daily lives -
Freedom of mobility
Freed from anxiety
about driving,
Everyone can enjoy
mobility at any time
New joy of mobility
Provide new private space
and time during movement
Everyone can discover new
joy
Collision free society
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved



World after 2030
HONDA
Reduce moving time ⇔ Labor shortage
Lack of TAXI driver
Busy businessman
Issue
Limitation of service area of Non-driver taxi/bus
Move in short time by a taxi
Increase in logistics ⇔ Labor shortage
Lack of delivery staff
Increase online shopping
Issue
I want to go outdoor concert
Narrow passage
Ego car
I'm not good at driving
Issue
New generation people seek realistic and valuable experience through internet
source : https://www.dod.camp/
New generation people's movement requirement ⇔ Unskilled driver
Good
Risk recognition level
28-54
55-64
65-74
75-
Issue
Senior people want to move actively
Reduce risk-prediction ability ⇒ Increase traffic accidents
Senior people's movement requirement ⇔ Unskilled driver
Normal evolution of current IOT and autonomous technologies will resolve various issues after 2030.
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

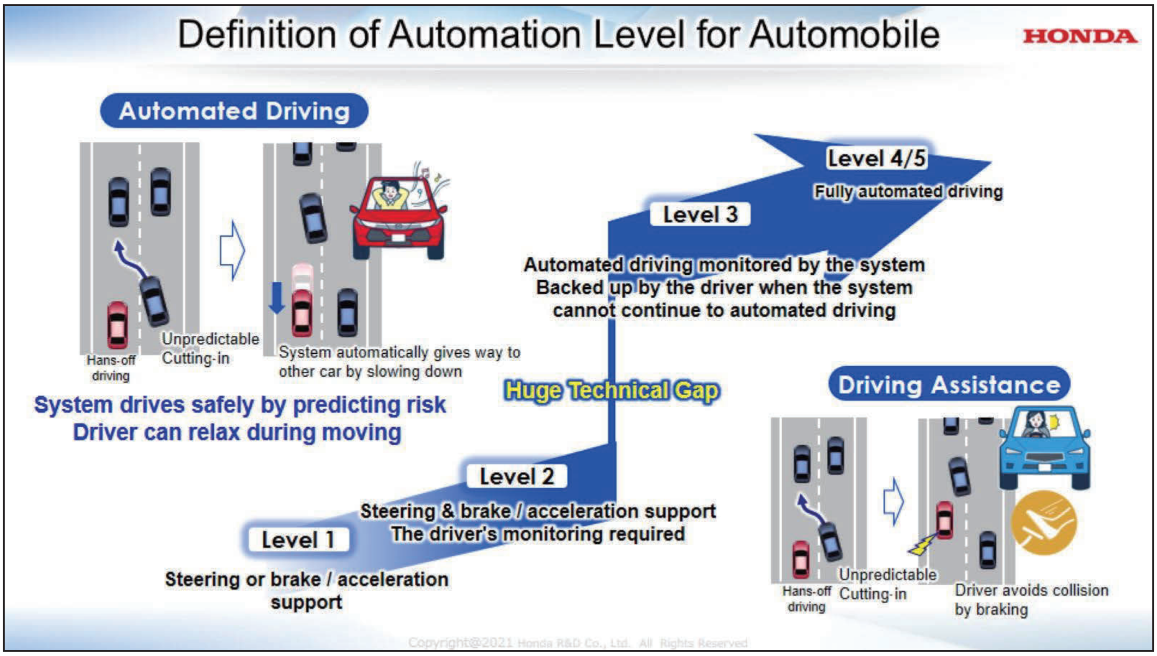


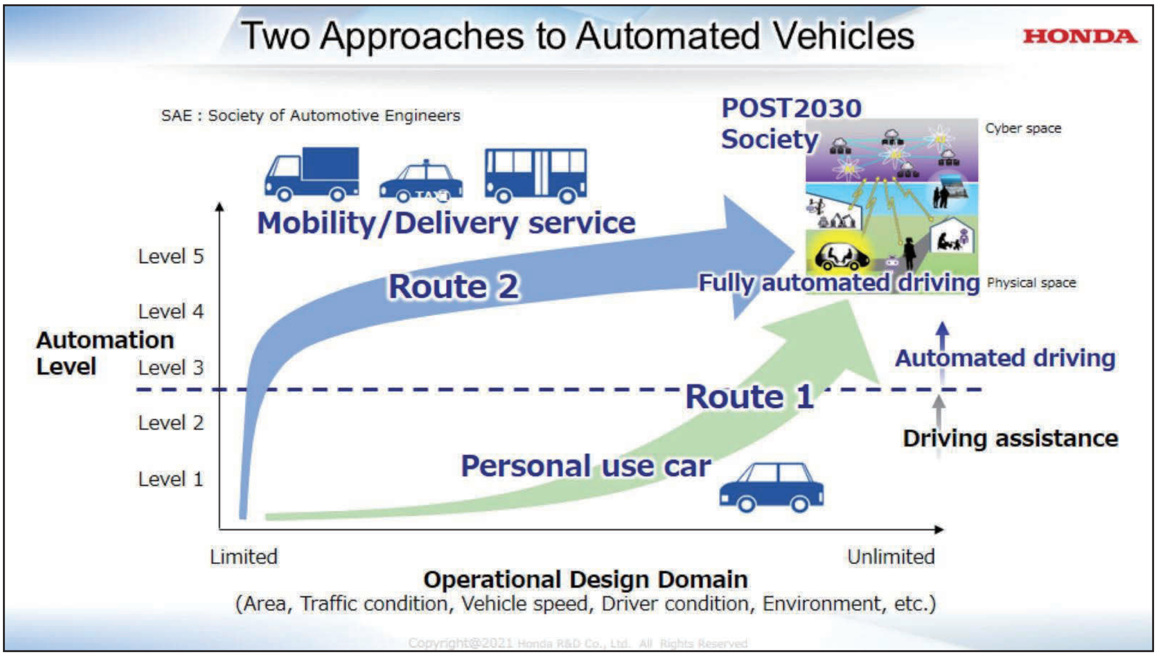
Society Realized by Automated Driving and Driving Assistance Systems in Future
HONDA
"Move as I wish" even in difficult driving scenes by driver assistance including automated passing
Pass each other
Pass in narrow road
Pass in crowds
Pass in junction
Comfortable long driving
Safe and Secure driving by anticipation and prediction
Highway
Tourist area
Level3
Suburb road
Human Driving + Driving Assistance ~ Level2
Station
Safe and comfortable daily short driving
Level2~Level3
Grocery Store
Public transportation
Home
Office
Station
Automated avoidance for emergency case
Hospital
City road
Human Driving + Driving Assistance ~ Level2
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

