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Hakata Workshop 2014
~Discrete Mathematics and its Applications~

Editors : Yoshihiro Mizoguchi, Hayato Waki, Takafumi Shibuta, Tetsuji Taniguchi
Osamu Shimabukuro, Makoto Tagami, Hirotake Kurihara, Shuya Chiba

九州大学マス・フォア・インダストリ研究所

MI Lecture Note Vol.56 : Kyushu University

Hakata Workshop 2014

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About MI Lecture Note Series

The Math-for-Industry (MI) Lecture Note Series is the successor to the COE Lecture Notes, which were published for the 21st COE Program "Development of Dynamic Mathematics with High Functionality," sponsored by Japan's Ministry of Education, Culture, Sports, Science and Technology (MEXT) from 2003 to 2007. The MI Lecture Note Series has published the notes of lectures organized under the following two programs: "Training Program for Ph.D. and New Master's Degree in Mathematics as Required by Industry," adopted as a Support Program for Improving Graduate School Education by MEXT from 2007 to 2009; and "Education-and-Research Hub for Mathematics-for-Industry," adopted as a Global COE Program by MEXT from 2008 to 2012.

In accordance with the establishment of the Institute of Mathematics for Industry (IMI) in April 2011 and the authorization of IMI's Joint Research Center for Advanced and Fundamental Mathematics-for-Industry as a MEXT Joint Usage / Research Center in April 2013, hereafter the MI Lecture Notes Series will publish lecture notes and proceedings by worldwide researchers of MI to contribute to the development of MI.

September 2013
Masato Wakayama
Director
Institute of Mathematics for Industry

Hakata Workshop 2014

~ Discrete Mathematics and its Applications ~

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Preface

Computer science, we use the word computer mathematics, is a kind of study about computers. Sometimes it relates to the fields such as natural science, economics and philosophy. Discrete mathematics is an important section in computer science including graph theory, combinatorics theory and optimization theory. Practical applications of these theory using computer software are more and more expected in the industrial world.

This lecture note is collecting several presentation slides of the

Hakata Workshop 2014

~ Discrete Mathematics and its Applications ~

held on 8th February 2014 at Fukuoka. Six speakers gave talks and seven speakers gave demonstrations of software in Mathematics. The workshop had about 40 participants including overseas researchers and international students.

The topics talked in this workshop was planned to include combinatorics theories, numerical computation and symbolical computations. We also have software in Mathematics demonstration track, which main goal is to provide a forum to discuss with a live presentation of any kinds of software program in Mathematics for research students and practitioners. We do not limit the area of Mathematics in the software track to extend the topics to not only Mathematics but also a novel idea and a prototype program itself in early stages. After attractive talks and demonstrations, we spent the time for endless discussions including a dream of future Mathematics, developments of computer programs and combinatorics theory.

We hope that all the participants, including speakers enjoyed this workshop and found new discovery both theories and communications.

This workshop is supported by

- Laboratory of Advanced Software in Mathematics, Institute of Mathematics for Industry, Kyushu University,
- JSPS Grant-in-Aid for Exploratory Research No.25610034,
- JSPS Grant-in-Aid for Scientific Research (C) No.25400217.

We also would like to express our gratitude to the secretarial staffs for their helping in editing this lecture notes.

Yoshihiro Mizoguchi (Kyushu University)
Hayato Waki (Kyushu University)
Takafumi Shibuta (Kyushu University)
Tetsuji Taniguchi (Matsue College of Technology)
Osamu Shimabukuro (Nagasaki University)
Makoto Tagami (Kyushu Institute of Technology)
Hirotake Kurihara (Kitakyushu National College of Technology)
Shuya Chiba (Kumamoto Univeersity)

Fukuoka, February 2014

Hakata Workshop 2014
~ Discrete Mathematics and its Applications ~

Date: February 8, 2014. 9:15-17:30
Venue: Seminar Room R, Reference Eki Higashi Building
(1-16-14 Hakata-Eki-Higashi, Hakata-ku, Fukuoka)

Program:

9:15～9:20	Opening (Tetsuji Taniguchi)
9:20～10:00	On Halin graphs and generalized Halin graphs Shoichi Tsuchiya (Tokyo University of Science)
10:10～10:50	On the number of components of 2-factors in claw-free graphs Shuya Chiba (Kumamoto University)
11:00～11:40	On complementary Ramsey numbers Masashi Shinohara (Shiga University)
13:10～14:40	***Software in Mathematics Demonstration Track***
15:00～15:40	Graph homomorphisms and de Branges Rovnyak theory Michio Seto (Shimane University)
15:50～16:30	The convergence of relaxed functional iterations for solving quadratic matrix equations with an M-matrix Jong Hyeon Seo (Pusan National University)
16:40～17:20	Computing resultant matrix of general multivariate polynomials and its determinant using Magma Shun'ichi Yokoyama (Kyushu University)
17:20～17:30	Closing (Yoshihiro Mizoguchi)

Software in Mathematics Demonstration Track

HEAP モデル法によるプル型スケジューリングプログラム
岩下 寛弥（九州大学大学院工学府海洋システム工学専攻）

構文解析に特化した翻訳ソフト
山岡 幸高（九州大学数理学府）

TRDRD に基づくサッカーの分析プログラム
大塚 寛（愛媛大学理工学研究科）

A Java-based simulation-oriented design for robust financial portfolios
Omar Rifki (Department of Economic Engineering, Kyushu University)

ラプラシアン固有マップ法における評価方法及びその応用
吉野 聖人（松江工業高等専門学校 電子制御工学科）

Coq Modules for Automata and Sticker Systems
田中 久治（佐賀大学大学院工学系研究科）

The monotone convergence of Newton’s method for differentiable convex matrix functions
Sang-Hyup Seo(Department of Mathematics, Pusan National University)


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Software in Mathematics Demonstration Track

Workshop

On Halin graphs and generalized Halin graphs

Shoichi Tsuchiya（Tokyo University of Science）

This talk is based on joint works with
Guantao Chen (Georgia State University)
Hikoe Enomoto (Waseda University)
Suil O (Georgia State University)
Kenta Ozeki (National Institute of Informatics)

Halin graph

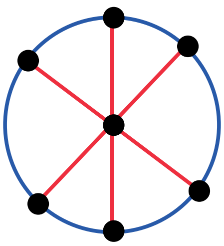
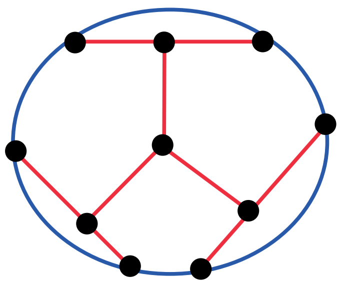
HIST : a spanning tree with no vertices of deg. 2.

Halin graph *H* :

a plane graph $T \cup C$ with order at least 4 s.t.

(i) *T* is a HIST of *H*,

(ii) *C* is a cycle where *V*(*C*) is the set of leaves of *T*.

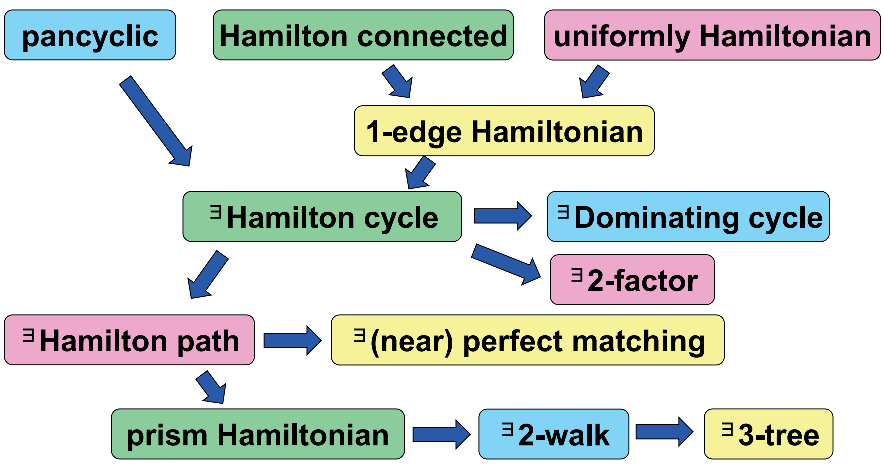
Problem

Problem 1
Which graph has a Halin graph as a span. sub.?

spanning Halin subgraph

Theorem 2 (Horton, Parker and Borie, 1992)
Deciding whether a graph has a span. Halin subgraph is NP-complete.

***G* : graph**



Known result 1

Theorem 3 (Bondy, 1973)
Every Halin graph is Hamiltonian.

Theorem 4 (Barefoot, 1987)
Every Halin graph is Hamilton connected.

Theorem 5 (Skupien, 1990)
Every Halin graph H is uniformly Hamiltonian.

For each $e \in E(H)$, H has two Hamilton cycles C_1, C_2 such that $e \in E(C_1)$ and $e \notin E(C_2)$.

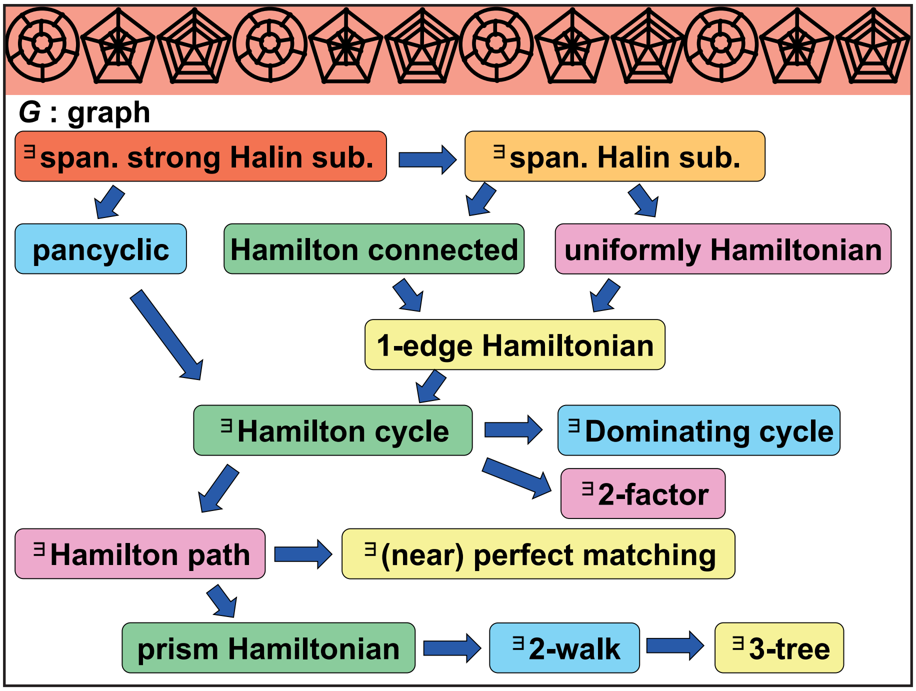
Known result 2

Thm 6 (Bondy and Lovász 1985, Skowrońska 1985)
Every Halin graph H is almost pancyclic.

For each l ($3 \leqq l \leqq |H|$), H has a cycle of length l other than one exception.

Thm 7 (Bondy and Lovász 1985, Skowrońska 1985)
H ($=T \cup C$) : Halin graph.
T has no vertices of deg. 3 $\Rightarrow H$ is pancyclic.

strong Halin graph



Application of Halin graph

Thm 8 (Cornuéjols, Naddef and Pulleyblank, 1983)
In Halin graphs, a Hamilton cycle can be found in polynomial time.

Chang, Wang, Tsai and Yang (2012) use a Halin graph in order to reduce the total energy consumption of the Wireless Sensor Network.

Nes. conditions for span. Halin sub.

G : simple graph.
G has a spanning Halin subgraph ⇒

(i) G has K_3 as a subgraph,
(ii) G is Hamiltonian, and
(iii) G has a HIST.

∃ 3-connected plane triangulation with no Hamilton cycle. (Moon and Moser, 1963)

∃ 3-connected plane graph with no HIST. (Joffe,1982)

∀ 4-connected plane graph is Hamiltonian. (Tutte, 1956)

∀ plane triangulation has a HIST. (Albertson, Berman, Hutchinson and Thomassen, 1990)

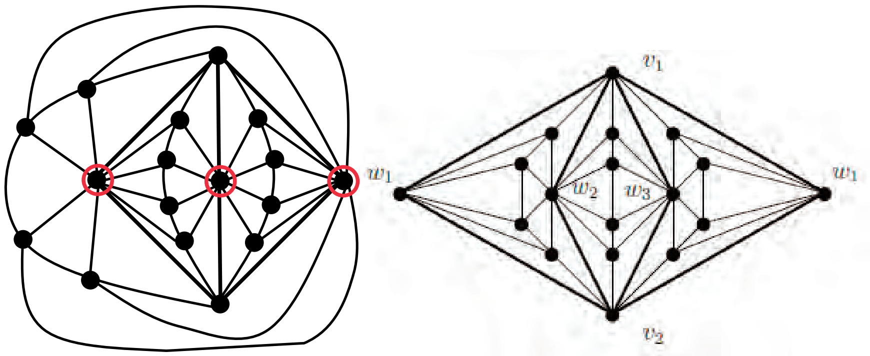
Lovász-Plummer conjecture

Conjecture 9 (Lovász and Plummer, 1975)
Every 4-connected plane triangulation has a spanning Halin subgraph.

Theorem 10 (Chen, Enomoto, Ozeki and T, 2014+)
∃ an infinite family of 4-connected plane tri. with no spanning Halin subgraph.

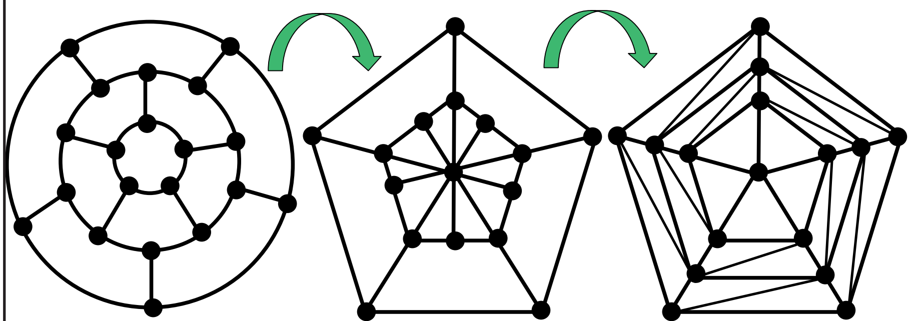
Counterexample

Theorem 10 (Chen, Enomoto, Ozeki and T, 2014+)
∃ an infinite family of 4-connected plane tri.
with no spanning Halin subgraph.

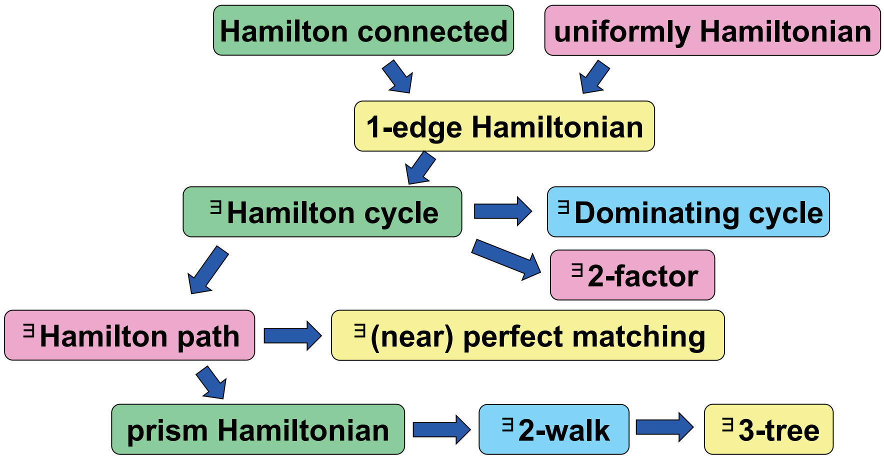


Counterexample 2

Theorem 11 (Chen, Enomoto, Ozeki and T, 2014+)
∃ an infinite family of 5-connected plane tri.
with no spanning Halin subgraph.



Properties of Halin graphs



Generalized Halin graph

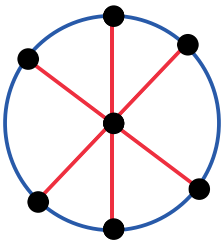
HIST : a spanning tree with no vertices of deg. 2.

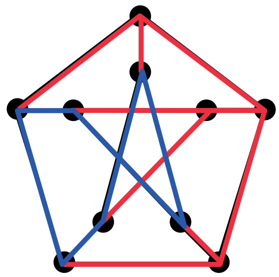
Generalized Halin graph *H* :

a ~~plane~~ graph $T \cup C$ with order at least 4 s.t.

(i) *T* is a HIST of *H*,

(ii) *C* is a cycle where *V*(*C*) is the set of leaves of *T*.



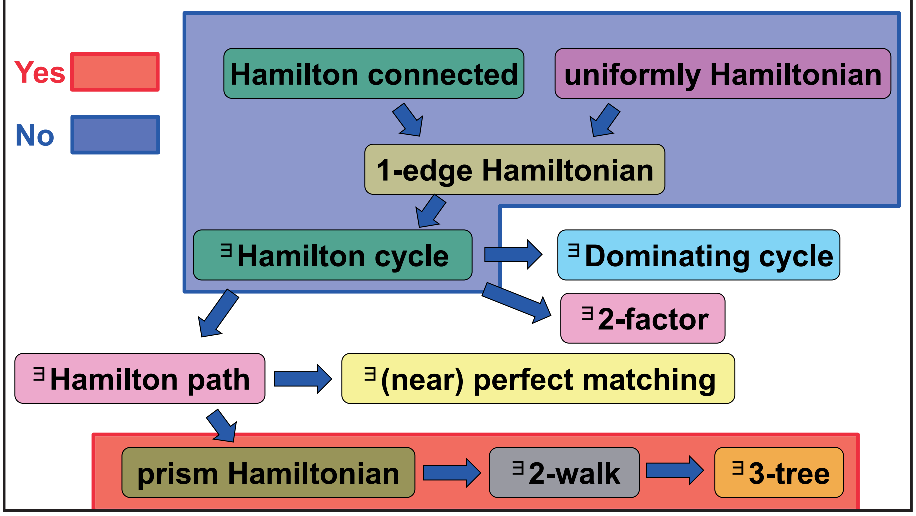


Known result 3

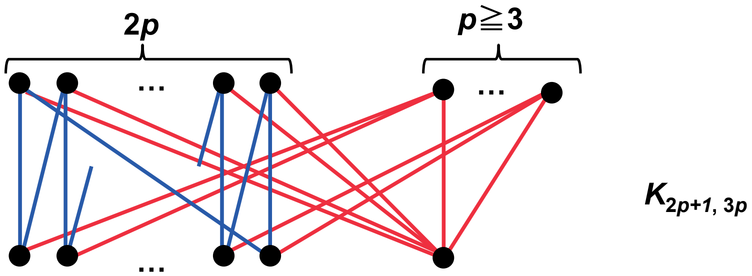
Definition
A graph G is prism Hamiltonian
$\underset{\text{def}}{\Leftrightarrow}$ cartesian product of G and K_2 is Hamiltonian.