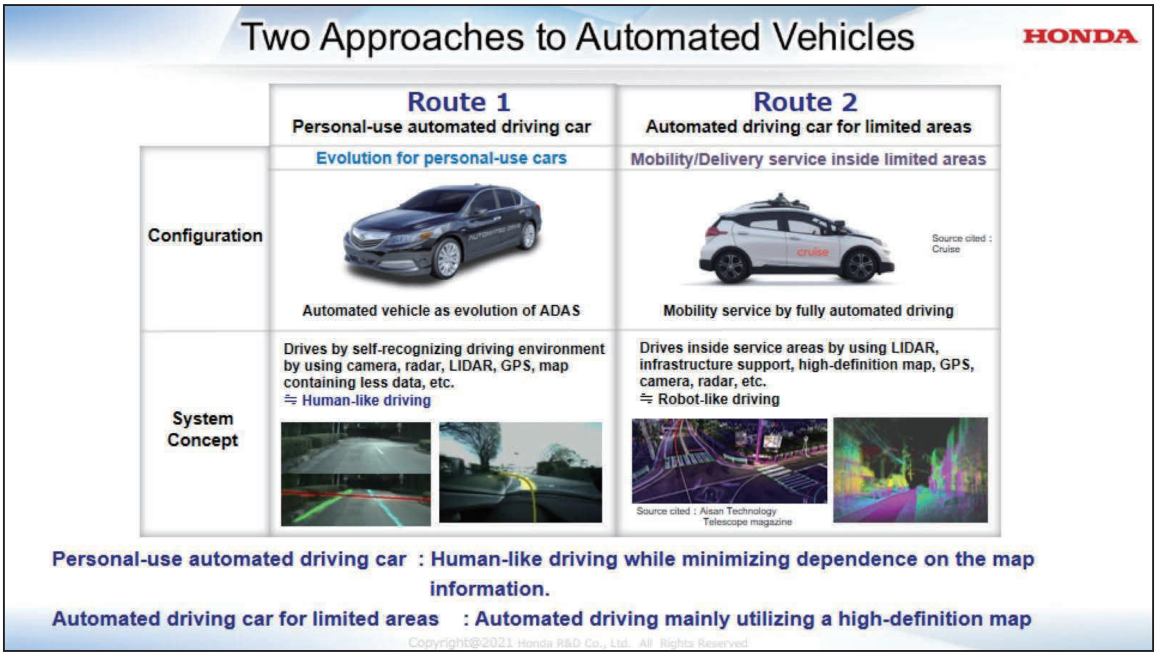
Contents

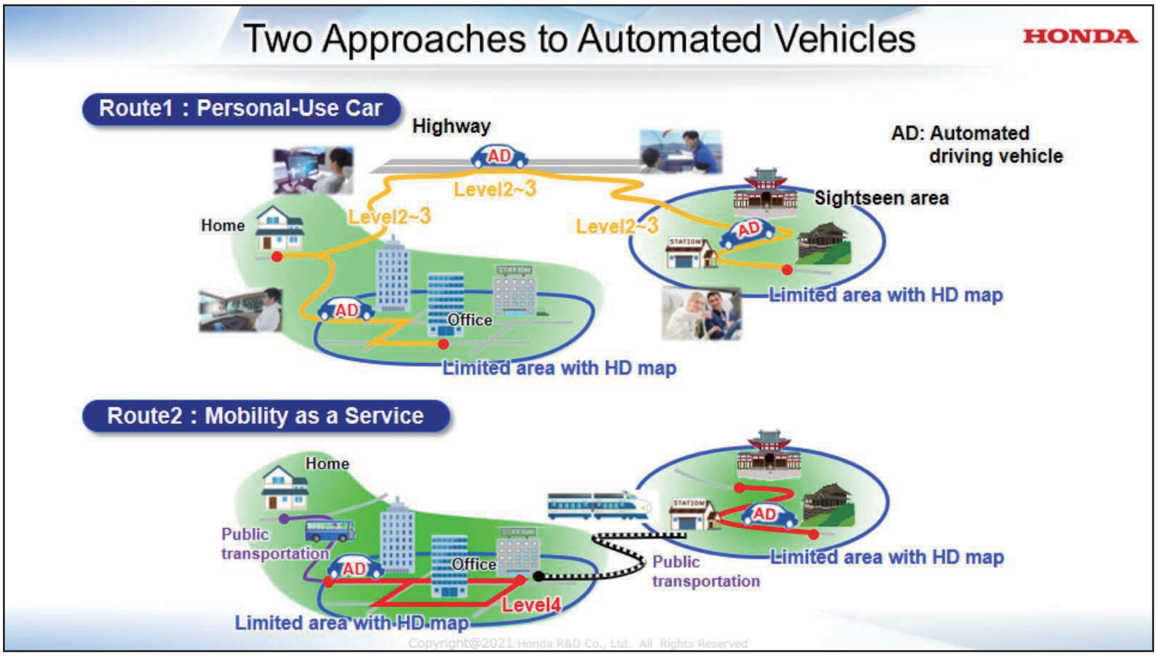
HONDA

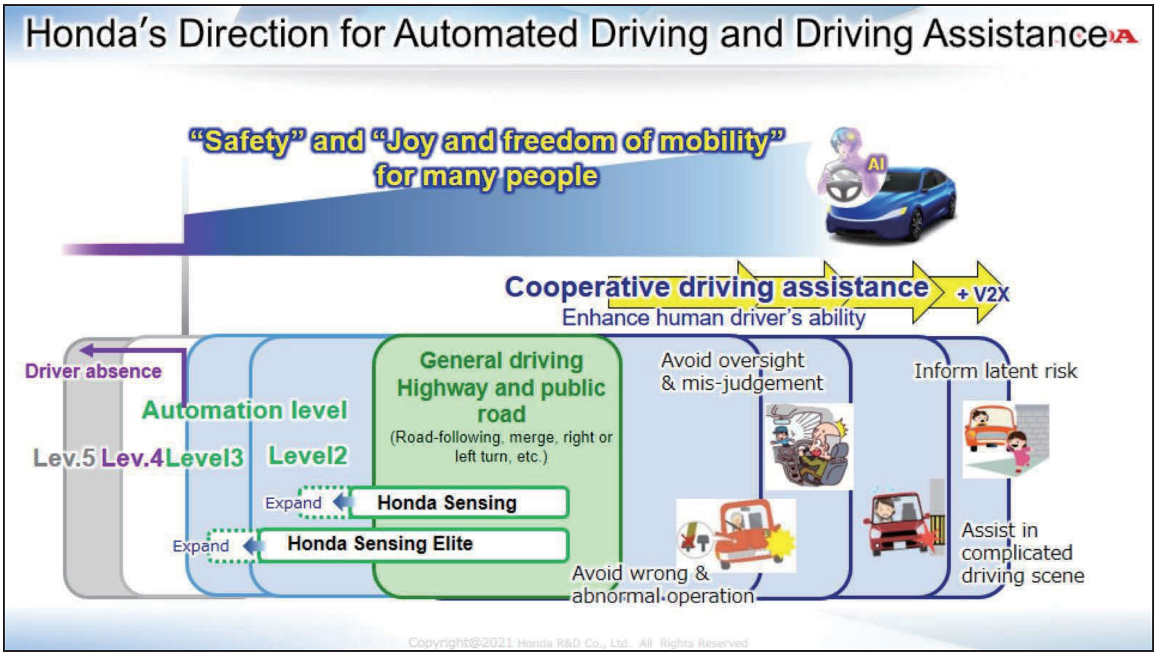
- Honda's 2030 Vision
- **Honda's Automated Driving and Driving Assistance Technology**
  - Automated Driving on Highways
  - Further Evolution through Utilization of AI Technologies
- Conclusion




Two Approaches to Automated Vehicles
HONDA
SAE : Society of Automotive Engineers
POST2030
Society
Cyber space
Physical space
Mobility/Delivery service
Route 2
Fully automated driving
Level 5
Level 4
Automation Level
Level 3
Level 2
Level 1
Automated driving
Route 1
Driving assistance
Personal use car
Limited
Unlimited
Operational Design Domain
(Area, Traffic condition, Vehicle speed, Driver condition, Environment, etc.)
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Two Approaches to Automated Vehicles
HONDA
Route 1
Personal-use automated driving car
Evolution for personal-use cars
Route 2
Automated driving car for limited areas
Mobility/Delivery service inside limited areas
Configuration
Automated vehicle as evolution of ADAS
Source cited : Cruise
Mobility service by fully automated driving
System Concept
Drives by self-recognizing driving environment by using camera, radar, LIDAR, GPS, map containing less data, etc.
≒ Human-like driving
Drives inside service areas by using LIDAR, infrastructure support, high-definition map, GPS, camera, radar, etc.
≒ Robot-like driving
Source cited : Aisan Technology Telescope magazine
Personal-use automated driving car : Human-like driving while minimizing dependence on the map information.
Automated driving car for limited areas : Automated driving mainly utilizing a high-definition map
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Two Approaches to Automated Vehicles
HONDA
Route1 : Personal-Use Car
Highway
AD: Automated driving vehicle
AD
Level2~3
Level2~3
Level2~3
Home
Sightseen area
Office
Limited area with HD map
Limited area with HD map
Route2 : Mobility as a Service
Home
Public transportation
Office
Level4
Limited area with HD map
Public transportation
Limited area with HD map
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Honda's Direction for Automated Driving and Driving Assistance
"Safety" and "Joy and freedom of mobility" for many people
AI
Cooperative driving assistance
+ V2X
Enhance human driver's ability
Driver absence
Automation level
Lev.5 Lev.4 Level3 Level2
General driving
Highway and public road
(Road-following, merge, right or left turn, etc.)
Expand
Honda Sensing
Expand
Honda Sensing Elite
Avoid oversight & mis-judgement
Inform latent risk
Assist in complicated driving scene
Avoid wrong & abnormal operation
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

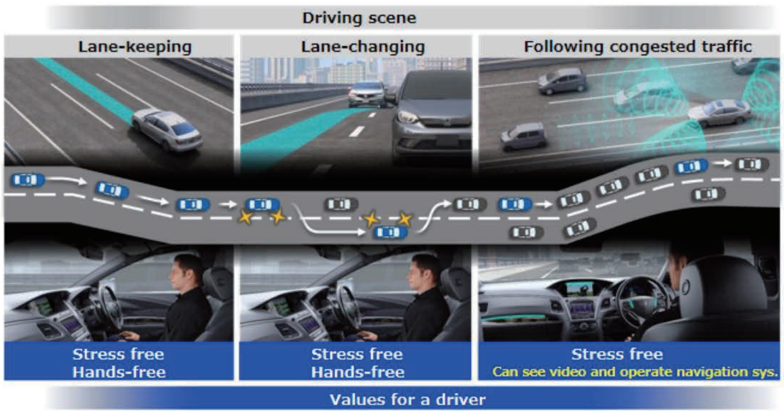
Contents

HONDA

- Honda's 2030 Vision
- **Honda's Automated Driving and Driving Assistance Technology**
  - **Automated Driving on Highways**
  - Further Evolution through Utilization of AI Technologies
- Conclusion

Honda SENSING Elite

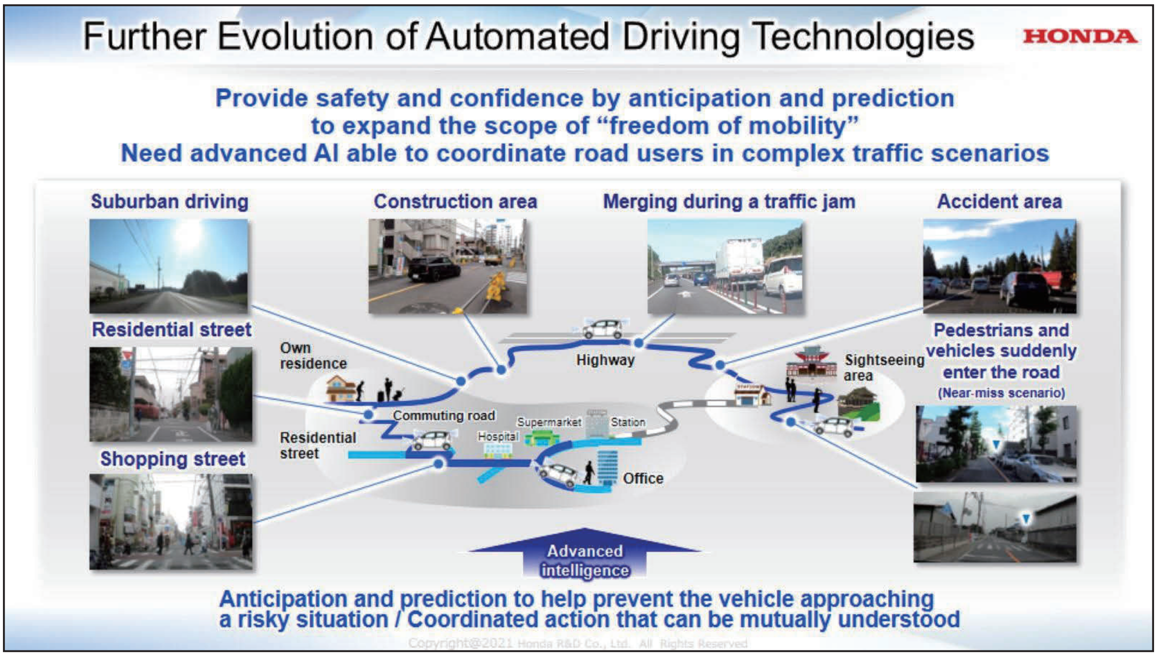
Provides support for staying in the lane, advanced support for lane changes, and automated driving (Level3) in traffic jams when driving on the main line of a highway