Theorem 12 (Kaiser et. al. , 1990)
Every gene. Halin graph is prism Hamiltonian.

Properties of gene. Halin graphs



Construction



Definition
Matching number of a graph *G*
$\overset{\Leftrightarrow}{\text{def}}$ maximum size of a matching in *G*.

***G* has a perfect matching**
⇔ matching number of *G* is |*G*|/2.

Matching in generalized Halin graphs

Theorem 13 (Chen, O and T, 2014+) *Best possible!*
***G* : generalized Halin graph of order *n*.**
The matching number of *G* is at least $(2n+3)/5$.

Theorem 14 (Chen, O and T, 2014+)
***G* : generalized Halin graph of order *n*.**
$\lambda_1(G)<3 \Rightarrow G$ has a (near) perfect matching.

Corollary 15
Every cubic generalized Halin graph has a perfect matching.

Lemma

Theorem 14 (Chen, O and T, 2014+)
G : generalized Halin graph of order n.
$\lambda_1(G)<3 \Rightarrow G$ has a (near) perfect matching.

Lemma 16
G : a graph such that $|V(G)|=n$ and $|E(G)|=m$.
$\lambda_1(G) \geqq 2m/n$

Theorem 14 may not be best possible.

Thank you for attention!

On the number of components of 2-factors in claw-free graphs

Kumamoto University
Shuya Chiba

based on joint-work with
Roman Čada, Kenta Ozeki, Petr Vrána, Kiyoshi Yoshimoto

February 8, 2014

- Definitions
- Matthews and Sumner's Conjecture
- Upper bounds on the number of components of 2-factors
- Implications and related results

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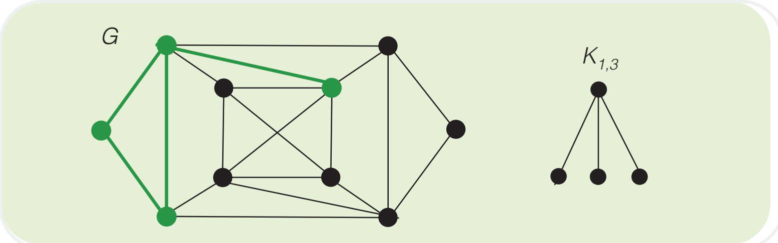
Definitions

G : graph

- $V(G)$: the vertex set of G ($n := |V(G)|$)
- $\delta := \delta(G)$: the minimum degree of G
- G : *Hamiltonian* $\overset{\text{def.}}{\Leftrightarrow}$ G has a Hamilton cycle (cycle containing all vertices)
- G : *claw-free* $\overset{\text{def.}}{\Leftrightarrow}$ G has no induced subgraph isomorphic to $K_{1,3}$
- *2-factor* of G $\overset{\text{def.}}{\Leftrightarrow}$ span. subgraph of G in which $\forall$compo. is a cycle



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- Definitions
- Matthews and Sumner's Conjecture
- Upper bounds on the number of components of 2-factors
- Implications and related results

MS-Conjecture

Conj. (Matthews&Sumner1984)

G : 4-conn. claw-free ⇒ G : Hamiltonian

Conj. (Chvátal1973)

$\exists t_0$ s.t. G : t_0 - tough ⇒ G : Hamiltonian

Conj. (Chvátal1973)

For $\forall t > 3/2$, G : t - tough ⇒ G : Hamiltonian

- G : t-tough $\overset{\text{def.}}{\Leftrightarrow}$ $t \cdot \omega(G - S) \leq |S|$ for $\forall S \subseteq V(G)$
- G : Hamiltonian ⇒ $\omega(G - S) \leq |S|$ for $\forall S \subseteq V(G)$
- G : claw-free

  G : 2-tough ⇔ G : 4-connected

MS-Conjecture

Conj. (Matthews&Sumner1984)

G : 4-conn. claw-free ⇒ G : Hamiltonian

【claw-free graph G has a 2-factor ??】

- if $\delta \geq 4$ ⇒ Yes (Egawa&Ota1991)
- if G : 2-conn. $\delta \geq 3$ ⇒ Yes (Yoshimoto2007)

Thm. (Broersma et al.2001)

If $^{\exists}$function $f(n)$ of n s.t.

$\lim_{n \to \infty} f(n) / n = 0$

$^{\forall}$4-conn. claw-free graph of order n has a 2-factor with $\leq f(n)$ compo.s

⇒ MS-conjecture is true

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Upper bounds

Conj. (Matthews&Sumner1984)

G : 4-conn. claw-free ⇒ G : Hamiltonian

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- if G : 2-conn. $\delta \geq 3$ ⇒ Yes (Yoshimoto2007)

【Upper bounds on # of compo.s of 2-factors】

	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta \geq 6$
conn.	×	×	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)
2-conn.	×	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)
3-conn.	-	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)

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Upper bounds

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Thm. (Faudree et al.1999)

$\delta \geq 4$ ⇒ $\exists$2-factor

$\leq$ $6(n-1)/(\delta+2)$ compo.s

【Upper bounds on # of compo.s of 2-factors】

	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta \geq 6$
conn.	×	×	n / 3 (trivial)	n / 3 (trivial)	$6(n-1)/(\delta+2)$ (Faudree et al.1999)
2-conn.	×	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)
3-conn.	-	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)

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Upper bounds

Conj. (Matthews&Sumner1984)

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- if $\delta \geq 4$ ⇒ Yes (Egawa&Ota1991)
- if *G* : 2-conn. $\delta \geq 3$ ⇒ Yes (Yoshimoto2007)

Thm. (Broersma et al.2009)

$\delta \geq 5$ ⇒ ∃2-factor $\leq$ (n-3) / (δ - 1) compo.s

【Upper bounds on # of compo.s of 2-factors】

	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta \geq 6$
conn.	×	×	n / 3 (trivial)	n / 4 (Broersma et al.2009)	(n-3)/(δ - 1) (Broersma et al.2009)
2-conn.	×	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)
3-conn.	-	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)	n / 3 (trivial)

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- if *G* : 2-conn. $\delta \geq 3$ ⇒ Yes (Yoshimoto2007)

Thm. (Jackson&Yoshimoto2007)

- *G* : 2-conn. $\delta \geq 4$
  ⇒ ∃2-factor $\leq$ (n+1) / 4 compo.s
- *G* : 3-conn.
  ⇒ ∃2-factor $\leq$ 2n / 15 compo.s

【Upper bounds on # of compo.s of 2-factors】

	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta \geq 6$
conn.	×	×	n / 3 (trivial)	n / 4 (Broersma et al.2009)	(n-3)/(δ - 1) (Broersma et al.2009)
2-conn.	×	n / 3 (trivial)	(n+1) / 4 (Jackson et al.2007)	(n+1) / 4 (Jackson et al.2007)	(n+1) / 4 (Jackson et al.2007)
3-conn.	-	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)

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Upper bounds

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- if G : 2-conn. $\delta \geq 3$ ⇒ Yes (Yoshimoto2007)

Thm. (Kužel et al.2012)

G : 3-conn.

⇒ $\exists$2-factor ≤ $n / (\delta + 2)$ compo.s

【Upper bounds on # of compo.s of 2-factors】

	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta \geq 6$
conn.	×	×	n / 3 (trivial)	n / 4 (Broersma et al.2009)	(n-3)/(δ - 1) (Broersma et al.2009)
2-conn.	×	n / 3 (trivial)	(n+1) / 4 (Jackson et al.2007)	(n+1) / 4 (Jackson et al.2007)	(n+1) / 4 (Jackson et al.2007)
3-conn.	-	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	n / (δ + 2) (Kužel et al.2012)

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Upper bounds

Thm. (Čada, C., Yoshimoto)

G : claw-free, $\delta \geq 4$ ⇒ $\exists$2-factor s.t. $\forall$cycle contains ≥ (δ - 1) /2 vertices

≥ (δ + 1) /2 ???

Thm. (Čada, C., Yoshimoto)

G : 2-conn. claw-free, $\delta \geq 3$ ⇒ $\exists$2-factor s.t. $\forall$cycle contains ≥ δ vertices

Thm. (Ando et al. 2002)

G : claw-free ⇒ $\exists$path-factor s.t. $\forall$path contains ≥ δ + 1 vertices

Conj. (Ando et al. 2002)

G : 2-conn. claw-free ⇒ $\exists$path-factor s.t. $\forall$path contains ≥ 3 δ + 3 vertices

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Upper bounds

Conj. (Matthews&Sumner1984)

G : 4-conn. claw-free ⇒ G : Hamiltonian

【claw-free graph G has a 2-factor ??】

・if δ ≥ 4 ⇒ Yes (Egawa&Ota1991)

・if G : 2-conn. δ ≥ 3 ⇒ Yes (Yoshimoto2007)

∃2-factor s.t. ∀cycle contains ≥ δ vertices

Thm. (Čada, C. Yoshimoto)

G : 2-conn.

⇒ ∃2-factor ≤ n / δ compo.s

【Upper bounds on # of compo.s of 2-factors】

	δ = 2	δ = 3	δ = 4	δ = 5	δ ≥ 6
conn.	×	×	n / 3 (trivial)	n / 4 (Broersma et al.2009)	(n-3)/(δ - 1) (Broersma et al.2009)
2-conn.	×	n / 3 (trivial)	n / 4 (CCY)	n / 5 (CCY)	n / δ (CCY)
3-conn.	-	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	n / (δ + 2) (Kužel et al.2012)

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Conj. (Matthews&Sumner1984)

G : 4-conn. claw-free ⇒ G : Hamiltonian

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・if δ ≥ 4 ⇒ Yes (Egawa&Ota1991)

・if G : 2-conn. δ ≥ 3 ⇒ Yes (Yoshimoto2007)

Thm. (Ozeki, Ryjáček, Yoshimoto)

G : 3-conn.

⇒ ∃2-factor ≤ 4n / 5(δ + 2) compo.s

【Upper bounds on # of compo.s of 2-factors】

	δ = 2	δ = 3	δ = 4	δ = 5	δ ≥ 6
conn.	×	×	n / 3 (trivial)	n / 4 (Broersma et al.2009)	(n-3)/(δ - 1) (Broersma et al.2009)
2-conn.	×	n / 3 (trivial)	n / 4 (CCY)	n / 5 (CCY)	n / δ (CCY)
3-conn.	-	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	4n / 5(δ + 2) (Ozeki et al.)