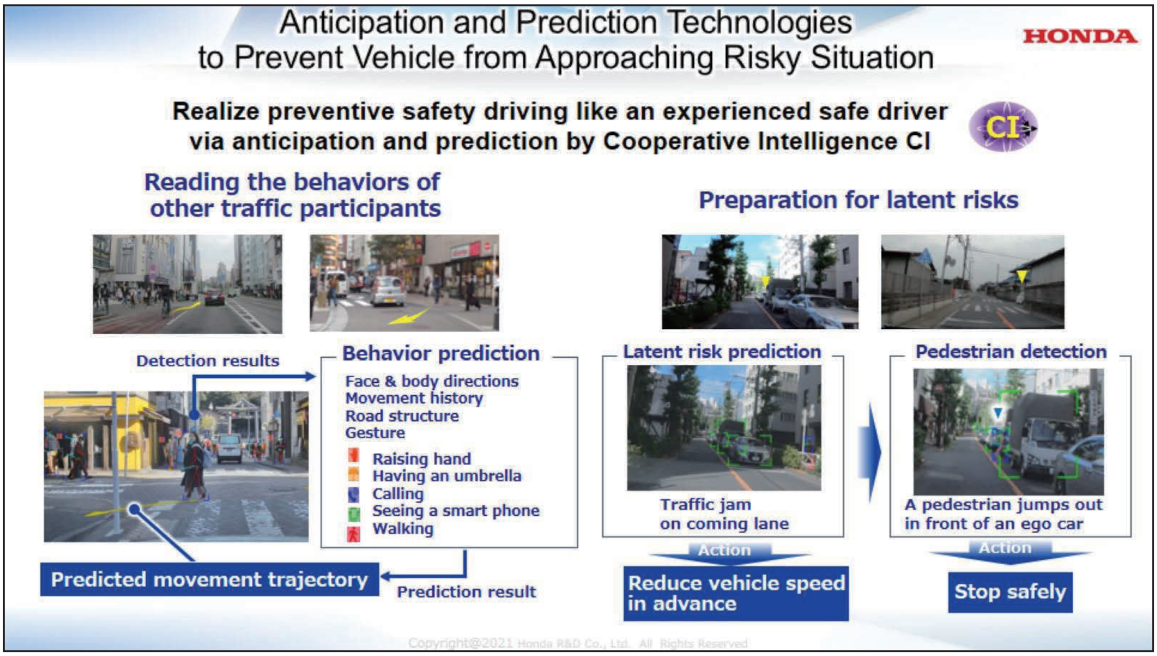
High-way Automated Driving ~ Level 3 ~

Contents

- Honda's 2030 Vision
- **Honda's Automated Driving and Driving Assistance Technology**
  - Automated Driving on Highways
  - **Further Evolution through Utilization of AI Technologies**
- Conclusion


Further Evolution of Automated Driving Technologies
HONDA
Provide safety and confidence by anticipation and prediction
to expand the scope of "freedom of mobility"
Need advanced AI able to coordinate road users in complex traffic scenarios
Suburban driving
Construction area
Merging during a traffic jam
Accident area
Residential street
Own residence
Highway
Sightseeing area
Pedestrians and vehicles suddenly enter the road
(Near-miss scenario)
Commuting road
Supermarket
Station
Hospital
Residential street
Shopping street
Office
Advanced intelligence
Anticipation and prediction to help prevent the vehicle approaching
a risky situation / Coordinated action that can be mutually understood
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

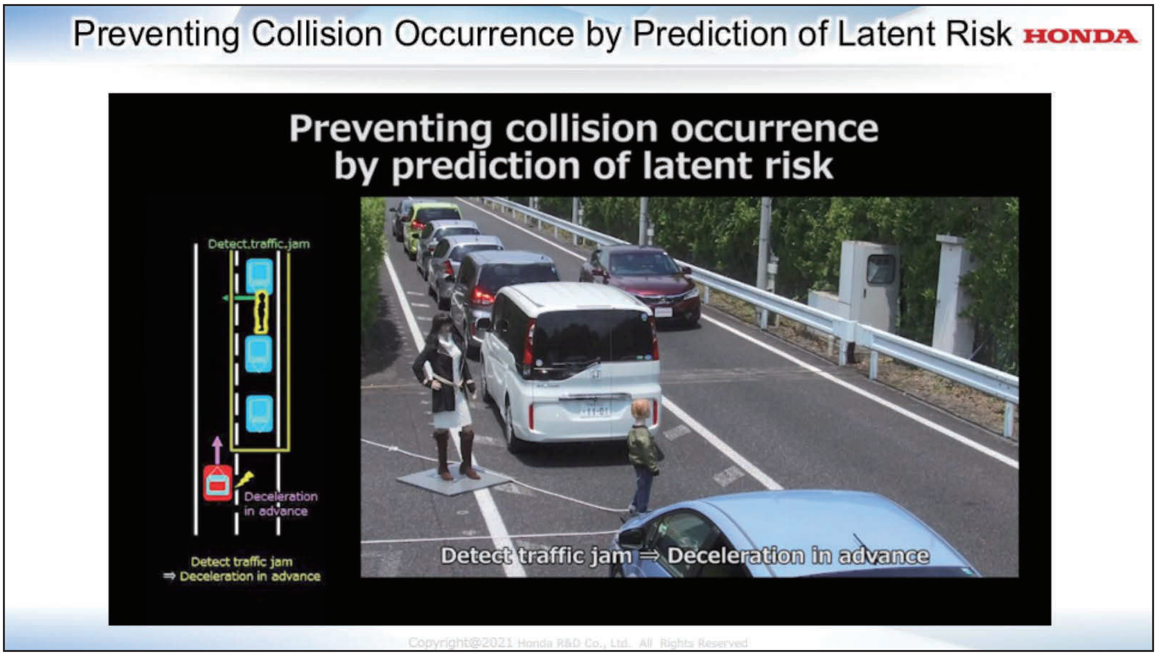

Cooperative Intelligence Technology
HONDA
CI
Cooperative Intelligence
Cooperative action
Human
Cooperative action
Environmental
Intention/
Environment
understanding
Inform intention
(Encouragement)
Machine
Cooperative intelligence
harmonizes with human
in order to realize cooperative
actions and various supports
Environmental understanding
Scene understanding
Behavior prediction
Latent risk prediction
Intention understanding
& Communication
Cooperative action
(Negotiation/Yielding)
Safety and Reliability
from Automated Driving Technology
Safety and Reliability from Automated Driving Technology
will expand human's ability and support cooperative action with others in various scenes
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved



Anticipation and Prediction Technologies
to Prevent Vehicle from Approaching Risky Situation
HONDA
Realize preventive safety driving like an experienced safe driver
via anticipation and prediction by Cooperative Intelligence CI
CI
Reading the behaviors of
other traffic participants
Preparation for latent risks
Detection results
Behavior prediction
Face & body directions
Movement history
Road structure
Gesture
Raising hand
Having an umbrella
Calling
Seeing a smart phone
Walking
Predicted movement trajectory
Prediction result
Latent risk prediction
Traffic jam
on coming lane
Action
Reduce vehicle speed
in advance
Pedestrian detection
A pedestrian jumps out
in front of an ego car
Action
Stop safely
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

Prediction of Pedestrian's Behavior at Shopping District
HONDA
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

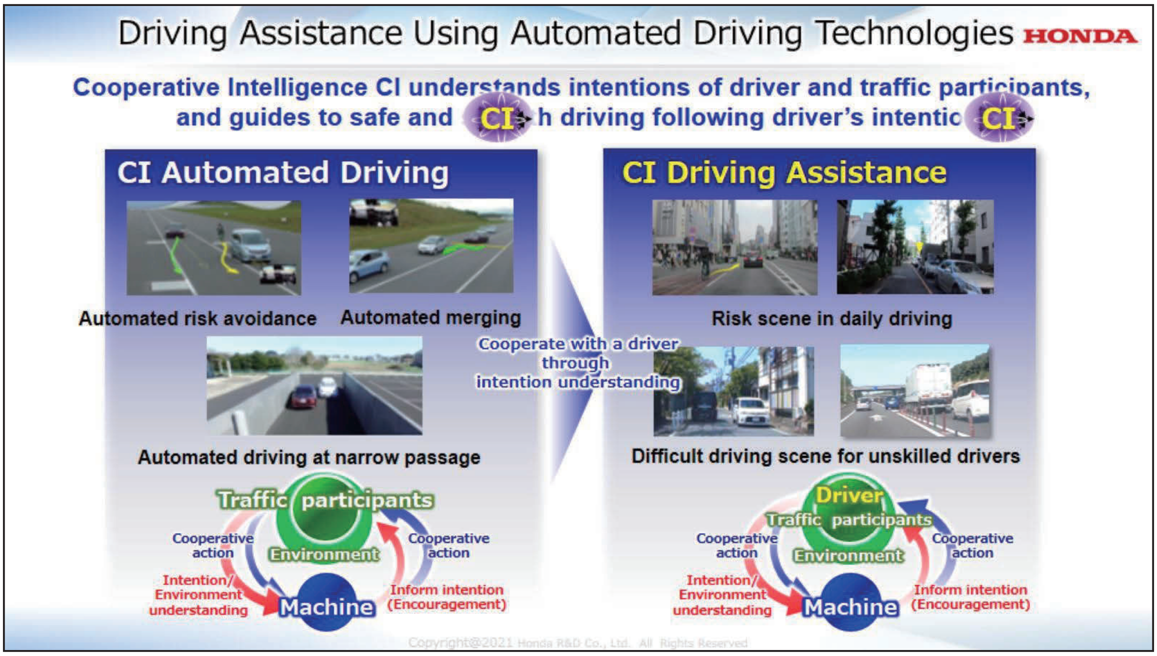

Going Through Shopping District
HONDA
Going Through Shopping District
Cyclist
Slow down and stop
Predict cyclist's crossing road ⇒ Slow down and stop in advance
Predict cyclist's crossing road
⇒ Slow down and stop in advance
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

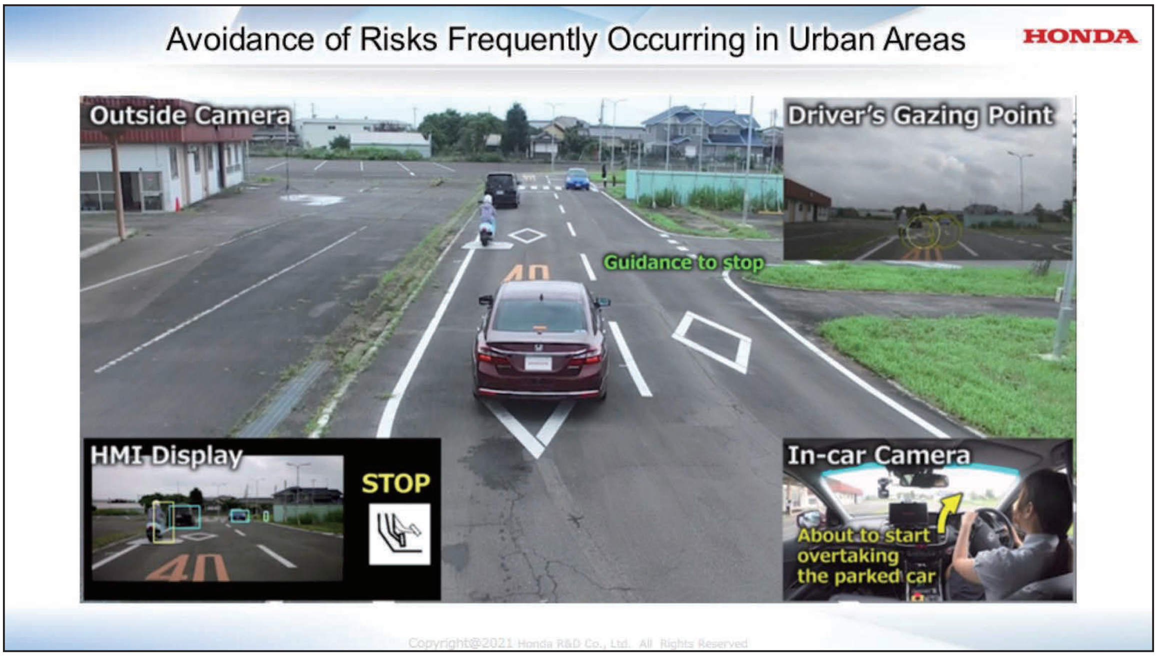

Preventing Collision Occurrence by Prediction of Latent Risk
HONDA
Preventing collision occurrence by prediction of latent risk
Detect traffic jam
Deceleration in advance
Detect traffic jam ⇒ Deceleration in advance
Detect traffic jam ⇒ Deceleration in advance
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Cooperative Action That Can Be Mutually Understood
HONDA
Realize safe and smooth driving like that of an experienced driver via cooperative action with other road users
Negotiating merging during a traffic jam
Mutual giving way in construction area
Conventional
Human
Environmental
Machine
Intention/ Environment understanding
Action
Act in coordination with other road users
Make an appeal to get mutual understanding
Cooperative Intelligence
CI
Cooperative action
Human
Cooperative action
Environmental
Intention/ Environment understanding
Machine
Inform intention (Encouragement)
Make an appeal to surrounding road users and understand intentions mutually, coordinating with surrounding traffics
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Automated Lane-changing to Traffic-jam Lane
HONDA
Negotiation
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Turning Right at Congested Intersection
HONDA
Outside Camera
Go slowly
In-car Camera
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Driving Assistance Using Automated Driving Technologies
HONDA
Cooperative Intelligence CI understands intentions of driver and traffic participants,
and guides to safe and
CI
h driving following driver's intentic
CI
CI Automated Driving
Automated risk avoidance
Automated merging
Automated driving at narrow passage
Traffic participants
Environment
Cooperative action
Cooperative action
Intention/ Environment understanding
Machine
Inform intention (Encouragement)
Cooperate with a driver through intention understanding
CI Driving Assistance
Risk scene in daily driving
Difficult driving scene for unskilled drivers
Driver
Traffic participants
Environment
Cooperative action
Cooperative action
Intention/ Environment understanding
Machine
Inform intention (Encouragement)
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Avoidance of Risks Frequently Occurring in Urban Areas
HONDA
Outside Camera
Driver's Gazing Point
Guidance to stop
HMI Display
STOP
In-car Camera
About to start
overtaking
the parked car
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved