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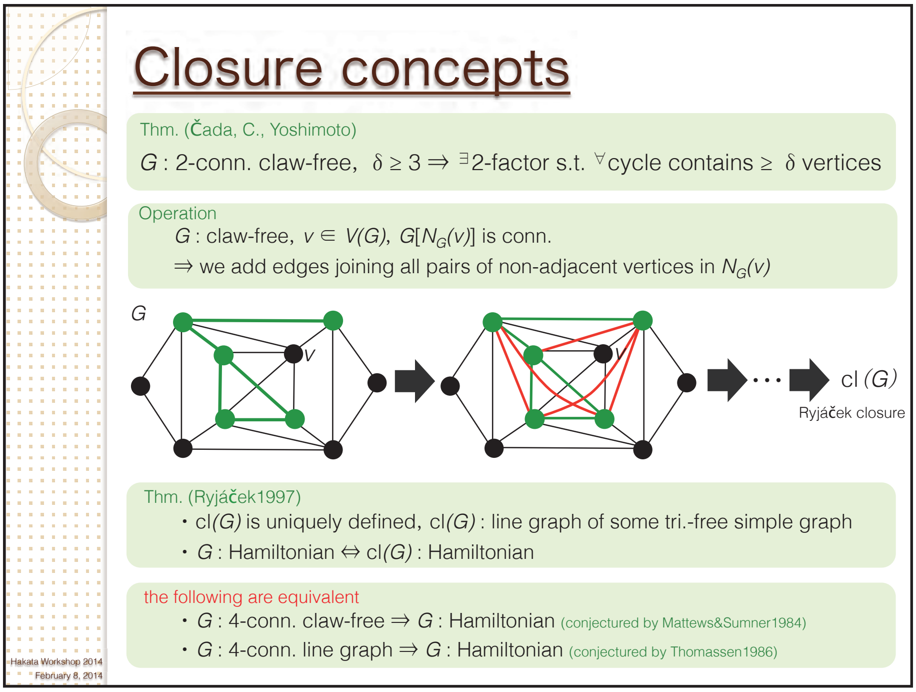
Closure concepts
Thm. (Čada, C., Yoshimoto)
G : 2-conn. claw-free, $\delta \geq 3 \Rightarrow$ $^{\exists}$2-factor s.t. $^{\forall}$cycle contains $\geq \delta$ vertices
Operation
G : claw-free, $v \in V(G)$, $G[N_G(v)]$ is conn.
$\Rightarrow$ we add edges joining all pairs of non-adjacent vertices in $N_G(v)$
G
v
v
cl (G)
Ryjáček closure
Thm. (Ryjáček1997)
• cl(G) is uniquely defined, cl(G) : line graph of some tri.-free simple graph
• G : Hamiltonian ⇔ cl(G) : Hamiltonian
the following are equivalent
• G : 4-conn. claw-free ⇒ G : Hamiltonian (conjectured by Mattews&Sumner1984)
• G : 4-conn. line graph ⇒ G : Hamiltonian (conjectured by Thomassen1986)
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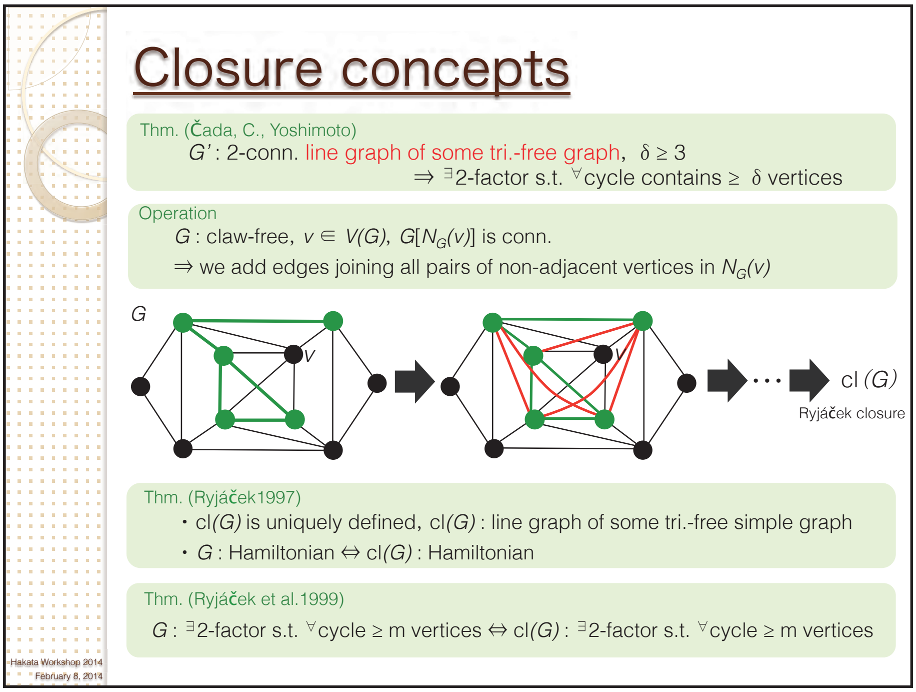
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Thm. (Ryjáček et al.1999)
G : $^{\exists}$2-factor s.t. $^{\forall}$cycle $\geq$ m vertices ⇔ cl(G) : $^{\exists}$2-factor s.t. $^{\forall}$cycle $\geq$ m vertices
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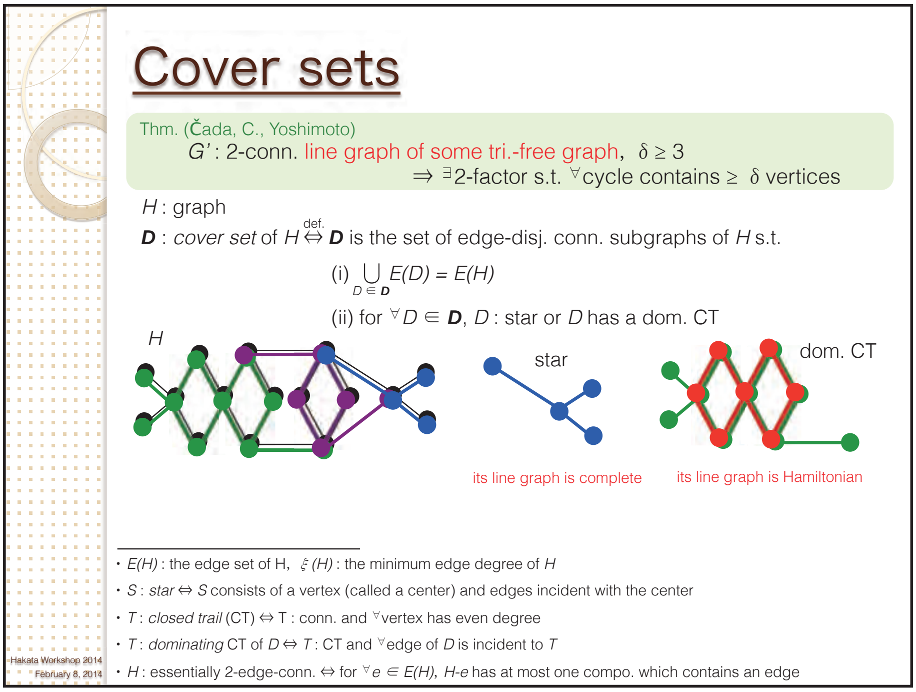
Cover sets
Thm. (Čada, C., Yoshimoto)
G' : 2-conn. line graph of some tri.-free graph, δ ≥ 3
⇒ ∃2-factor s.t. ∀cycle contains ≥ δ vertices
H : graph
D : cover set of H ⇔(def.) D is the set of edge-disj. conn. subgraphs of H s.t.
(i) ⋃_{D ∈ D} E(D) = E(H)
(ii) for ∀D ∈ D, D : star or D has a dom. CT
H
star
dom. CT
its line graph is complete
its line graph is Hamiltonian
• E(H) : the edge set of H, ξ(H) : the minimum edge degree of H
• S : star ⇔ S consists of a vertex (called a center) and edges incident with the center
• T : closed trail (CT) ⇔ T : conn. and ∀vertex has even degree
• T : dominating CT of D ⇔ T : CT and ∀edge of D is incident to T
• H : essentially 2-edge-conn. ⇔ for ∀e ∈ E(H), H-e has at most one compo. which contains an edge
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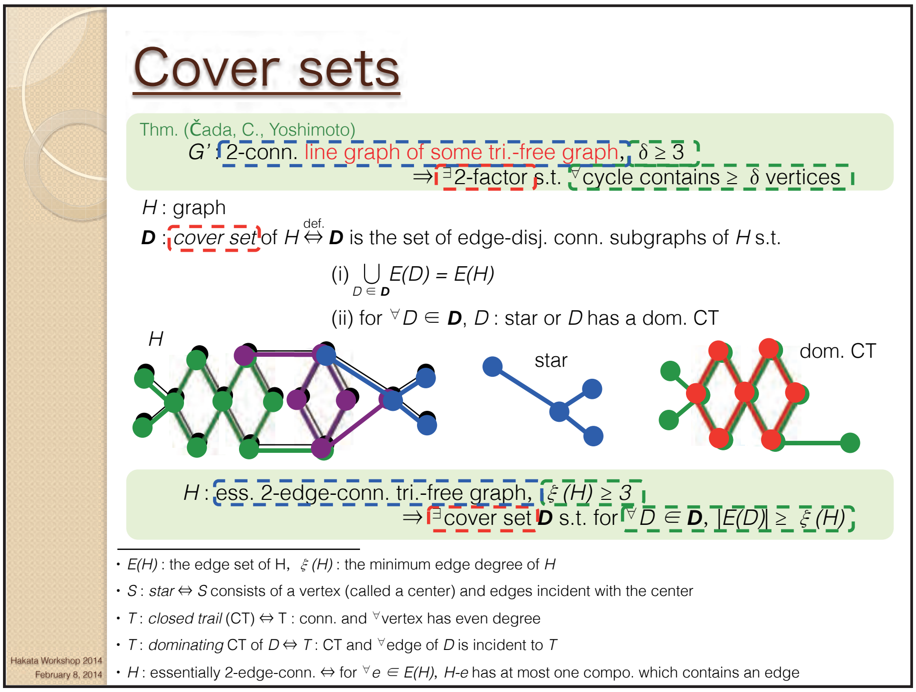
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H
star
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H : ess. 2-edge-conn. tri.-free graph, ξ(H) ≥ 3
⇒ ∃cover set D s.t. for ∀D ∈ D, |E(D)| ≥ ξ(H)
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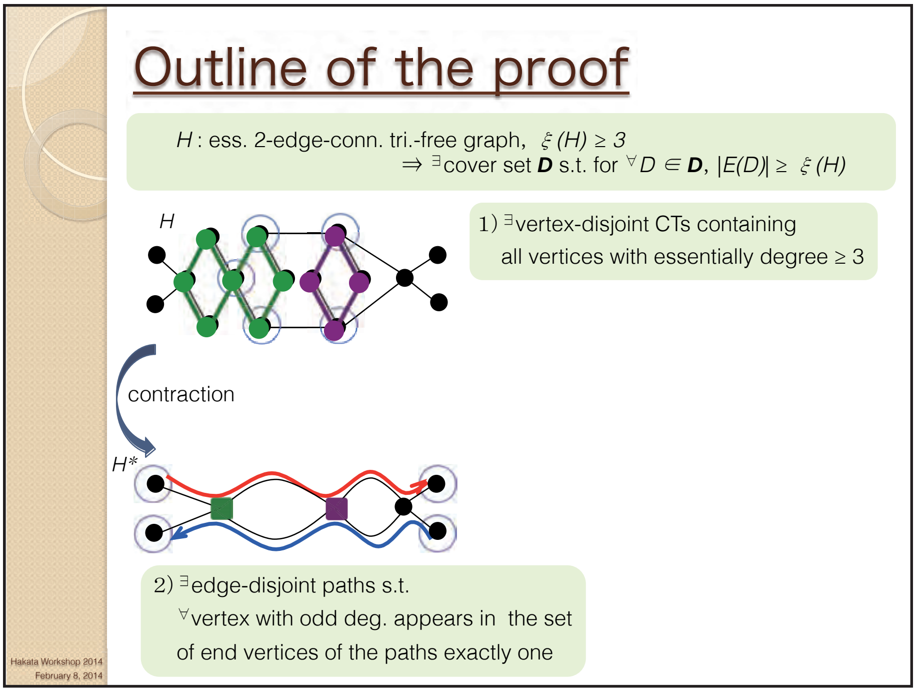
Outline of the proof
H : ess. 2-edge-conn. tri.-free graph, ξ (H) ≥ 3
⇒ ∃cover set D s.t. for ∀D ∈ D, |E(D)| ≥ ξ (H)
H
1) ∃vertex-disjoint CTs containing
all vertices with essentially degree ≥ 3
contraction
H*
2) ∃edge-disjoint paths s.t.
∀vertex with odd deg. appears in the set
of end vertices of the paths exactly one
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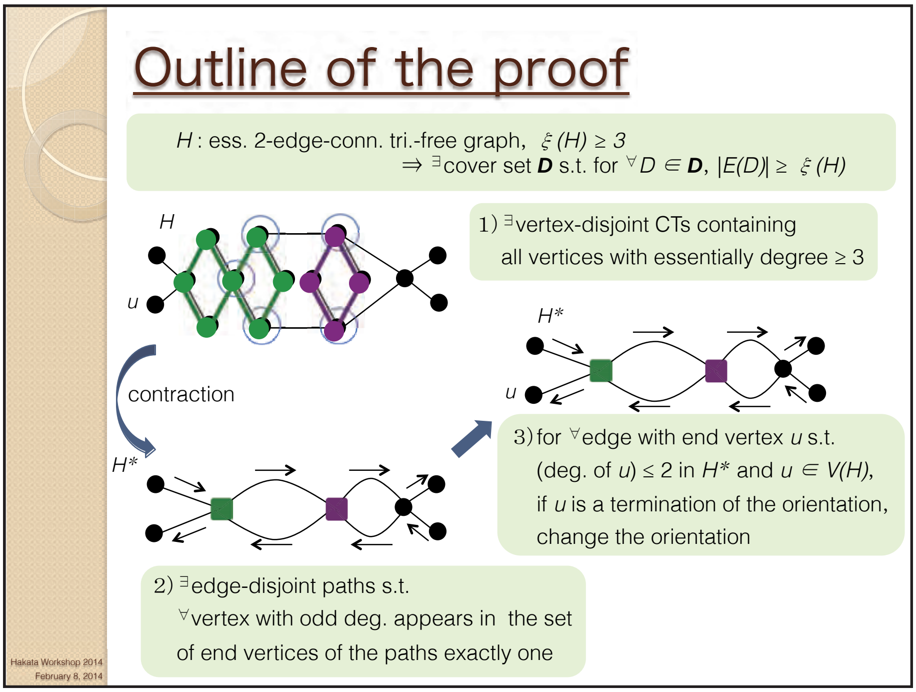
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all vertices with essentially degree ≥ 3
H*
u
contraction
H*
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(deg. of u) ≤ 2 in H* and u ∈ V(H),
if u is a termination of the orientation,
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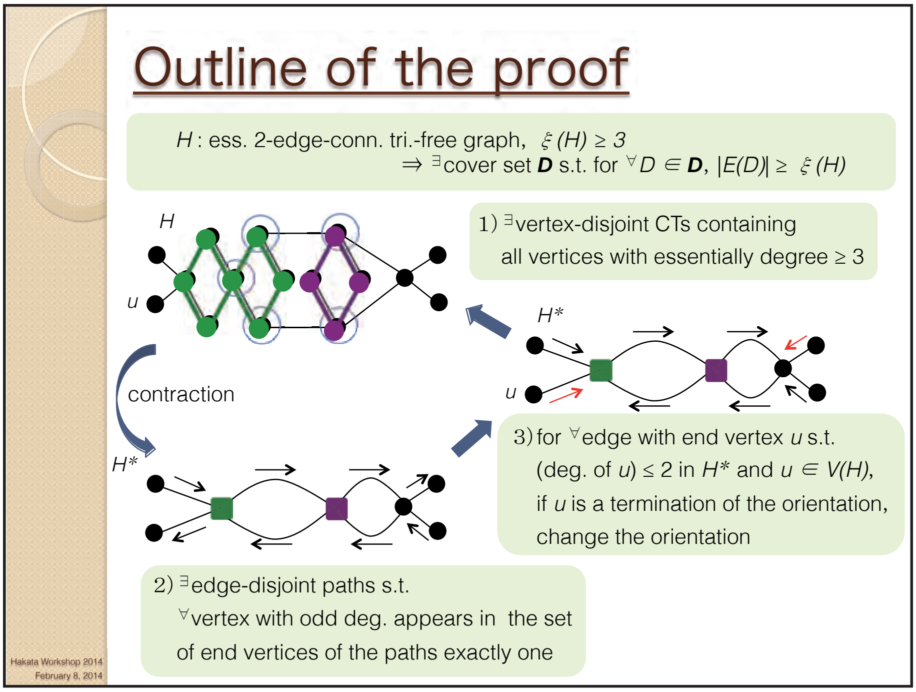
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H : ess. 2-edge-conn. tri.-free graph, $\xi(H) \geq 3$
$\Rightarrow$ $^\exists$cover set $\boldsymbol{D}$ s.t. for $^\forall D \in \boldsymbol{D}$, $|E(D)| \geq \xi(H)$
H
u
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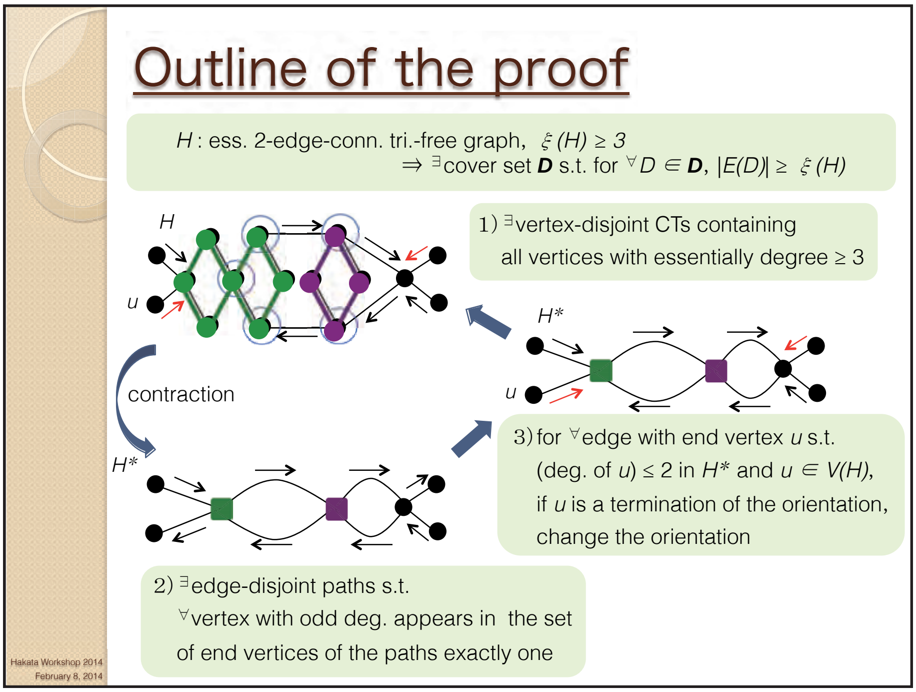
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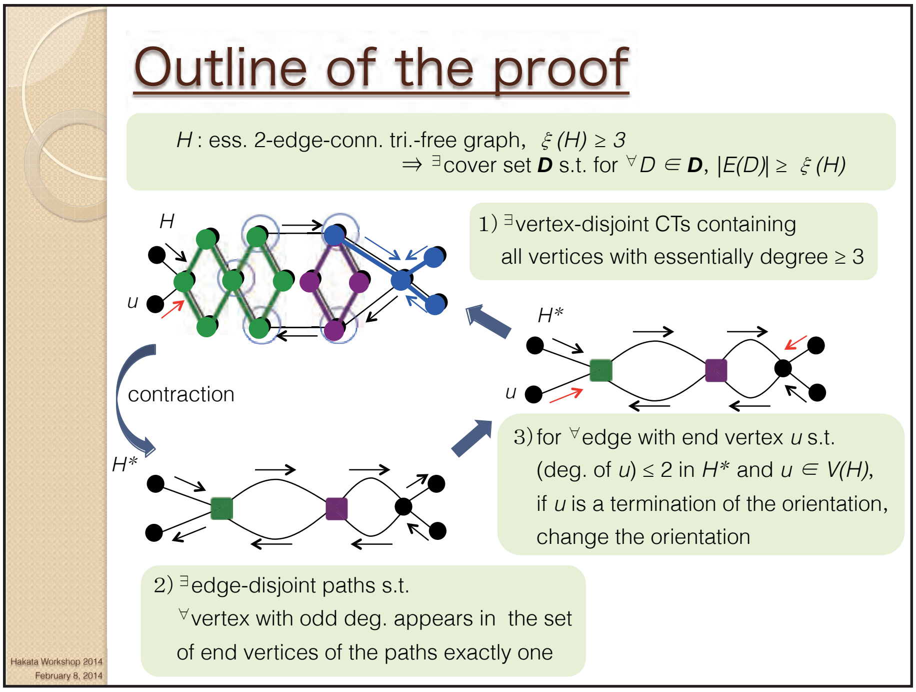
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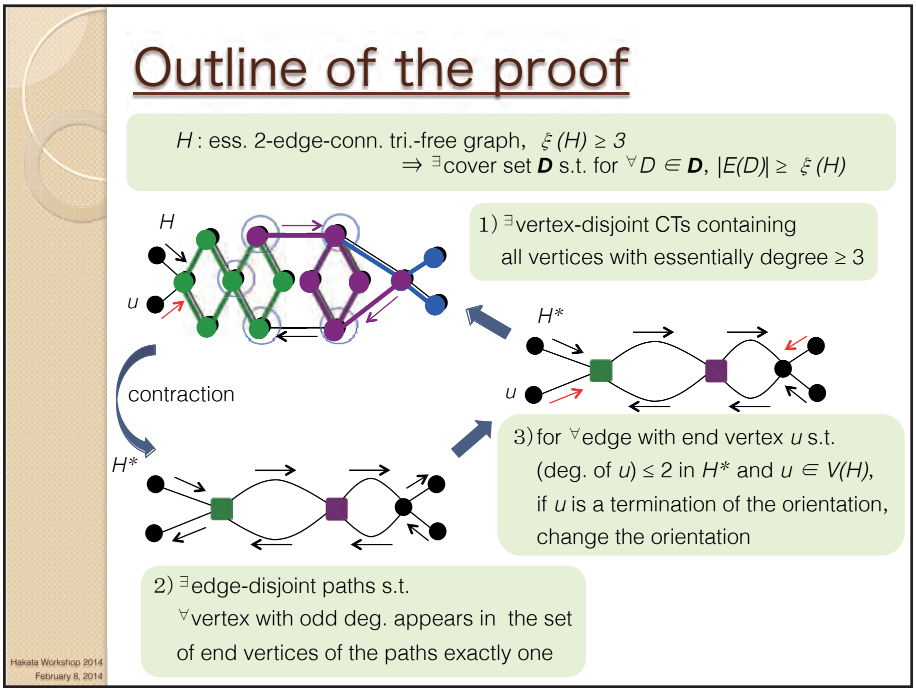
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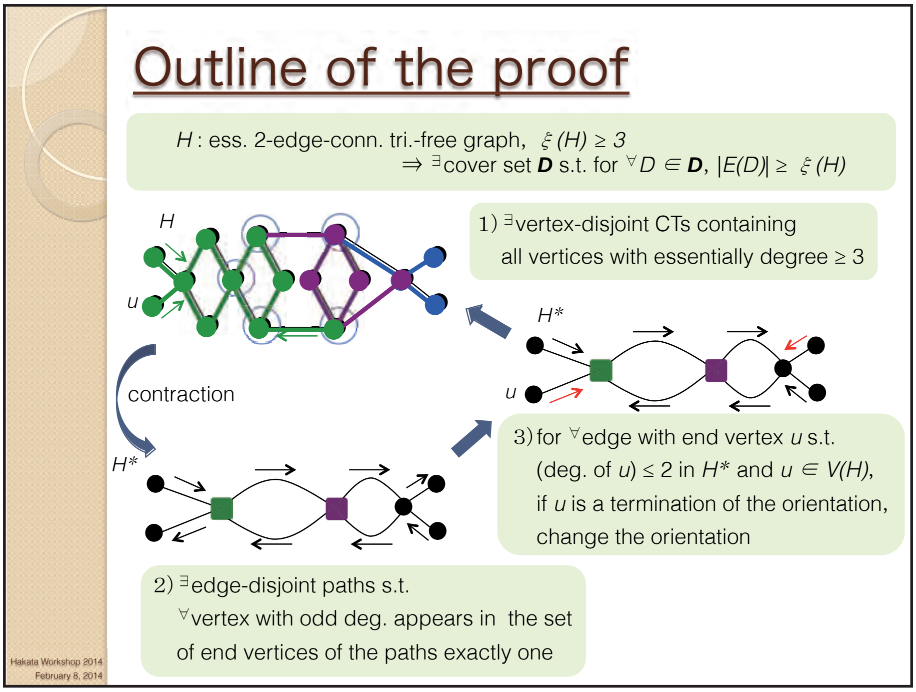
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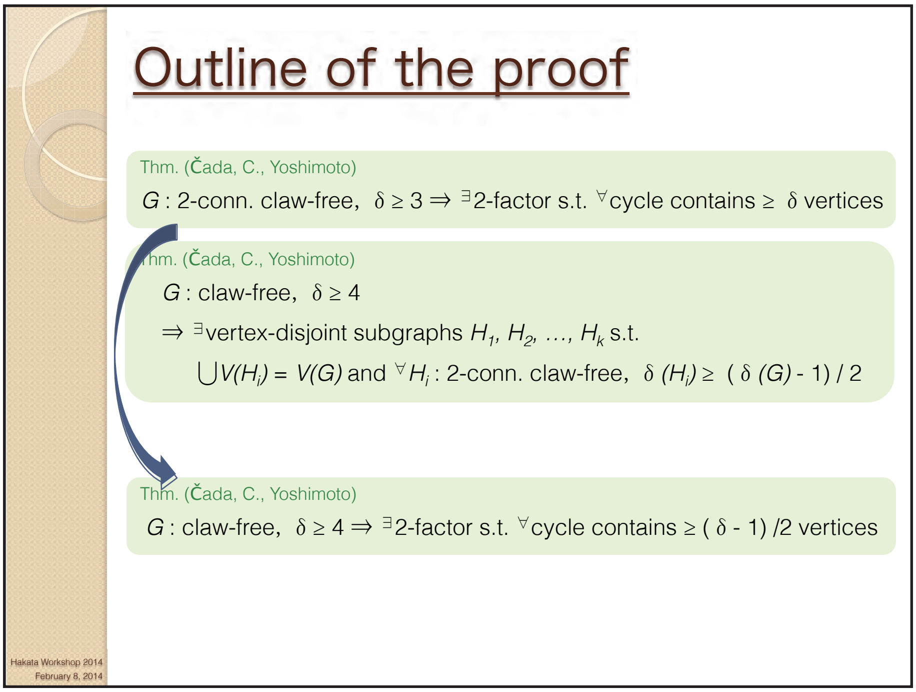
Outline of the proof
Thm. (Čada, C., Yoshimoto)
G : 2-conn. claw-free, δ ≥ 3 ⇒ ∃2-factor s.t. ∀cycle contains ≥ δ vertices
Thm. (Čada, C., Yoshimoto)
G : claw-free, δ ≥ 4
⇒ ∃vertex-disjoint subgraphs H1, H2, …, Hk s.t.
⋃V(Hi) = V(G) and ∀Hi : 2-conn. claw-free, δ (Hi) ≥ (δ (G) - 1) / 2
Thm. (Čada, C., Yoshimoto)
G : claw-free, δ ≥ 4 ⇒ ∃2-factor s.t. ∀cycle contains ≥ (δ - 1) /2 vertices
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Upper bounds

Conj. (Matthews&Sumner1984)

G : 4-conn. claw-free ⇒ *G* : Hamiltonian

【Upper bounds on # of compo.s of 2-factors】

	δ = 2	δ = 3	δ = 4	δ = 5	δ ≥ 6
conn.	×	×	n / 3 (trivial)	n / 4 (Broersma et al.2009)	(n-3)/(δ - 1) (Broersma et al.2009)
2-conn.	×	n / 3 (trivial)	n / 4 (CCY)	n / 5 (CCY)	n / δ (CCY)
3-conn.	-	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	2n / 15 (Jackson et al.2007)	4n / 5(δ + 2) (Ozeki et al.)

- *G* : 2-conn. claw-free ⇒ the length of the longest cycle ≥ 3 δ + 2 (Gao et al.1997)
- *G* : 3-conn. claw-free ⇒ the length of the longest cycle ≥ 6 δ - 15 (Li et al.2007)

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Upper bounds

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Thm. (Broersma et al.2001)

If ∃constant *c* s.t. ∀4-conn. claw-free graph has a 2-factor with ≤ *c* compo.s
⇒ MS-conjecture is true

Thm. (Čada, C., Ozeki, Vrána, Yoshimoto)

If ∃constant *c* s.t. ∀4-conn. claw-free graph *G* has a cycle of length ≥ *c*・|*V(G)*|
⇒ ∀4-conn. claw-free graph with δ ≥ 5 is Hamiltonian (slightly weaker version)

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- Definitions
- Matthews and Sumner's Conjecture
- Upper bounds on the number of components of 2-factors
- Implications and related results

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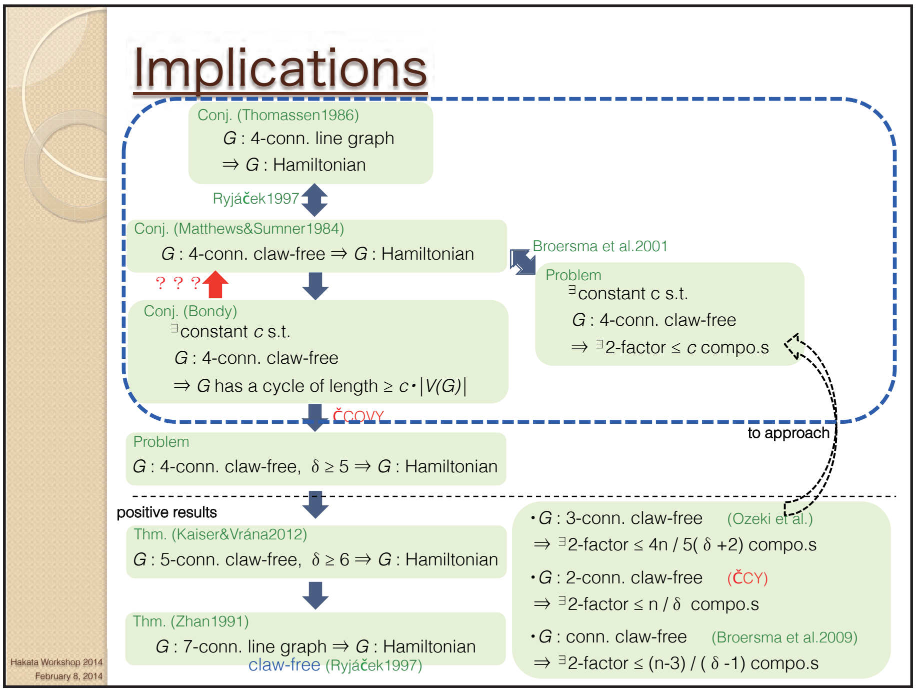

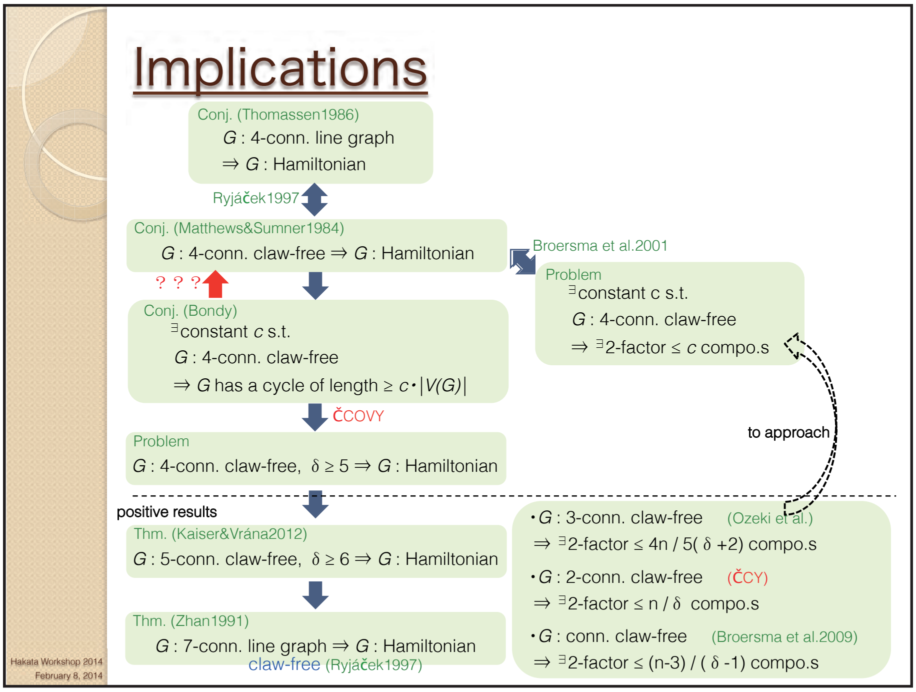
Implications
Conj. (Thomassen1986)
G : 4-conn. line graph
⇒ G : Hamiltonian
Ryjáček1997
Conj. (Matthews&Sumner1984)
G : 4-conn. claw-free ⇒ G : Hamiltonian
? ? ?
Conj. (Bondy)
∃constant c s.t.
G : 4-conn. claw-free
⇒ G has a cycle of length ≥ c·|V(G)|
ČCOVY
Problem
G : 4-conn. claw-free, δ ≥ 5 ⇒ G : Hamiltonian
Broersma et al.2001
Problem
∃constant c s.t.
G : 4-conn. claw-free
⇒ ∃2-factor ≤ c compo.s
to approach
positive results
Thm. (Kaiser&Vrána2012)
G : 5-conn. claw-free, δ ≥ 6 ⇒ G : Hamiltonian
Thm. (Zhan1991)
G : 7-conn. line graph ⇒ G : Hamiltonian
claw-free (Ryjáček1997)
・G : 3-conn. claw-free (Ozeki et al.)
⇒ ∃2-factor ≤ 4n / 5(δ +2) compo.s
・G : 2-conn. claw-free (ČCY)
⇒ ∃2-factor ≤ n / δ compo.s
・G : conn. claw-free (Broersma et al.2009)
⇒ ∃2-factor ≤ (n-3) / (δ -1) compo.s
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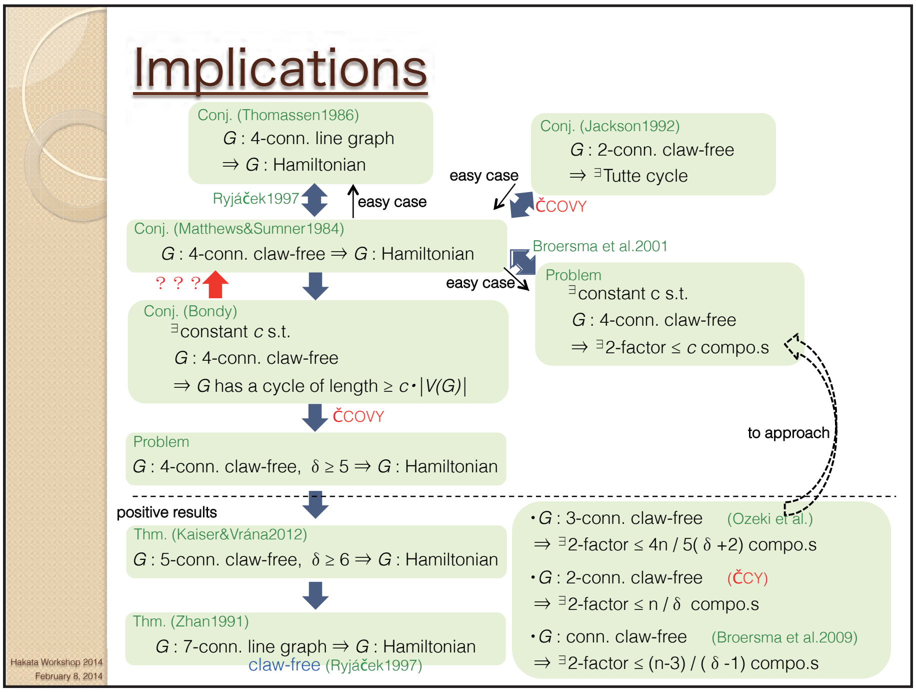
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easy case
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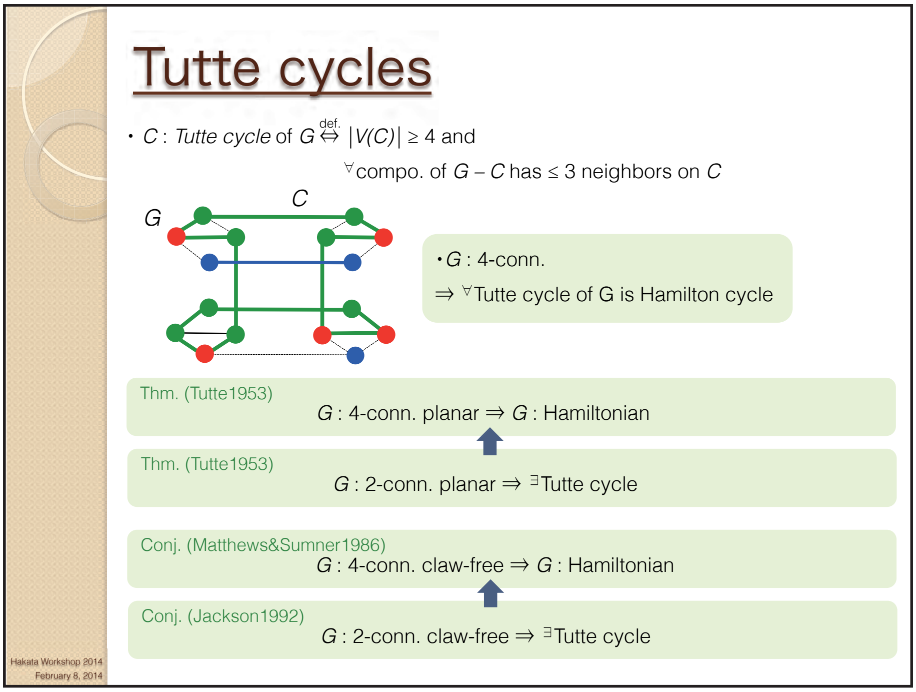
Tutte cycles
• C : Tutte cycle of G ⇔(def.) |V(C)| ≥ 4 and
∀compo. of G – C has ≤ 3 neighbors on C
G
C
• G : 4-conn.
⇒ ∀Tutte cycle of G is Hamilton cycle
Thm. (Tutte1953)
G : 4-conn. planar ⇒ G : Hamiltonian
Thm. (Tutte1953)
G : 2-conn. planar ⇒ ∃Tutte cycle
Conj. (Matthews&Sumner1986)
G : 4-conn. claw-free ⇒ G : Hamiltonian
Conj. (Jackson1992)
G : 2-conn. claw-free ⇒ ∃Tutte cycle
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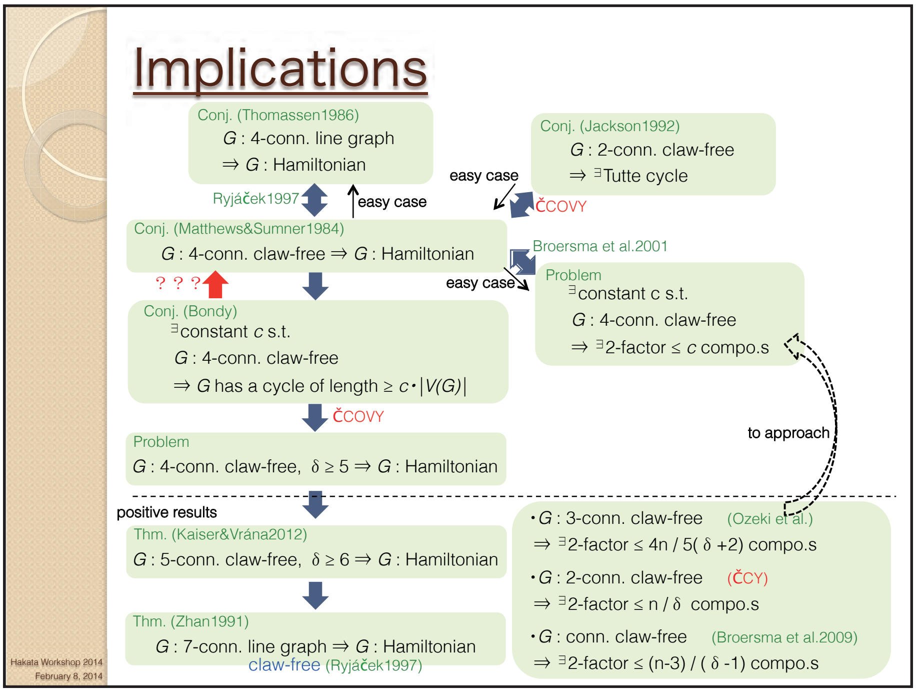
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On complementary Ramsey numbers

Masashi Shinohara (Shiga University)

(joint work with Akihiro Munemasa)

Hakata Workshop 2014
(February 8, 2014)

Edge coloring and decomposition of a complete graph

Definition

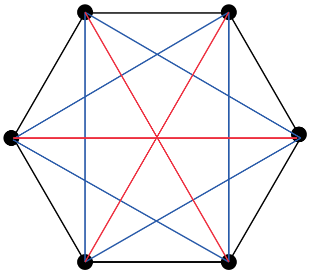
For $W \subset V(G)$

$$W\text{: independent set of } G \overset{\text{def}}{\Longleftrightarrow} \{v, w\} \notin E(G) \text{ for } \forall v, w \in W$$

$\alpha(G) := \max\{|W| \mid W : ind. set\ of\ G\}$ *(independence number of G),*

Given: an edges coloring of complete graph K_n by three colors

$$K_n = G_{black} \cup G_{red} \cup G_{blue}$$



- $G_{black} \cong C_6$. Then $\alpha(G_{black}) = 3$
  $G_{red} \cong 3K_2$. Then $\alpha(G_{red}) = 3$
  $G_{blue} \cong 2K_3$. Then $\alpha(G_{black}) = 2$

Complementary Ramsey number

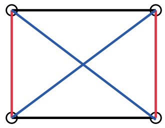
Example

(i) Find a 3-edge coloring of K_4 such that

$$\alpha(G_{black}) < 3,\ \alpha(G_{red}) < 3 \text{ and } \alpha(G_{blue}) < 3.$$

(ii) Prove that for any 3-edge coloring of K_5, there exists $i \in \{black, red, blue\}$ such that $\alpha(G_i) \geq 3$.