Avoidance of Risks Frequently Occurring in Urban Areas
HONDA
Outside Camera
Driver's Gazing Point
Overlooked risk
Risk prediction
& notification
HMI Display
In-car Camera
Copyright@2021 Honda R&D Co., Ltd. All Rights Reserved

Contents

HONDA

- Honda's 2030 Vision
- Honda's Automated Driving and Driving Assistance Technology
  - Automated Driving on Highways
  - Further Evolution through Utilization of AI Technologies
- **Conclusion**

Conclusion

HONDA

Serve people worldwide with the "joy of expanding their life's potential"

- Lead the advancement of mobility and enable people everywhere in the world to improve their daily lives -

At anytime, from anywhere, to anywhere, we realize "collision-free" and "stress-free" movement of people, and lead the society where individuals can have "joy and freedom of mobility"

World's first High-way automated driving (Level3) was realized in 2021.

Cooperative Intelligence CI assists various people's movement scenes and provide "joy and freedom of mobility."

Freedom of mobility

New joy of mobility


HONDA
The Power of Dreams
Thank you for your kind Attention!

MI レクチャーノートシリーズ刊行にあたり

本レクチャーノートシリーズは、文部科学省 21 世紀 COE プログラム「機能数理学の構築と展開」（H.15-19 年度）において作成した COE Lecture Notes の続刊であり、文部科学省大学院教育改革支援プログラム「産業界が求める数学博士と新修士養成」（H19-21 年度）および、同グローバル COE プログラム「マス・フォア・インダストリ教育研究拠点」（H.20-24 年度）において行われた講義の講義録として出版されてきた。平成 23 年 4 月のマス・フォア・インダストリ研究所（IMI）設立と平成 25 年 4 月の IMI の文部科学省共同利用・共同研究拠点として「産業数学の先進的･基礎的共同研究拠点」の認定を受け、今後、レクチャーノートは、マス・フォア・インダストリに関わる国内外の研究者による講義の講義録、会議録等として出版し、マス・フォア・インダストリの本格的な展開に資するものとする。

平成 30 年 10 月
マス・フォア・インダストリ研究所
所長　佐伯修

SICE-JSAE-AIMaP Tutorial

Advanced Automotive Control and Mathematics

発　行　2021年12月27日
編　集　Taketoshi Kawabe, Yoshihiro Mizoguchi, Junichi Kako, Masakazu Mukai, Yuji Yasui
発　行　九州大学マス・フォア・インダストリ研究所
九州大学大学院数理学府
〒819-0395 福岡市西区元岡744
九州大学数理・IMI 事務室
TEL 092-802-4402　FAX 092-802-4405
URL https://www.imi kyushu-u.ac.jp/

印　刷　城島印刷株式会社
〒810-0012 福岡市中央区白金 2 丁目 9 番 6 号
TEL 092-531-7102　FAX 092-524-4411

シリーズ既刊

Issue	Author／Editor	Title	Published
COE Lecture Note	Mitsuhiro T NAKAO Kazuhiro YOKOYAMA	Computer Assisted Proofs - Numeric and Symbolic Approaches - 199pages	August 22, 2006
COE Lecture Note	M J Shai HARAN	Arithmetical Investigations - Representation theory, Orthogonal polynomials and Quantum interpolations- 174pages	August 22, 2006
COE Lecture Note Vol 3	Michal BENES Masato KIMURA Tatsuyuki NAKAKI	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2005 155pages	October 13, 2006
COE Lecture Note Vol 4	宮田　健治	辺要素有限要素法による磁界解析 - 機能数理学特別講義 21pages	May 15, 2007
COE Lecture Note Vol 5	Francois APERY	Univariate Elimination Subresultants - Bezout formula, Laurent series and vanishing conditions - 89pages	September 25, 2007
COE Lecture Note Vol 6	Michal BENES Masato KIMURA Tatsuyuki NAKAKI	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2006 209pages	October 12, 2007
COE Lecture Note Vol 7	若山　正人 中尾　充宏	九州大学産業技術数理研究センター キックオフミーティング 138pages	October 15, 2007
COE Lecture Note Vol 8	Alberto PARMEGGIANI	Introduction to the Spectral Theory of Non-Commutative Harmonic Oscillators 233pages	January 31, 2008
COE Lecture Note Vol 9	Michael I TRIBELSKY	Introduction to Mathematical modeling 23pages	February 15, 2008
COE Lecture Note Vol 10	Jacques FARAUT	Infinite Dimensional Spherical Analysis 74pages	March 14, 2008
COE Lecture Note Vol 11	Gerrit van DIJK	Gelfand Pairs And Beyond 60pages	August 25, 2008
COE Lecture Note Vol 12	Faculty of Mathematics, Kyushu University	Consortium “MATH for INDUSTRY” First Forum 87pages	September 16, 2008
COE Lecture Note Vol 13	九州大学大学院 数理学研究院	プロシーディング「損保数理に現れる確率モデル」 — 日新火災・九州大学 共同研究2008年11月 研究会 — 82pages	February 6, 2009

シリーズ既刊

Issue	Author／Editor	Title	Published
COE Lecture Note Vol 14	Michal Beneš, Tohru Tsujikawa Shigetoshi Yazaki	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2008 77pages	February 12, 2009
COE Lecture Note Vol 15	Faculty of Mathematics, Kyushu University	International Workshop on Verified Computations and Related Topics 129pages	February 23, 2009
COE Lecture Note Vol 16	Alexander Samokhin	Volume Integral Equation Method in Problems of Mathematical Physics 50pages	February 24, 2009
COE Lecture Note Vol 17	矢嶋　　徹 及川　正行 梶原　健司 辻　　英一 福本　康秀	非線形波動の数理と物理　66pages	February 27, 2009
COE Lecture Note Vol 18	Tim Hoffmann	Discrete Differential Geometry of Curves and Surfaces　75pages	April 21, 2009
COE Lecture Note Vol 19	Ichiro Suzuki	The Pattern Formation Problem for Autonomous Mobile Robots —Special Lecture in Functional Mathematics—　23pages	April 30, 2009
COE Lecture Note Vol 20	Yasuhide Fukumoto Yasunori Maekawa	Math-for-Industry Tutorial: Spectral theories of non-Hermitian operators and their application　184pages	June 19, 2009
COE Lecture Note Vol 21	Faculty of Mathematics, Kyushu University	Forum “Math-for-Industry” Casimir Force, Casimir Operators and the Riemann Hypothesis 95pages	November 9, 2009
COE Lecture Note Vol 22	Masakazu Suzuki Hoon Hong Hirokazu Anai Chee Yap Yousuke Sato Hiroshi Yoshida	The Joint Conference of ASCM 2009 and MACIS 2009: Asian Symposium on Computer Mathematics Mathematical Aspects of Computer and Information Sciences　436pages	December 14, 2009
COE Lecture Note Vol 23	荒川　恒男 金子　昌信	多重ゼータ値入門　111pages	February 15, 2010
COE Lecture Note Vol 24	Fulton B Gonzalez	Notes on Integral Geometry and Harmonic Analysis　125pages	March 12, 2010
COE Lecture Note Vol 25	Wayne Rossman	Discrete Constant Mean Curvature Surfaces via Conserved Quantities 130pages	May 31, 2010
COE Lecture Note Vol 26	Mihai Ciucu	Perfect Matchings and Applications　66pages	July 2, 2010