(i)



(ii)

- Since $|E(G_{black})| + |E(G_{red})| + |E(G_{blue})| = |E(K_5)| = 10 = 4 + 3 + 3$, there exists $i \in \{black, red, blue\}$ such that $|E(G_i)| \leq 3$.
- If G satisfies $|V(G)| = 5$ and $|E(G)| \leq 3$, then $\alpha(G) \geq 3$.

Distance sets and their substructures

For $X \subset \mathbb{R}^d$, $A(X) := \{d(x,y) \mid x, y \in X, x \neq y\} = \{d_i\}$.
X is a k-distance set if $|A(X)| = k$.

$$V(G_i) = X, \quad \{x,y\} \in E(G_i) \iff d(x,y) = d_i.$$

$$\begin{array}{ccccccc} d_1 & > & d_2 & > & \cdots & > & d_k \\ \updownarrow & & \updownarrow & & & & \updownarrow \\ G_1 & & G_2 & & & & G_k \end{array}$$

d_1: diameter of X, G_1: diameter graph of X

Theorem

Let $X \subset \mathbb{R}^3$ and $|X| = 12$. Then $\alpha(G_1) \geq 5$.
($\alpha(G)$: the independence number of G)

Corollary

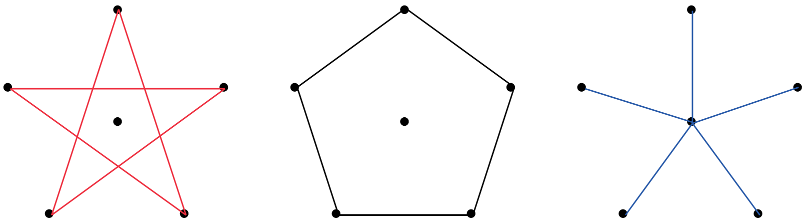
Every 12-point 3-distance set in $\mathbb{R}^3$ contains 5-point 2-distance set in $\mathbb{R}^3$.

Theorem (S.)

Every 3-distance set in $\mathbb{R}^3$ with 12 points is similar to an icosahedron.

Distance sets and their substructures

X: the vertex set of regular pentagon and its center.



$\alpha(G_1) = 3$ $\quad$ $\alpha(G_2) = 3$ $\quad$ $\alpha(G_3) = 5$

$$\max_{1 \le i \le 3} \alpha(G_i) = 5.$$

X contains a 5-point (1- or) 2-distance set.

Distance sets and their substructures

- $[n] := \{1, 2, \cdots, n\}$.
  $\binom{X}{n}$: the set of all n-element subsets of a set X.
  $C(n,k) = \left\{ f \mid f : \binom{[n]}{2} \to [k] \right\}$.
  $G_i = G_i(f) = ([n], f^{-1}(i))$ and $\alpha_i(f) = \alpha(G_i)$.
  $\alpha(f) := \max\{\alpha_i(f) \mid 1 \le i \le k\}$.

$$\bar{R}(m;k) := \min\{n \in \mathbb{N} \mid \alpha(f) \ge m \text{ for } \forall f \in C(n,k)\}.$$

Proposition

Every $\bar{R}(m;k)$-point k-distance set in any space contains an m-point k'-distance set for $\exists k' \le k-1$.

$\bar{R}(5;3) \le 12$? $\quad$ false $\quad$ (In fact, $\bar{R}(5;3) \ge 17$)
$\bar{R}(4;3) = 10$.

Complementary Ramsey number $\bar{R}(m; k)$

k	2	3	4	5	6	7	8	9	10	$11 \sim 15$	16
$\bar{R}(3; k)$	6	5	3	$\cdots$							
$\bar{R}(4; k)$	18	10	10	7	5	4	$\cdots$				
$\bar{R}(5; k)$	?	?	?	17	10	9	6	6	6	5	$\cdots$
$\bar{R}(6; k)$	?	?	?	?	26	16	11	11	8	7	6

complementary Ramsey number for $\bar{R}(m; k)$

Theorem

A complete set of MOLS of order n exists $\Longleftrightarrow \bar{R}(n+1; n+1) = n^2 + 1$.

Designs and complementary Ramsey numbers

- Two Latin squares L_1, L_2 of order n are orthogonal if $\{(L_1(i,j), L_2(i,j)) \mid (i,j) \in [n]^2\} = [n]^2$
  $\{L_i\}_{i=1}^m$: MOLS if L_i and L_j are orthogonal for any $i \neq j$

1	2	3
2	3	1
3	1	2

L_1

1	2	3
3	1	2
2	3	1

L_2

MOLS is said to be complete set if $m = n - 1$.

Define a coloring $f \in C([n]^2, m+2)$ by

$$f(\{(i,j),(i',j')\}) = \begin{cases} l & \text{if} \quad L_l(i,j) = L_l(i'j') \\ \infty & \text{otherwise} \end{cases}$$

Since $\alpha_i(f) \leq n$, $\bar{R}(n+1; m+2) > n^2$.

Coloring and MOLS

1	1	1
2	2	2
3	3	3

L_0

1	2	3
1	2	3
1	2	3

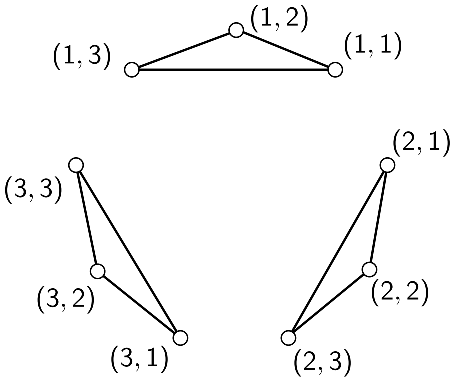
L_∞

1	2	3
2	3	1
3	1	2

L_1

1	2	3
3	1	2
2	3	1

L_2



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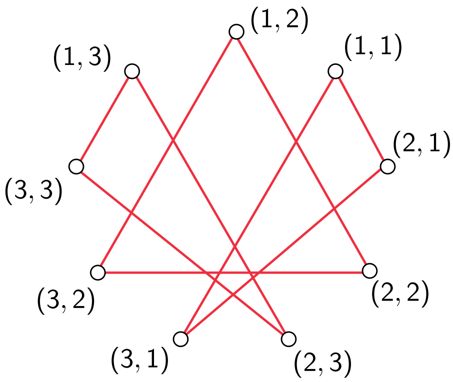

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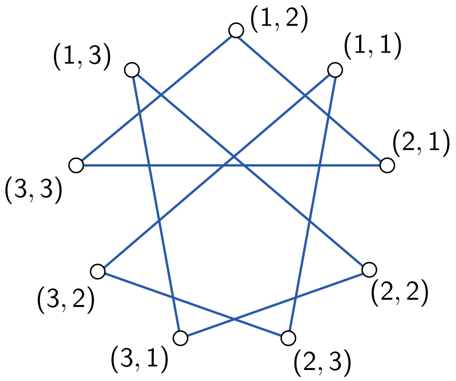

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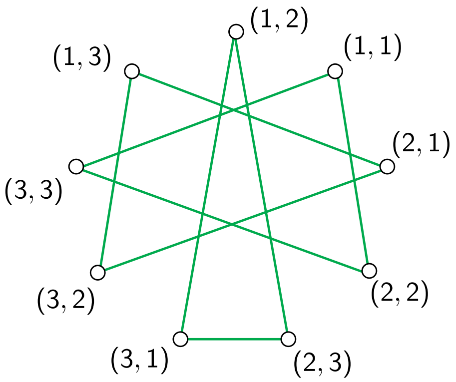

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2	2	2
3	3	3

L_0

1	2	3
1	2	3
1	2	3

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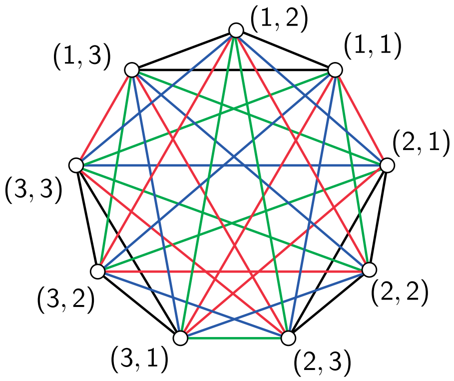


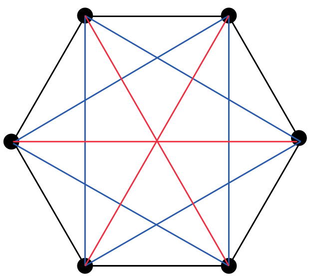
Complementary Ramsey numbers (general cases)

Let $m_1, \ldots, m_k$ be natural numbers greater than 1.

- Ramsey number
  $R(m_1, \ldots, m_k) = \min\{n \in \mathbb{N} \mid \forall f \in C(n,k),\ \exists i \in [k],\ \omega_i(f) \geq m_i\}$.

  Complementary Ramsey number
  $\bar{R}(m_1, \ldots, m_k) = \min\{n \in \mathbb{N} \mid \forall f \in C(n,k),\ \exists i \in [k],\ \alpha_i(f) \geq m_i\}$.

  $\bar{R}(m;k) = \bar{R}(\underbrace{m, m, \ldots, m}_{k})$



$\bar{R}(4, 4, 3) = ?$

$G_1 \cong C_6$, $G_2 \cong 3K_2$ and $G_3 \cong 2K_3$

$\alpha_1 = \alpha_2 = 3$ and $\alpha_3 = 2$.

$\bar{R}(4, 4, 3) \geq 7$.

We can also prove $\bar{R}(4, 4, 3) \leq 7$.

Property of complementary Ramsey numbers

- Since $\alpha(G) = \omega(\bar{G})$, $\bar{R}(m_1, m_2) = R(m_2, m_1) = R(m_1, m_2)$.
  If $m_i \le m'_i$ for $\forall i \in [k]$, then $\bar{R}(m_1, \dots, m_k) \le \bar{R}(m'_1, \dots, m'_k)$.

k	3	4	5	6	7	8	$9 \sim 13$	$14 \sim$
$\bar{R}(k,3,3)$	5			6				
$\bar{R}(k,4,3)$	(5)	7	8		9			
$\bar{R}(k,5,3)$	(5)	(8)	9	11	12		13	14

$\bar{R}(m_1, \dots, m_k) \le \bar{R}(m_1, \dots, m_{k-1})$.
Moreover, equality holds if $m_k \ge \bar{R}(m_1, \dots, m_{k-1})$.

Problem

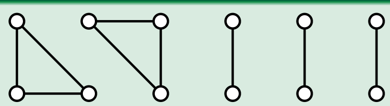
Determine $\bar{R}(k,4,4)$, $\bar{R}(k,5,4)$ and $\bar{R}(k,5,5)$(difficult).
($R(4,4) = 18$, $R(5,4) = 25$ and $R(5,5)$: unknown)

Minimum size for given independence number

For given n and m,

$$\bar{t}_m(n) = \min\{|E(G)| : G \text{ is a graph with } |V(G)| = n, \alpha(G) \le m\},$$
$$\bar{T}_m(n) = \{G : |V(G)| = n, \alpha(G) \le m \text{ and } |E(G)| = \bar{t}_m(n)\}$$

Example ($\bar{T}_5(12) = 2K_3 \cup 3K_2$)



Lemma (Turán, 1940)

$$\bar{T}_m(n) = \{rK_{q+1} \cup (m-r)K_q\}, \quad \bar{t}_m(n) = r\binom{q+1}{2} + (m-r)\binom{q}{2}$$

where $q = \left\lfloor \frac{n}{m} \right\rfloor$ *and* $r = n - mq$.

Factorizations and c-Ramsey numbers

Definition

K_n is said to be factorable into the factors H_1, H_2, ..., H_k if these factors are pairwise edge-disjoint and $E(K_n) = \bigcup_{i=1}^{k} E(H_i)$.

Theorem

Suppose that K_n is factorable into H_1, H_2, ..., H_k where

$$H_i \cong r_i K_{q_i+1} \cup (m_i - r_i) K_{q_i} \qquad (\alpha(H_i) = m_i)$$

for some non-negative integers m_i, q_i, r_i which satisfy $n = m_i q_i + r_i$ and $0 \leq r_i < m_i$ for any $i \in [k]$. Assume further that $m_i - r_i - 1 > 0$ for some $i \in [k]$. Then

$$\bar{R}(m_1 + 1, m_2 + 1, \ldots, m_k + 1) \geq n + 1.$$

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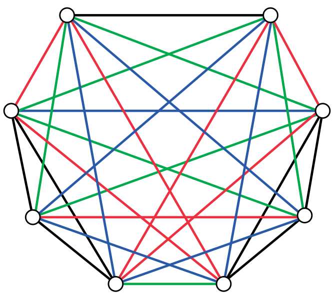
Assumption: $m_i - r_i - 1 > 0$ for some $i \in [k]$

- $$\begin{aligned} m_i - r_i - 1 > 0 &\iff r_i < m_i - 1 \\ &\iff r_i \neq m_i - 1 \end{aligned}$$

$H_i = (m_i - 1)K_{q_i+1} \cup K_{q_i}$ for any $i \in [k]$.



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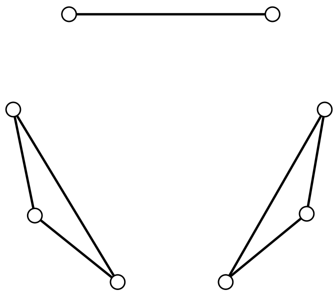

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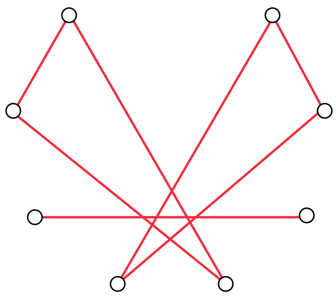

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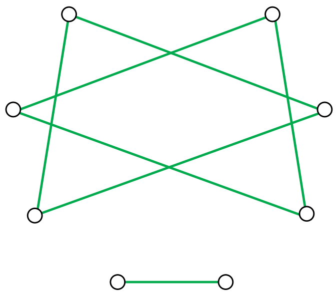

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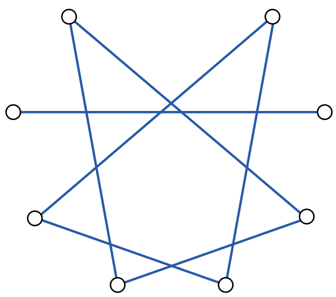

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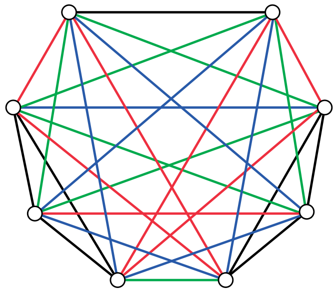

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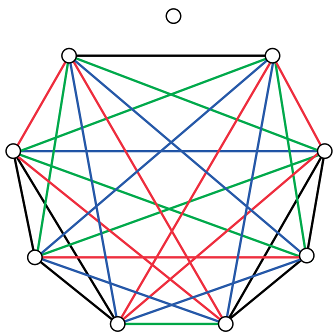

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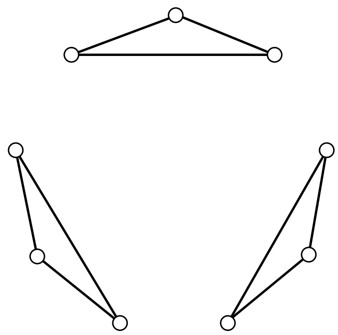

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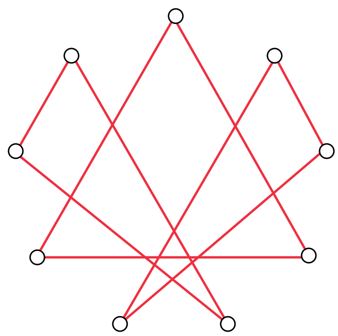

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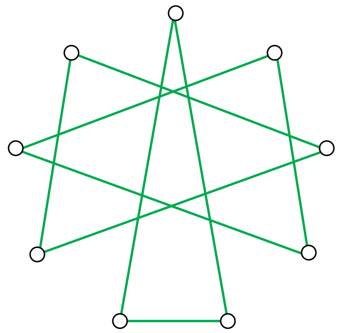

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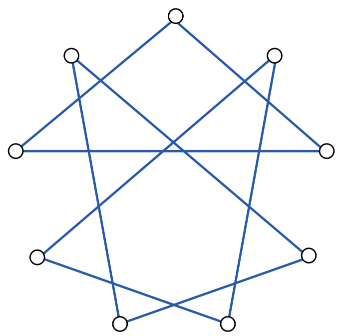

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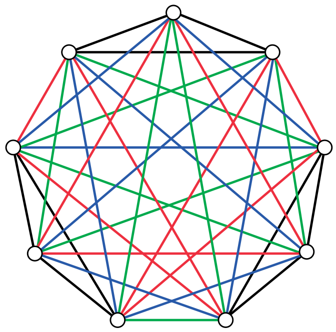

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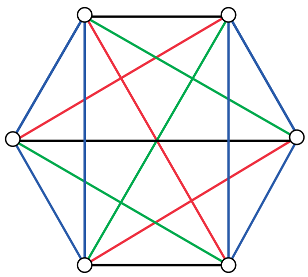
Examples of factorizations



$H_i \cong 3K_2$ for $i \in [3]$
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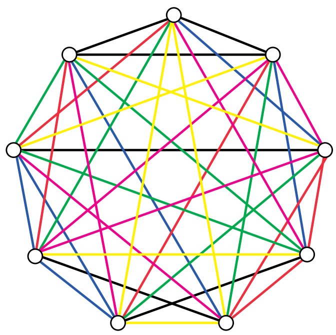
$\alpha_1 = \alpha_2 = \alpha_3 = 3, \alpha_4 = 2$
$\overline{R}(4,4,4,3) = 7$



$H_i \cong K_3 \cup 3K_2$ for $i \in [6]$

$\alpha_i = 4$ for $i \in [6]$
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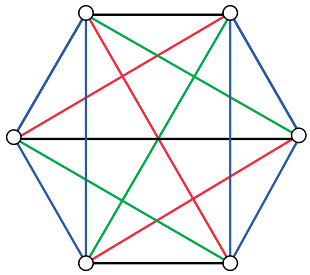

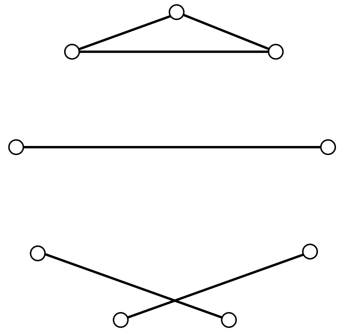

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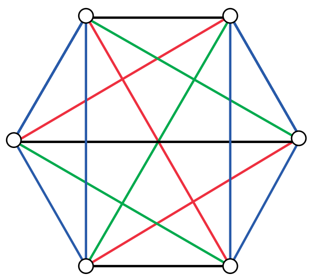

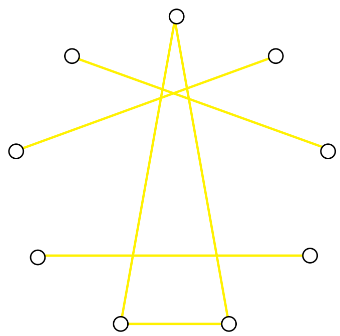

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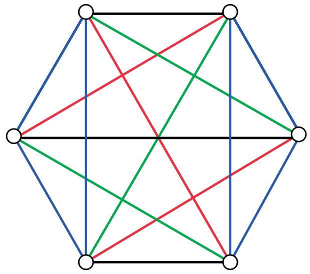

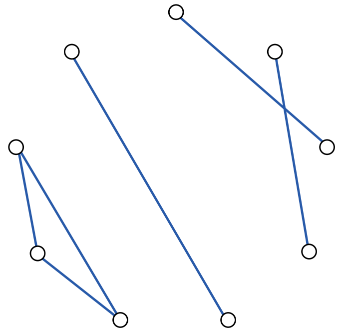

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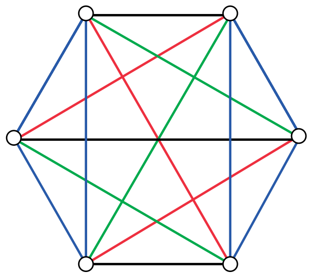
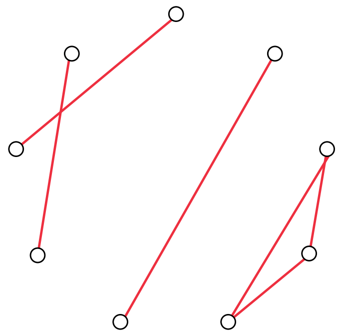
Examples of factorizations



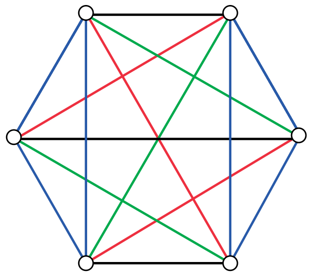


$H_i \cong 3K_2$ for $i \in [3]$
$H_4 \cong 2K_3$
$\alpha_1 = \alpha_2 = \alpha_3 = 3, \alpha_4 = 2$
$\overline{R}(4,4,4,3) = 7$

$H_i \cong K_3 \cup 3K_2$ for $i \in [6]$

$\alpha_i = 4$ for $i \in [6]$
$\overline{R}(5,5,5,5,5,5) = 10$

Examples of factorizations



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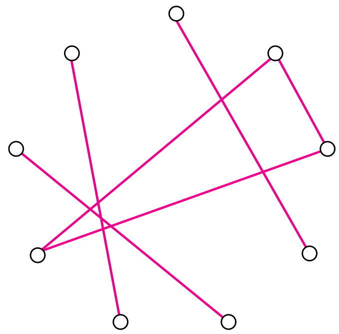

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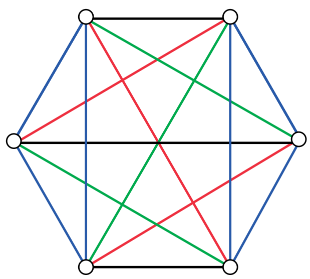
Examples of factorizations



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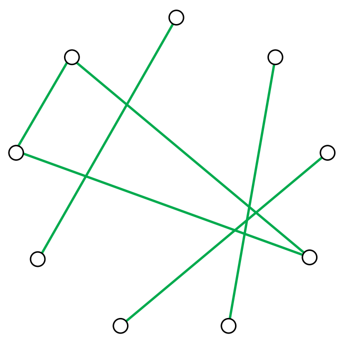

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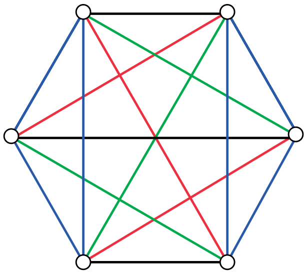

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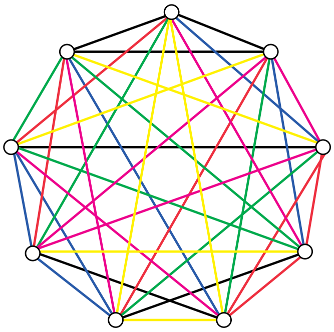

$H_i \cong K_3 \cup 3K_2$ for $i \in [6]$

$\alpha_i = 4$ for $i \in [6]$
$\overline{R}(5,5,5,5,5,5) = 10$

Known factorizations of K_n into $H_1, \ldots, H_k$

$$H_i \cong r_i K_{q_i+1} \cup (m_i - r_i) K_{q_i}$$

$r_i = 0$ for any $i \in [k]$

- $\exists$ perfect matching ($q_i = 2$ for $\forall i \in [k]$) if and only if $n \equiv 0 \mod 2$.
- $\exists$ Kirkman triple system ($q_i = 3$ for $\forall i \in [k]$) if and only if $n \equiv 3 \mod 6$.
- $\exists$ Nearly Kirkman triple system ($q_i = 3$ for $\forall i \in [k-1]$, $q_k = 2$) if and only if $n \equiv 0 \mod 6 \quad (n \neq 6, 12)$.
- $\exists$ Complete set of MOLS ($m_i = q_i = q$ for $\forall i \in [k]$, k=q+1) if q is prime power.

Others

- uniformly resolvable design, class-uniformly resolvable design, restricted resolvable design, class-uniformly resolvable group divisible design.

Degree condition

For $f \in C(n,k)$, $x \in [n]$, $i \in [k]$,

$$f_i(x) := |\{y \in [n] \mid f(\{x,y\}) = i\}| \quad (\text{degree of } x \text{ in } G_i)$$

Proposition

$m_i \in \mathbb{Z}_{z>0}$, $f \in C(n,k)$. If

$$\sum_{i=1}^{t} f_i(x) \geq \bar{R}(m_1, \ldots, m_t, m_{t+1}-1, \ldots, m_k - 1)$$

Then there exists $j \in [k]$ such that $\alpha_j(f) \geq m_j$.

Proof.

Let $Y = \{y \in [n] \mid f(\{x,y\}) \in [t]\}$ and $g = f|_{\binom{Y}{2}}$.
By the assumption, (i) $1 \leq \exists j \leq t$ such that $\alpha_j(g) \geq m_j$ (then OK) or (ii) $t+1 \leq \exists j \leq k$ such that $\alpha_j(g) \geq m_j - 1$ holds.

(ii) If Z be an ind. set of $g^{-1}(j)$, then $Z \cup \{x\}$ is also an ind. set. Therefore $\alpha_j(f) \geq m_j$.

□

Degree condition and $\bar{R}(4,4,4)$

Corollary

Let $f \in C(10,3)$. If $\alpha_i(f) < 4$ for $\forall i \in [3]$, then $f^{-1}(i)$ is 3-regular ($\forall i$).

Lemma

Let G be a K_4-free 3-regular graph of order 10. Then $\alpha(G) \geq 4$.

Theorem

$\bar{R}(4,4,4) = 10$.

Proof.

Since $\bar{R}(4,4,4) \geq \bar{R}(4,4,4,4) = 10$, we prove $\bar{R}(4,4,4) \leq 10$.
Let $f \in C(10,3)$ satisfy $\alpha_i(f) < 4$ ($\forall i$). Then $f^{-1}(1)$ is 3-regular.

- If $f^{-1}(1)$ contains K_4, then $\alpha_1(f) \geq 4$.
- If $f^{-1}(1)$ is K_4-free, then $\alpha_1(f) \geq 4$.

□

Tables

k	2	3	4	5	6	7	8	9	10	$11 \sim 15$	16
$\bar{R}(3;k)$	6	5	3	$\cdots$							
$\bar{R}(4;k)$	18	10	10	7	5	4	$\cdots$				
$\bar{R}(5;k)$	?	?	?	17	10	9	6	6	6	5	$\cdots$
$\bar{R}(6;k)$	?	?	?	?	26	16	11	11	8	7	6

complementary Ramsey number for $\bar{R}(m;k)$

k	3	4	5	6	7	8	$9 \sim 13$	$14 \sim$
$\bar{R}(k,3,3)$	5			6				
$\bar{R}(k,4,3)$	(5)	7	8		9			
$\bar{R}(k,5,3)$	(5)	(8)	9	11	12		13	14

complementary Ramsey number for $\bar{R}(m_1, m_2, m_3)$

Masashi Shinohara (Shiga University) On complementary Ramsey numbers Hakata Workshop 2014 43 / 1

Graph homomorphisms
and
de Branges-Rovnyak theory

(Gram matrices of RKHS's over graphs II)

Michio Seto (Shimane University)

joint work with S. Suda and T. Taniguchi

1

1. RKHS

$\mathcal{H}$: a Hilbert space consisting of functions on Ω.

If $f \mapsto f(\lambda)$ $(f \in \mathcal{H})$ is continuous,

then $\exists k_\lambda \in \mathcal{H}$ (unique) s.t. $f(\lambda) = \langle f, k_\lambda \rangle$

by the Riesz representation theorem.

$\mathcal{H}$ is a RKHS $\overset{\text{def}}{\Leftrightarrow}$ (i) and (ii)

(i) $\mathcal{H}$ is a Hilbert space consisting of functions on a set Ω.

(ii) $\forall \lambda \in \Omega$, $\exists k_\lambda \in \mathcal{H}$ s.t. $f(\lambda) = \langle f, k_\lambda \rangle$ $(\forall f \in \mathcal{H})$.

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One of the origins is Cauchy's integral formula

$$f(\lambda) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-\lambda}\,dz \quad (C : |z| = 1)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{f(e^{i\theta})}{1-\lambda e^{-i\theta}}\,d\theta$$

$$= \left\langle f(\theta), \frac{1}{1-\overline{\lambda}e^{i\theta}} \right\rangle_{L^2(d\theta)}$$

$\Rightarrow \mathcal{H}$ **is the Hardy space** H^2, k_λ **is called the Szegö kernel.**

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2. RKHS over G

$G = (V, E)$: **a finite graph**

V: **the vertex set of** G,

E: **the edge set of** G.

In this talk,

a graph is always connected, non-directed,

has neither loops nor mulitiedges.

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A table in my mind

graph theory	complex analysis
a vertex	a point
an edge	a n.b.d.
graphs	regions
⋮	⋮

My Interest
Can we give a new point of view
from RKHS theory to graph theory?

13

G: a graph with adjacency matrix A,

$\mathcal{F}$: the set of all real valued functions on V.

$\forall u, v \in \mathcal{F}$

$$\mathcal{E}(u,v) := \frac{1}{2}\sum_{x,y\in V} A_{xy}(u(x)-u(y))(v(x)-v(y)),$$

$$\langle u, v\rangle := \sum_{x\in V} u(x)v(x) + \mathcal{E}(u,v).$$

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G: a graph

$$\mathcal{H}_G := (\mathcal{F}, \langle \cdot, \cdot \rangle).$$

- $\mathcal{H}_G$ is a real Hilbert space and $\dim \mathcal{H}_G = |V|$,
- $\forall x \in V$, $\exists^1 k_x \in \mathcal{H}_G$ s.t. $u(x) = \langle u, k_x \rangle$ $(\forall u \in \mathcal{H})$,
- k_x is called the reproducing kernel of $\mathcal{H}_G$ at x.

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A table in my mind

graph theory	complex analysis
a vertex	a point
an edge	a n.b.d.
graphs	regions
$\mathcal{E}(u, u)$	$\int_\Omega \|f'(z)\|^2\, dxdy$
$\mathcal{H}_G$	Dirichle space
$\vdots$	$\vdots$

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3. Homomorphisms

$G_j = (V_j, E_j)$ $(j = 1, 2)$: graphs

$\varphi : G_1 \to G_2$ is a homomorphism $\overset{\text{def}}{\Leftrightarrow}$ (i) and (ii):

(i) $\varphi : V_1 \to V_2$,

(ii) $\{x, y\} \in E_1 \Rightarrow \{\varphi(x), \varphi(y)\} \in E_2$.

- φ is a hom. $\Leftrightarrow A_{xy}(G_1) \leq A_{\varphi(x)\varphi(y)}(G_2)$.

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graph theory	complex analysis
a vertex	a point
an edge	a n.b.d.
graphs	regions
$\mathcal{E}(u, u)$	$\int_\Omega \lvert f'(z)\rvert^2 \, dxdy$
$\mathcal{H}_G$	Dirichlet space
homomorphisms	holomorphic maps
$\vdots$	$\vdots$

My Interest
Applying de Branges-Rovnyak theory to graph theory, what can we obtain?

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Brief History of Bieberbach conjecture

- **Bieberbach conjectured that for any normalized univalent function on $\{z : |z| < 1\}$**

$$|a_n| \le n? \quad (f(z) = z + \sum_{n=2}^{\infty} a_n z^n)$$

(he proved $|a_2| \le 2$).

- **Löwner proved $|a_3| \le 3$ with his theory.**
- **Later, $n = 4, 5, 6$ cases were solved.**
- **Finally, in 1984, de Branges gave the complete solution.**

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For a graph hom. $\varphi : G_1 \to G_2$

$$C_\varphi u := u \circ \varphi \quad (u \in \mathcal{H}_{G_2}), \quad C_\varphi : \mathcal{H}_{G_2} \to \mathcal{H}_{G_1},$$

$$N_\varphi := \max_{x_2 \in V(G_2)} |\varphi^{-1}(x_2)|$$

- $\|C_\varphi u\|_{\mathcal{H}_{G_1}} \le N_\varphi \|u\|_{\mathcal{H}_{G_2}}$,
- $T := C_\varphi^* / N_\varphi$, $T : \mathcal{H}_{G_1} \to \mathcal{H}_{G_2}$ **and** $\|T\| \le 1$,
- T **is an onto isometry** $\Leftrightarrow$ φ **is an isomorphism.**

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$T : \mathcal{H}_{G_1} \to \mathcal{H}_{G_2}, \quad P : \mathcal{H}_{G_1} \to (\ker T)^\perp \quad$ **proj.**

$$\langle Tu_1, Tv_1\rangle_T := \langle Pu_1, Pv_1\rangle_{\mathcal{H}_{G_1}} \quad (u_1, v_1 \in \mathcal{H}_{G_1}),$$

$$\mathcal{M}(T) := (T\mathcal{H}_{G_1}, \|\cdot\|_T)$$

We are interested in the following Hilbert space inclusion:

$$\mathcal{M}(T) \subset \mathcal{H}_{G_2}.$$

Why?

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We are interested in the following Hilbert space inclusion:

$$\mathcal{M}(T) \subset \mathcal{H}_{G_2}.$$

Why?

- $\mathcal{M}(T)$ **inherits its inner product from** $\mathcal{H}_{G_1}$.
- **The structure of** $\mathcal{H}_G$ **is equivalent to that of** G.
- **Hence we will be able to deal with** $\varphi : G_1 \to G_2$ **via Hilbert space inclusion** $\mathcal{M}(T) \subset \mathcal{H}_{G_2}$.

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7. Quasi-orthogonal decomposition

$$\mathcal{H}_{G_2} = \mathcal{M}(T) \dot{+} \mathcal{H}(T), \quad \mathcal{H}(T) := \mathcal{M}(\sqrt{I - TT^*}).$$

- $\forall u \in \mathcal{M}(T)$ and $\forall v \in \mathcal{H}(T)$,

$$\|u+v\|^2_{\mathcal{H}_{G_2}} \leq \|u\|^2_{\mathcal{M}(T)} + \|v\|^2_{\mathcal{H}(T)},$$

- $\forall w \in \mathcal{H}_{G_2}, \exists u \in \mathcal{M}(T)$ and $\exists v \in \mathcal{H}(T)$ (uniquely) s.t. $\quad w = u + v \quad$ and $\quad \|w\|^2_{\mathcal{H}_{G_2}} = \|u\|^2_{\mathcal{M}(T)} + \|v\|^2_{\mathcal{H}(T)}$,

- $\mathcal{M}(T)$ and $\mathcal{H}(T)$ are RKHS's.

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φ, ψ: homorphisms, $T := C^*_\varphi / N_\varphi$, $S := C^*_\psi / N_\psi$,

Φ, Ψ: isomorphisms.

Then the commuting diagram

$$\begin{array}{ccc} G_1 & \xrightarrow{\varphi} & G_2 \\ {\scriptstyle\Phi}\big\downarrow & & \big\downarrow{\scriptstyle\Psi} \\ H_1 & \xrightarrow[\psi]{} & H_2 \end{array}$$

can be dealt with by Gram matrices of $\mathcal{M}(T)$, $\mathcal{H}(T)$, $\mathcal{M}(S)$ and $\mathcal{H}(S)$ (however, a litte complicated).

($K = (\langle k_{x_j}, k_{x_i} \rangle)_{i,j}$: the Gram matrix of k_x)

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Defn.

$$\text{ind } T := \dim \mathcal{H}(T^*) - \dim \mathcal{H}(T)$$

Thm.

$$|\varphi(V_1)| - |V_2| \leq \text{ind } T \leq |V_1| - |\varphi(V_1)|.$$

Further, if φ is injective then ind $T = |V_1| - |V_2|$.

Cor.