シリーズ既刊

Issue	Author／Editor	Title	Published
COE Lecture Note Vol 27	九州大学大学院 数理学研究院	Forum "Math-for-Industry" and Study Group Workshop Information security, visualization, and inverse problems, on the basis of optimization techniques　100pages	October 21, 2010
COE Lecture Note Vol 28	ANDREAS LANGER	MODULAR FORMS, ELLIPTIC AND MODULAR CURVES LECTURES AT KYUSHU UNIVERSITY 2010　62pages	November 26, 2010
COE Lecture Note Vol 29	木田　雅成 原田　昌晃 横山　俊一	Magma で広がる数学の世界　157pages	December 27, 2010
COE Lecture Note Vol 30	原　　　隆 松井　　卓 廣島　文生	Mathematical Quantum Field Theory and Renormalization Theory 201pages	January 31, 2011
COE Lecture Note Vol 31	若山　正人 福本　康秀 高木　　剛 山本　昌宏	Study Group Workshop 2010 Lecture & Report　128pages	February 8, 2011
COE Lecture Note Vol 32	Institute of Mathematics for Industry, Kyushu University	Forum "Math-for-Industry" 2011 "TSUNAMI-Mathematical Modelling" Using Mathematics for Natural Disaster Prediction, Recovery and Provision for the Future　90pages	September 30, 2011
COE Lecture Note Vol 33	若山　正人 福本　康秀 高木　　剛 山本　昌宏	Study Group Workshop 2011 Lecture & Report　140pages	October 27, 2011
COE Lecture Note Vol 34	Adrian Muntean Vladimír Chalupecký	Homogenization Method and Multiscale Modeling　72pages	October 28, 2011
COE Lecture Note Vol 35	横山　俊一 夫　　紀恵 林　　卓也	計算機代数システムの進展　210pages	November 30, 2011
COE Lecture Note Vol 36	Michal Beneš Masato Kimura Shigetoshi Yazaki	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2010 107pages	January 27, 2012
COE Lecture Note Vol 37	若山　正人 高木　　剛 Kirill Morozov 平岡　裕章 木村　正人 白井　朋之 西井　龍映 栄　伸一郎 穴井　宏和 福本　康秀	平成23年度 数学・数理科学と諸科学・産業との連携研究ワークショップ　拡がっていく数学　～期待される"見えない力"～ 154pages	February 20, 2012

シリーズ既刊

Issue	Author／Editor	Title	Published
COE Lecture Note Vol 38	Fumio Hiroshima Itaru Sasaki Herbert Spohn Akito Suzuki	Enhanced Binding in Quantum Field Theory　204pages	March 12, 2012
COE Lecture Note Vol 39	Institute of Mathematics for Industry, Kyushu University	Multiscale Mathematics; Hierarchy of collective phenomena and interrelations between hierarchical structures　180pages	March 13, 2012
COE Lecture Note Vol 40	井ノ口順一 太田　泰広 筧　　三郎 梶原　健司 松浦　　望	離散可積分系・離散微分幾何チュートリアル2012　152pages	March 15, 2012
COE Lecture Note Vol 41	Institute of Mathematics for Industry, Kyushu University	Forum “Math-for-Industry” 2012 “Information Recovery and Discovery”　91pages	October 22, 2012
COE Lecture Note Vol 42	佐伯　　修 若山　正人 山本　昌宏	Study Group Workshop 2012 Abstract, Lecture & Report　178pages	November 19, 2012
COE Lecture Note Vol 43	Institute of Mathematics for Industry, Kyushu University	Combinatorics and Numerical Analysis Joint Workshop　103pages	December 27, 2012
COE Lecture Note Vol 44	萩原　　学	モダン符号理論からポストモダン符号理論への展望　107pages	January 30, 2013
COE Lecture Note Vol 45	金山　　寛	Joint Research Workshop of Institute of Mathematics for Industry (IMI), Kyushu University “Propagation of Ultra-large-scale Computation by the Domain-decomposition-method for Industrial Problems (PUCDIP 2012)” 121pages	February 19, 2013
COE Lecture Note Vol 46	西井　龍映 栄　伸一郎 岡田　勘三 落合　啓之 小磯　深幸 斎藤　新悟 白井　朋之	科学・技術の研究課題への数学アプローチ —数学モデリングの基礎と展開—　325pages	February 28, 2013
COE Lecture Note Vol 47	SOO TECK LEE	BRANCHING RULES AND BRANCHING ALGEBRAS FOR THE COMPLEX CLASSICAL GROUPS　40pages	March 8, 2013
COE Lecture Note Vol 48	溝口　佳寛 脇　　隼人 平坂　　貢 谷口　哲至 島袋　　修	博多ワークショップ「組み合わせとその応用」　124pages	March 28, 2013

シリーズ既刊

Issue	Author／Editor	Title	Published
COE Lecture Note Vol 49	照井　　章 小原　功任 濱田　龍義 横山　俊一 穴井　宏和 横田　博史	マス・フォア・インダストリ研究所　共同利用研究集会 II 数式処理研究と産学連携の新たな発展　137pages	August 9, 2013
MI Lecture Note Vol 50	Ken Anjyo Hiroyuki Ochiai Yoshinori Dobashi Yoshihiro Mizoguchi Shizuo Kaji	Symposium MEIS2013: Mathematical Progress in Expressive Image Synthesis　154pages	October 21, 2013
MI Lecture Note Vol 51	Institute of Mathematics for Industry, Kyushu University	Forum “Math-for-Industry” 2013 “The Impact of Applications on Mathematics”　97pages	October 30, 2013
MI Lecture Note Vol 52	佐伯　　修 岡田　勘三 髙木　　剛 若山　正人 山本　昌宏	Study Group Workshop 2013 Abstract, Lecture & Report 142pages	November 15, 2013
MI Lecture Note Vol 53	四方　義啓 櫻井　幸一 安田　貴徳 Xavier Dahan	平成25年度　九州大学マス・フォア・インダストリ研究所 共同利用研究集会　安全・安心社会基盤構築のための代数構造 ～サイバー社会の信頼性確保のための数理学～　158pages	December 26, 2013
MI Lecture Note Vol 54	Takashi Takiguchi Hiroshi Fujiwara	Inverse problems for practice, the present and the future　93pages	January 30, 2014
MI Lecture Note Vol 55	栄　伸一郎 溝口　佳寛 脇　　隼人 渋田　敬史	Study Group Workshop 2013 数学協働プログラム Lecture & Report 98pages	February 10, 2014
MI Lecture Note Vol 56	Yoshihiro Mizoguchi Hayato Waki Takafumi Shibuta Tetsuji Taniguchi Osamu Shimabukuro Makoto Tagami Hirotake Kurihara Shuya Chiba	Hakata Workshop 2014 ~ Discrete Mathematics and its Applications ~　141pages	March 28, 2014
MI Lecture Note Vol 57	Institute of Mathematics for Industry, Kyushu University	Forum “Math-for-Industry” 2014: “Applications + Practical Conceptualization + Mathematics = fruitful Innovation”　93pages	October 23, 2014
MI Lecture Note Vol 58	安生健一 落合啓之	Symposium MEIS2014: Mathematical Progress in Expressive Image Synthesis　135pages	November 12, 2014