If $\varphi_j : G_j \to G_{j+1}$ $(j = 1, 2)$ are injective, then

$$\text{ind } T_2 T_1 = \text{ind } T_1 + \text{ind } T_2,$$

where

$$\mathcal{H}_{G_1} \overset{T_1 = C_{\varphi_1}^*}{\to} \mathcal{H}_{G_2} \overset{T_2 = C_{\varphi_2}^*}{\to} \mathcal{H}_{G_3}.$$

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graph theory	complex analysis
a vertex	a point
an edge	a n.b.d.
graphs	regions
$\mathcal{E}(u, u)$	$\int_\Omega \|f'(z)\|^2 \, dxdy$
$\mathcal{H}_G$	Dirichlet space
homomorphisms	holomorphic maps
injective homomorphisms	univalent functions
⋮	⋮

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Thm.
If $\varphi : G_1 \to G_2$ injective,

(i) $\dim \mathcal{H}(T) = |V_2| - |\{x \in V_1 : \deg x = \deg \varphi(x)\}|$

(ii) $\dim \mathcal{H}(T^*) = |V_1| - |\{x \in V_1 : \deg x = \deg \varphi(x)\}|.$

Thm.
If $\varphi : G_1 \to G_2$ bijective,

$$\frac{1}{2} \sum_{x \in V_1} \|(I - TT^*)\delta_{\varphi(x)}\|^2_{\mathcal{H}(T)} = |E_2| - |E_1|.$$

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graph theory	complex analysis
a vertex	a point
an edge	a n.b.d.
graphs	regions
$\mathcal{E}(u, u)$	$\int_\Omega |f'(z)|^2 \, dxdy$
$\mathcal{H}_G$	Dirichlet space
homomorphisms	holomorphic maps
injective homomorphisms	univalent functions
⋮	⋮
?	Löwner theory
???	Biberbach conjecture

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The Convergence of Relaxed Functional Iterations for Solving Quadratic Matrix Equations with an M-matrix

Pusan National University

Jong-Hyeon Seo

Introduction : Matrix Polynomial Equations 2

Quadratic Matrix Equation

Quadratic Matrix Equation (QME)

$$Q(X) := AX^2 - BX + C = 0$$

$$A, B, C \in \mathbb{C}^{m \times m}$$

$P(S) = 0 \Leftrightarrow S$ is a *solvent* [Dennis1976]

Contents

Notations of Matrices

- I_m : $m \times m$ identity matrix.
- 0 : zero matrix of any dimension.
- $\mathbf{1}_{m \times n}$: $m \times n$ matrix all of whose entries are 1.
- $\rho(A)$: the *spectral radius* of a square matrix A.

Preliminaries : Positive and Nonnegative Matrices 5

Positive and Nonnegative Matrices

Definition 2.4 Positive and Nonnegative Matrices

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be $m \times n$ real matrices.

1. A is called a *positive matrix* (-*nonnegative matrix*) if

$$a_{ij} > 0 \ (a_{ij} \geq 0), \quad \text{for all } 1 \leq i \leq m, \ 1 \leq j \leq n;.$$

2. $A > B$ ($A \geq B$) if $A - B$ is a positive matrix (-nonnegative matrix).

$$\mathbb{R}_{+}^{m \times n} := \{X \in \mathbb{R}^{m \times n} \ : \ X \geq 0\}.$$

Thus a nonnegative $m \times n$ matrix A is denoted by $A \in \mathbb{R}_{+}^{m \times n}$.

$\mathbb{N}_{+}$ denotes the set of all nonnegative integers.

Preliminaries : Positive and Nonnegative Matrices 6

Positive and Nonnegative Matrices

Theorem 2.8 Perron-Frobenius Theorem [Horn1985]

Let $A, B \in \mathbb{R}_{+}^{m \times m}$.

If $B \geq A$, $B \neq A$, then $\rho(B) \geq \rho(A)$, and if A is irreducible, then $\rho(B) > \rho(A)$.

Definition 2.9 Matrix Interval

Let $S, T \in \mathbb{R}^{m \times m}$, and suppose that $S \leq T$.

$$[S, T] := \{X \in \mathbb{R}^{m \times m} \ : \ S \leq X \leq T\}.$$

Definition 2.10 Stochastic Matrix [Horn1985]

Let $A \in \mathbb{R}_{+}^{m \times m}$.

A is called stochastic (substochastic) when $A\mathbf{1}_m = \mathbf{1}_m$ ($A\mathbf{1}_m \leq \mathbf{1}_m$).

Preliminaries : Elementwise Minimal Nonnegative Solvent 7

Elementwise Minimal Nonnegative Solvent

Definition 2.11 EMP-solvent or EMN-solvent [Kim2008]

Let F be a matrix function from $\mathbb{R}^{m\times n}$ to $\mathbb{R}^{m\times n}$.

A positive (-nonnegative) solution S_1 of the matrix equation $F(X) = 0$ is an elementwise minimal positive (-nonnegative) solution if for any positive (-nonnegative) solution S of $F(X) = 0$,

$$S_1 \leq S$$

Preliminaries : M-matrices 8

M-matrices

Definition 2.12 Z-matrix [Young1971]

Let $A \in \mathbb{R}^{m\times m}$. Then A is a Z-matrix if $Z = sI - B$ with $B \geq 0$

Definition 2.13 M-matrix [Guo2007]

$A \in \mathbb{R}^{m\times m}$ is an M-matrix if $\exists B \geq 0$ such that $A = sI - B$ and $s \geq \rho(B)$;

it is a singular M-matrix if $s = \rho(B)$ and a nonsingular M-matrix if $s > \rho(B)$.

Let $A \in \mathbb{R}^{m\times m}$. Then

- A is a Z-matrix $\Leftrightarrow A \in \mathbf{Z}$.
- A is an M-matrix $\Leftrightarrow A \in \mathbf{M}^{*}$.
- A is a nonsingular M-matrix $\Leftrightarrow A \in \mathbf{M}$.

M-matrices

Theorem 2.14 Standard Properties of a nonsingular M-matrix [Poole1974]

Let $A \in \mathbf{Z}$, the following are equivalent:

1. $A \in \mathbf{M}$.
2. $A^{-1} \geq 0$
3. $\exists v > 0$ s.t. $Av > 0$.

Theorem 2.15 Standard Properties of an M-matrix [Berman1994]

1. If $A \in \mathbf{Z}$ and $A \geq B$ for an M-matrix $B \neq A$, then $A \in \mathbf{M}$.
2. If $A \in \mathbf{Z}$ and $\exists v > 0$ such that $Av \geq 0$, then $A \in \mathbf{M}^{\star}$.

Theorem 2.16 If $A \in \mathbf{M}$, then $Av \geq 0$ implies $v \geq 0$. [Guo2009, Lem. 2]

Theorem 2.17 If $A \in \mathbf{M}$, then $Av > 0$ implies $v > 0$.

Theorem 2.18 Let $A, B \in \mathbf{M}$. Then if $A \leq B$, then $B^{-1} \leq A^{-1}$. [Horn1994]

Neumann Lemma

Lemma 2.25 **Neumann Lemma** [Ortega2000]

For $A, B \in \mathbb{C}^{m \times m}$, if A be nonsingular and $\rho(A^{-1}B) < 1$, then $A - B$ is also nonsingular and represented by

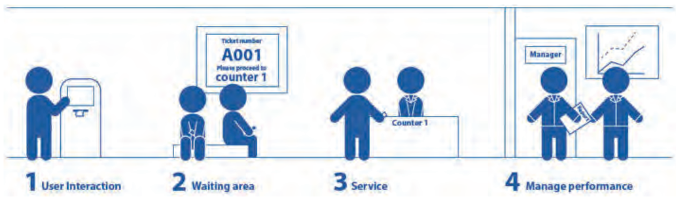
$$(A-B)^{-1} = A^{-1} + A^{-1}B(A-B)^{-1}.$$

$$(A-B)^{-1} = \left(\sum_{k=0}^{\infty}(A^{-1}B)^k\right)A^{-1} = A^{-1} + \left(\sum_{k=1}^{\infty}(A^{-1}B)^k\right)A^{-1}$$

Practical Examples : Queuing System 11

Queuing System

http://www.smartqueue.com.au



Queuing System

	Inter-Arrival Time	Service Time	# of Servers
M/M/1	Normal Distribution	Normal Distribution	1
PH/M/1	Erlang Distribution	Normal Distribution	1
M/G/1	Normal Distribution	General Distribution	1
$G^X/G/1$	General Dist. & Batch arr.	General Distribution	1

Practical Examples : Matrix-Analytic Method – QBD process 12

Quasi-Birth-Death Process

PH/M/1

$$\mathcal{P}_{QBD} = \begin{bmatrix} D & A & & & \\ C & B' & A & & \\ & C & B' & A & \\ & & C & B' & A \\ & & & \ddots & \ddots & \ddots \end{bmatrix},$$

Transition Matrix

Probabilistic Conditions

$$A, B', C, D, \in \mathbb{R}_+^{m\times m} \quad \text{and} \quad (A + B' + C)\mathbf{1}_m = \mathbf{1}_m.$$

Stationary Probability Vector

$$x^T \mathcal{P}_{QBD} = x^T \quad \|x\|_1 = 1.$$

Practical Examples : Matrix-Analytic Method – QBD process 13

Quasi-Birth-Death Process

PH/M/1

$$\mathcal{P}_{QBD} = \begin{bmatrix} D & A & & & \\ C & B' & A & & \\ & C & B' & A & \\ & & C & B' & A \\ & & & \ddots & \ddots & \ddots \end{bmatrix},$$

$$\begin{cases} x_0^T D + x_1^T C = x_0^T \\ x_k^T A + x_{k+1}^T B' + x_{k+2}^T C = x_k^T, \quad k = 1, 2, \ldots. \end{cases}$$

Let $x^T := \begin{bmatrix} x_0^T, & x_1^T, & \cdots \end{bmatrix}$, where $x_i \in \mathbb{R}_+^m$, $i = 0, 1, 2, \ldots$.

$$x^T \mathcal{P}_{QBD} = x^T, \quad \|x\|_1 = 1.$$

Practical Examples : Matrix-Analytic Method – QBD process 14

Quasi-Birth-Death Process

$$\begin{cases} x_0^T D + x_1^T C = x_0^T \\ x_k^T A + x_{k+1}^T B' + x_{k+2}^T C = x_k^T, \quad k = 1, 2, \ldots. \end{cases}$$

$x_{k+1}^T = x_k^T R$, $k = 0, 1, 2, \ldots$ [Latouche1999. Thm. 6.2.1]

$$\begin{cases} x_0^T (D + RC) = x_0^T \\ x_k^T (A - RB + R^2 C) = 0, \quad k = 0, 1, 2, \ldots, \end{cases}$$

$$B := I - B'.$$

$$A - XB + X^2 C = 0$$

Ramaswami's formula [Favati1998]

$$x_0^T (D + RC) = x_0^T$$
$$\left\| x_0 (I - R)^{-1} \right\|_1 = 1.$$

Practical Examples : Matrix-Analytic Method – QBD process 15

Quasi-Birth-Death Process

PH/M/1

$$\mathcal{P}_{QBD} = \begin{bmatrix} D & A & & & \\ C & B' & A & & \\ & C & B' & A & \\ & & C & B' & A \\ & & & \ddots & \ddots & \ddots \end{bmatrix},$$

If any one of the matrices G, or R is known, then we may determine the other.

Latouche and Ramaswami

$$Q(G) = AG^2 - BG + C = 0$$

$$\mathcal{P}_{QBD}x = x, \qquad \|x\|_1 = 1.$$

Applications : QME from the noisy Wiener-Hopf problems 16

QME from the noisy Wiener-Hopf problems

$$Z^2 \mp VZ + Q = 0$$

V : a diagonal matrix which has positive *and* negative diagonal elements

$$-Q \in \mathbf{M}$$

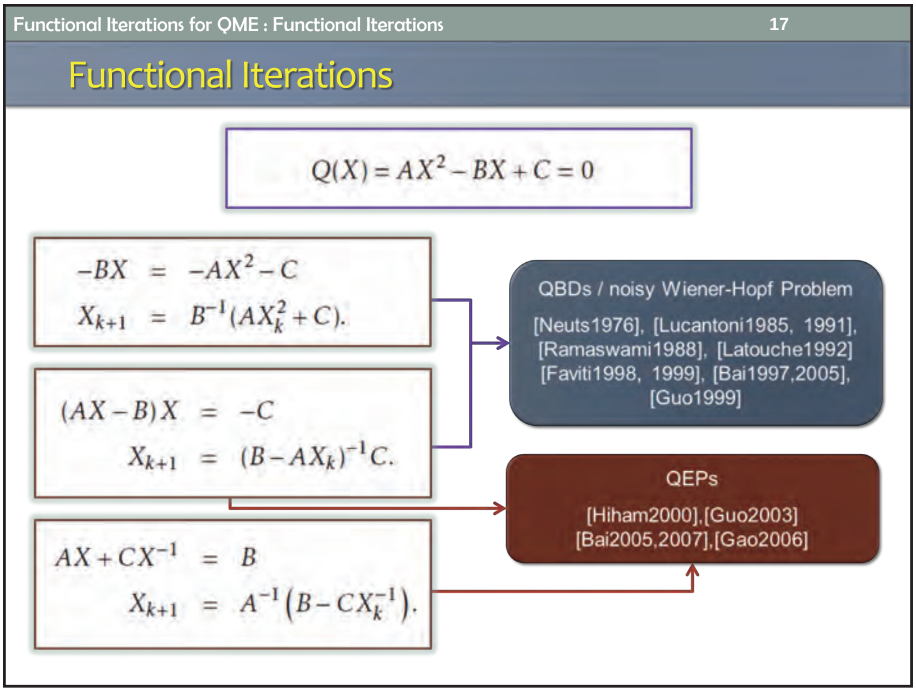
[Guo2003] $X = \sigma I - Z$

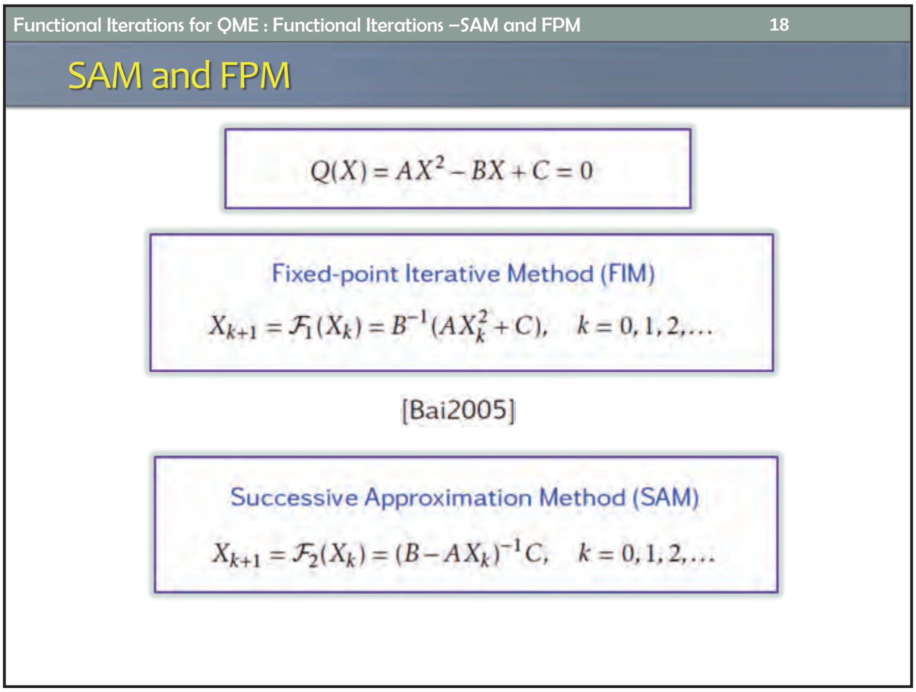
$$X^2 - (2\sigma I_m \mp V)X + (\sigma^2 I_m \mp \sigma V + Q) = 0$$