シリーズ既刊

Issue	Author／Editor	Title	Published
MI Lecture Note Vol 59	西井　龍映 岡田　勘三 梶原　健司 髙木　　剛 若山　正人 脇　　隼人 山本　昌宏	Study Group Workshop 2014 数学協働プログラム Abstract, Lecture & Report　196pages	November 14, 2014
MI Lecture Note Vol 60	西浦　　博	平成26年度九州大学 IMI 共同利用研究・研究集会（I） 感染症数理モデルの実用化と産業及び政策での活用のための新たな展開　120pages	November 28, 2014
MI Lecture Note Vol 61	溝口　佳寛 Jacques Garrigue 萩原　　学 Reynald Affeldt	研究集会 高信頼な理論と実装のための定理証明および定理証明器 Theorem proving and provers for reliable theory and implementations (TPP2014)　138pages	February 26, 2015
MI Lecture Note Vol 62	白井　朋之	Workshop on “β-transformation and related topics”　59pages	March 10, 2015
MI Lecture Note Vol 63	白井　朋之	Workshop on “Probabilistic models with determinantal structure” 107pages	August 20, 2015
MI Lecture Note Vol 64	落合　啓之 土橋　宜典	Symposium MEIS2015: Mathematical Progress in Expressive Image Synthesis　124pages	September 18, 2015
MI Lecture Note Vol 65	Institute of Mathematics for Industry, Kyushu University	Forum “Math-for-Industry” 2015 “The Role and Importance of Mathematics in Innovation”　74pages	October 23, 2015
MI Lecture Note Vol 66	岡田　勘三 藤澤　克己 白井　朋之 若山　正人 脇　　隼人 Philip Broadbridge 山本　昌宏	Study Group Workshop 2015 Abstract, Lecture & Report 156pages	November 5, 2015
MI Lecture Note Vol 67	Institute of Mathematics for Industry, Kyushu University	IMI-La Trobe Joint Conference “Mathematics for Materials Science and Processing” 66pages	February 5, 2016
MI Lecture Note Vol 68	古庄　英和 小谷　久寿 新甫　洋史	結び目と Grothendieck-Teichmüller 群 116pages	February 22, 2016
MI Lecture Note Vol 69	土橋　宜典 鍛治　静雄	Symposium MEIS2016: Mathematical Progress in Expressive Image Synthesis　82pages	October 24, 2016
MI Lecture Note Vol 70	Institute of Mathematics for Industry, Kyushu University	Forum “Math-for-Industry” 2016 “Agriculture as a metaphor for creativity in all human endeavors” 98pages	November 2, 2016
MI Lecture Note Vol 71	小磯　深幸 二宮　嘉行 山本　昌宏	Study Group Workshop 2016 Abstract, Lecture & Report　143pages	November 21, 2016

シリーズ既刊

Issue	Author／Editor	Title	Published
MI Lecture Note Vol 72	新井　朝雄 小嶋　　泉 廣島　文生	Mathematical quantum field theory and related topics　133pages	January 27, 2017
MI Lecture Note Vol 73	穴田　啓晃 Kirill Morozov 須賀　祐治 奥村　伸也 櫻井　幸一	Secret Sharing for Dependability, Usability and Security of Network Storage and Its Mathematical Modeling　211pages	March 15, 2017
MI Lecture Note Vol 74	QUISPEL, G Reinout W BADER, Philipp MCLAREN, David I TAGAMI, Daisuke	IMI-La Trobe Joint Conference Geometric Numerical Integration and its Applications　71pages	March 31, 2017
MI Lecture Note Vol 75	手塚　　集 田上　大助 山本　昌宏	Study Group Workshop 2017 Abstract, Lecture & Report　118pages	October 20, 2017
MI Lecture Note Vol 76	宇田川誠一	Tzitzéica 方程式の有限間隙解に付随した極小曲面の構成理論 ―Tzitzéica 方程式の楕円関数解を出発点として―　68pages	August 4, 2017
MI Lecture Note Vol 77	松谷　茂樹 佐伯　　修 中川　淳一 田上　大助 上坂　正晃 Pierluigi Cesana 濵田　裕康	平成29年度　九州大学マス・フォア・インダストリ研究所 共同利用研究集会（I） 結晶の界面，転位，構造の数理　148pages	December 20, 2017
MI Lecture Note Vol 78	瀧澤　重志 小林　和博 佐藤憲一郎 斎藤　　努 清水　正明 間瀬　正啓 藤澤　克樹 神山　直之	平成29年度　九州大学マス・フォア・インダストリ研究所 プロジェクト研究　研究集会（I） 防災・避難計画の数理モデルの高度化と社会実装へ向けて 136pages	February 26, 2018
MI Lecture Note Vol 79	神山　直之 畔上　秀幸	平成29年度　AIMaP チュートリアル 最適化理論の基礎と応用　96pages	February 28, 2018
MI Lecture Note Vol 80	Kirill Morozov Hiroaki Anada Yuji Suga	IMI Workshop of the Joint Research Projects Cryptographic Technologies for Securing Network Storage and Their Mathematical Modeling　116pages	March 30, 2018
MI Lecture Note Vol 81	Tsuyoshi Takagi Masato Wakayama Keisuke Tanaka Noboru Kunihiro Kazufumi Kimoto Yasuhiko Ikematsu	IMI Workshop of the Joint Research Projects International Symposium on Mathematics, Quantum Theory, and Cryptography　246pages	September 25, 2019
MI Lecture Note Vol 82	池森　俊文	令和２年度　AIMaP チュートリアル 新型コロナウイルス感染症にかかわる諸問題の数理 145pages	March 22, 2021

シリーズ既刊

Issue	Author／Editor	Title	Published
MI Lecture Note Vol 83	早川健太郎 軸丸　芳揮 横須賀洋平 可香谷　隆 林　　和希 堺　　雄亮	シェル理論・膜理論への微分幾何学からのアプローチとその建築曲面設計への応用　49pages	July 28, 2021

Institute of Mathematics for Industry
Kyushu University

九州大学マス・フォア・インダストリ研究所
九州大学大学院 数理学府
〒819-0395 福岡市西区元岡744　TEL 092-802-4402　FAX 092-802-4405
URL http://www.imi.kyushu-u.ac.jp/