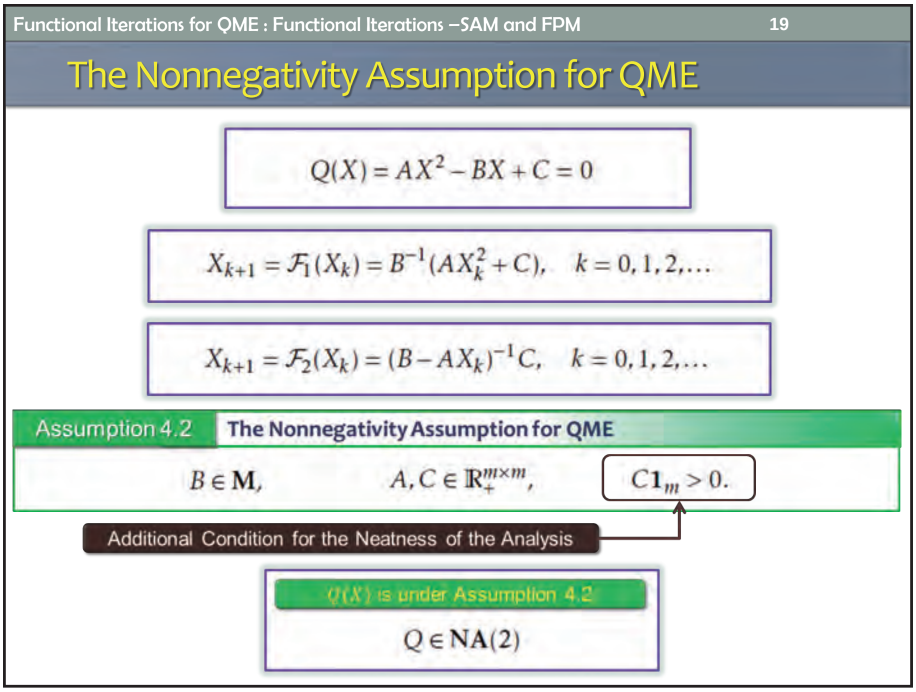
A nonsingular M-matrix

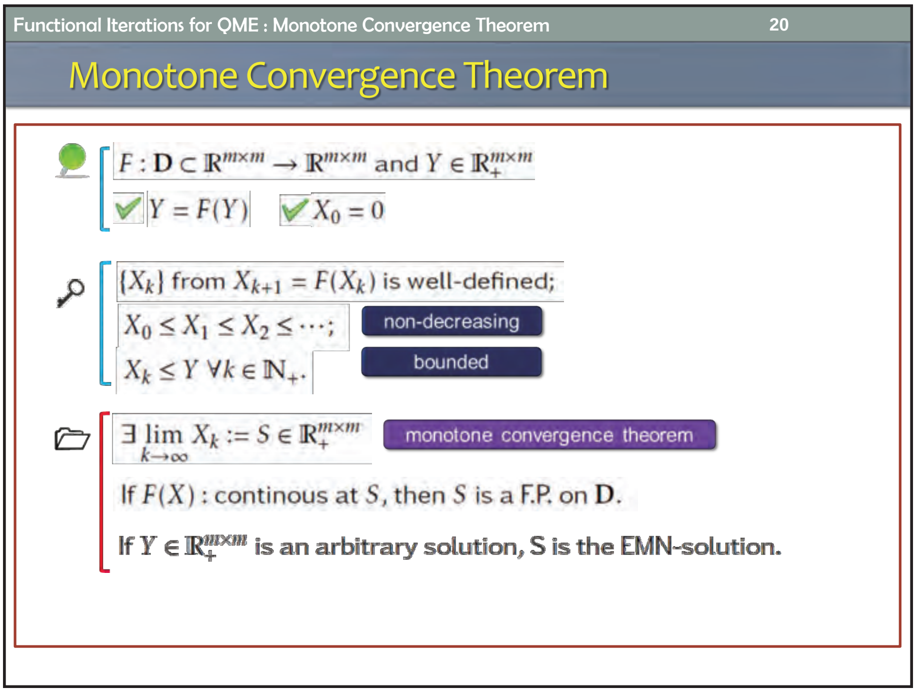
A nonnegative matrix

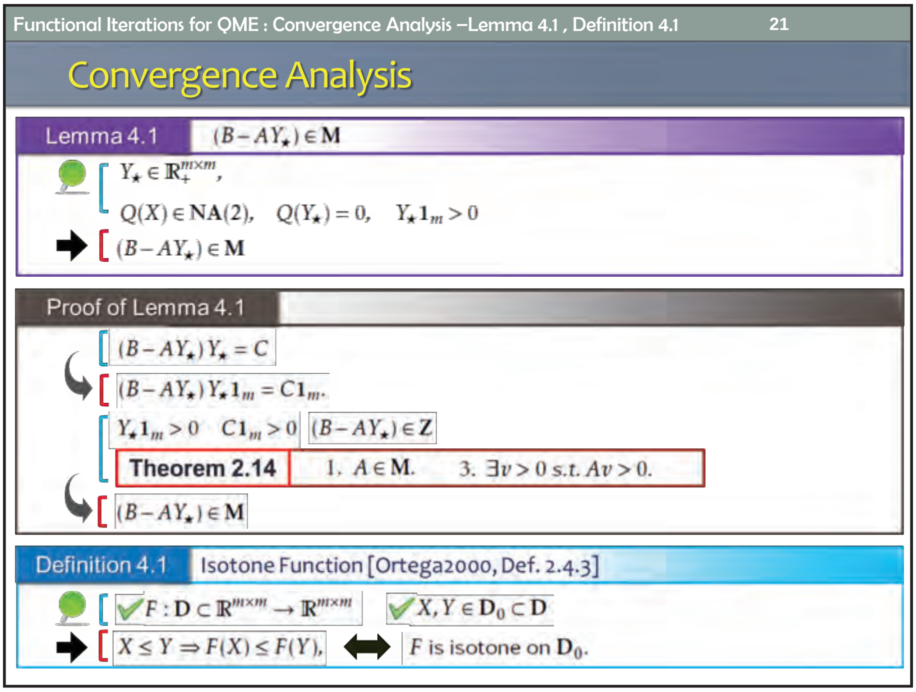
$$\sigma = \max_{1 \le i \le m} \left([V]_{ii} + \sqrt{[V]_{ii}^2 - 4[Q]_{ii}} \right) / 2.$$

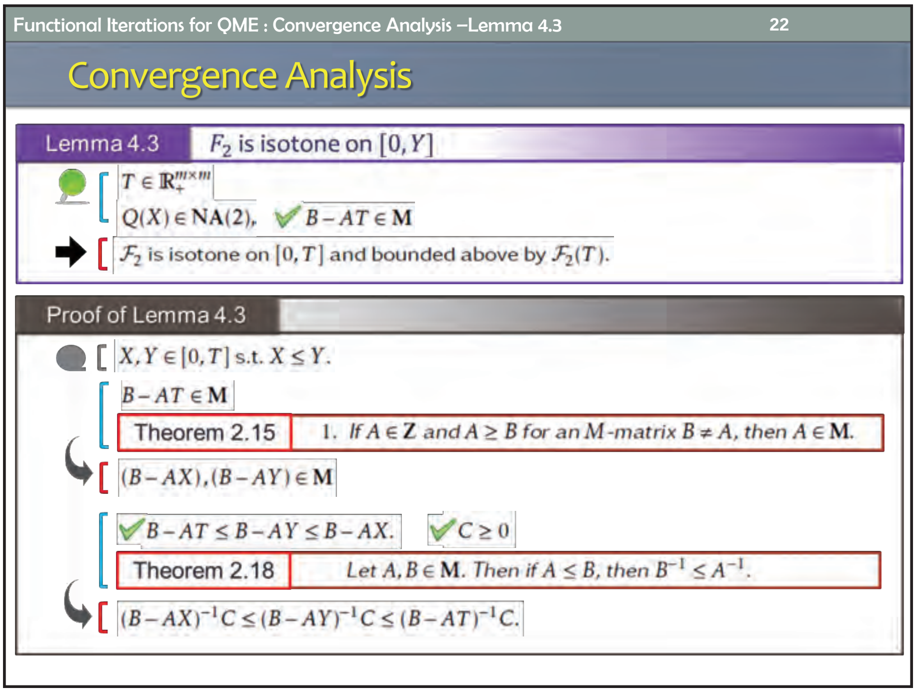

Functional Iterations for QME : Functional Iterations
17
Functional Iterations
$Q(X) = AX^2 - BX + C = 0$
$-BX = -AX^2 - C$
$X_{k+1} = B^{-1}(AX_k^2 + C).$
$(AX - B)X = -C$
$X_{k+1} = (B - AX_k)^{-1}C.$
$AX + CX^{-1} = B$
$X_{k+1} = A^{-1}(B - CX_k^{-1}).$
QBDs / noisy Wiener-Hopf Problem
[Neuts1976], [Lucantoni1985, 1991], [Ramaswami1988], [Latouche1992] [Faviti1998, 1999], [Bai1997,2005], [Guo1999]
QEPs
[Hiham2000],[Guo2003] [Bai2005,2007],[Gao2006]

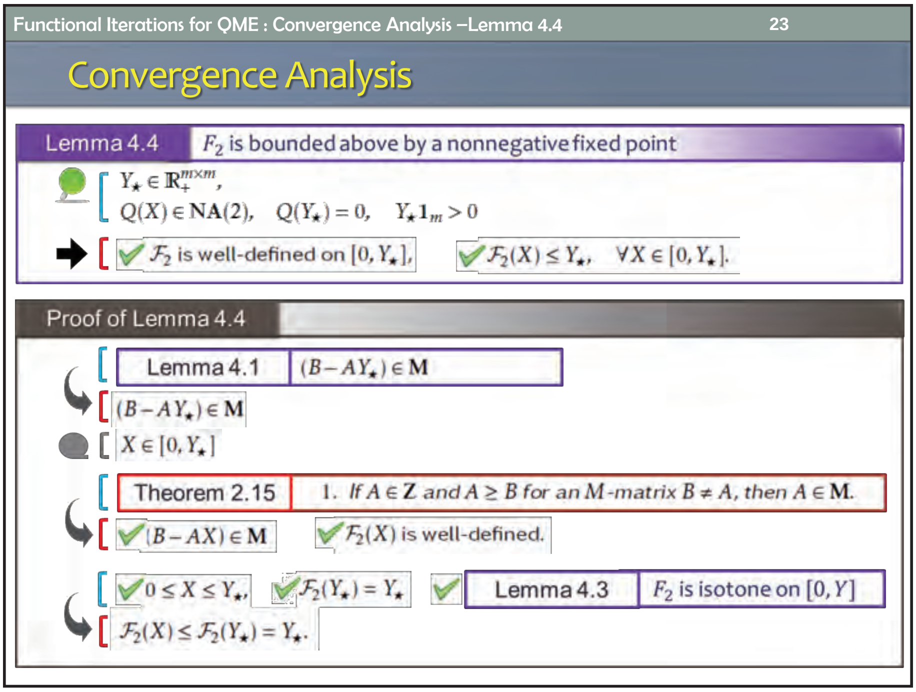

Functional Iterations for QME : Functional Iterations –SAM and FPM
18
SAM and FPM
$Q(X) = AX^2 - BX + C = 0$
Fixed-point Iterative Method (FIM)
$X_{k+1} = \mathcal{F}_1(X_k) = B^{-1}(AX_k^2 + C), \quad k = 0, 1, 2, \ldots$
[Bai2005]
Successive Approximation Method (SAM)
$X_{k+1} = \mathcal{F}_2(X_k) = (B - AX_k)^{-1}C, \quad k = 0, 1, 2, \ldots$

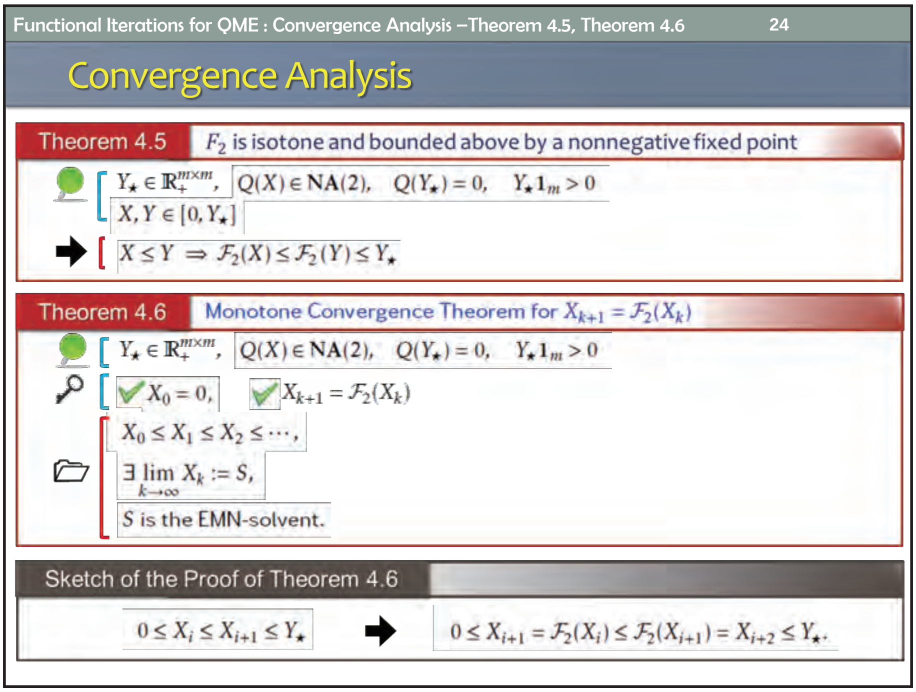

Functional Iterations for QME : Functional Iterations –SAM and FPM
19
The Nonnegativity Assumption for QME
$Q(X) = AX^2 - BX + C = 0$
$X_{k+1} = \mathcal{F}_1(X_k) = B^{-1}(AX_k^2 + C), \quad k = 0, 1, 2, \ldots$
$X_{k+1} = \mathcal{F}_2(X_k) = (B - AX_k)^{-1}C, \quad k = 0, 1, 2, \ldots$
Assumption 4.2
The Nonnegativity Assumption for QME
$B \in \mathbf{M}, \quad A, C \in \mathbb{R}_+^{m\times m}, \quad C\mathbf{1}_m > 0.$
Additional Condition for the Neatness of the Analysis
$Q(X)$ is under Assumption 4.2
$Q \in \mathbf{NA}(2)$

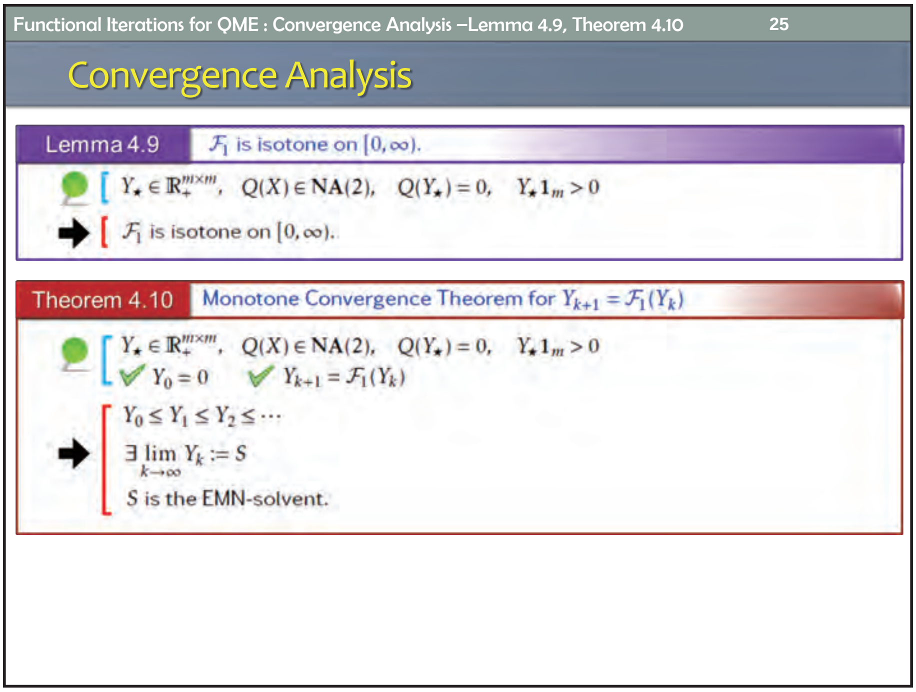

Functional Iterations for QME : Monotone Convergence Theorem
20
Monotone Convergence Theorem
$F : \mathbf{D} \subset \mathbb{R}^{m\times m} \to \mathbb{R}^{m\times m}$ and $Y \in \mathbb{R}_+^{m\times m}$
$Y = F(Y)$
$X_0 = 0$
$\{X_k\}$ from $X_{k+1} = F(X_k)$ is well-defined;
$X_0 \le X_1 \le X_2 \le \cdots;$
non-decreasing
$X_k \le Y \ \forall k \in \mathbb{N}_+.$
bounded
$\exists \lim_{k\to\infty} X_k := S \in \mathbb{R}_+^{m\times m}$
monotone convergence theorem
If $F(X)$: continous at S, then S is a F.P. on $\mathbf{D}$.
If $Y \in \mathbb{R}_+^{m\times m}$ is an arbitrary solution, S is the EMN-solution.


Functional Iterations for QME : Convergence Analysis –Lemma 4.1 , Definition 4.1
21
Convergence Analysis
Lemma 4.1
$(B-AY_\star)\in \mathbf{M}$
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X)\in \mathbf{NA}(2)$, $Q(Y_\star)=0$, $Y_\star \mathbf{1}_m > 0$
$(B-AY_\star)\in \mathbf{M}$
Proof of Lemma 4.1
$(B-AY_\star)Y_\star = C$
$(B-AY_\star)Y_\star \mathbf{1}_m = C\mathbf{1}_m$.
$Y_\star \mathbf{1}_m > 0$
$C\mathbf{1}_m > 0$
$(B-AY_\star)\in \mathbf{Z}$
Theorem 2.14
1. $A\in \mathbf{M}$.
3. $\exists v>0$ s.t. $Av>0$.
$(B-AY_\star)\in \mathbf{M}$
Definition 4.1
Isotone Function [Ortega2000, Def. 2.4.3]
$F: \mathbf{D}\subset \mathbb{R}^{m\times m} \to \mathbb{R}^{m\times m}$
$X,Y\in \mathbf{D}_0 \subset \mathbf{D}$
$X\le Y \Rightarrow F(X)\le F(Y)$,
F is isotone on $\mathbf{D}_0$.


Functional Iterations for QME : Convergence Analysis –Lemma 4.3
22
Convergence Analysis
Lemma 4.3
F_2 is isotone on $[0,Y]$
$T\in \mathbb{R}_+^{m\times m}$
$Q(X)\in \mathbf{NA}(2)$,
$B-AT\in \mathbf{M}$
$\mathcal{F}_2$ is isotone on $[0,T]$ and bounded above by $\mathcal{F}_2(T)$.
Proof of Lemma 4.3
$X,Y\in[0,T]$ s.t. $X\le Y$.
$B-AT\in \mathbf{M}$
Theorem 2.15
1. If $A\in \mathbf{Z}$ and $A\ge B$ for an M-matrix $B\ne A$, then $A\in \mathbf{M}$.
$(B-AX),(B-AY)\in \mathbf{M}$
$B-AT\le B-AY\le B-AX$.
$C\ge 0$
Theorem 2.18
Let $A,B\in \mathbf{M}$. Then if $A\le B$, then $B^{-1}\le A^{-1}$.
$(B-AX)^{-1}C\le (B-AY)^{-1}C\le (B-AT)^{-1}C$.


Functional Iterations for QME : Convergence Analysis –Lemma 4.4
23
Convergence Analysis
Lemma 4.4
F_2 is bounded above by a nonnegative fixed point
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in \mathbf{NA}(2)$, $Q(Y_\star) = 0$, $Y_\star \mathbf{1}_m > 0$
$\mathcal{F}_2$ is well-defined on $[0, Y_\star]$,
$\mathcal{F}_2(X) \le Y_\star$, $\forall X \in [0, Y_\star]$.
Proof of Lemma 4.4
Lemma 4.1
$(B - AY_\star) \in \mathbf{M}$
$(B - AY_\star) \in \mathbf{M}$
$X \in [0, Y_\star]$
Theorem 2.15
1. If $A \in \mathbf{Z}$ and $A \ge B$ for an M-matrix $B \ne A$, then $A \in \mathbf{M}$.
$(B - AX) \in \mathbf{M}$
$\mathcal{F}_2(X)$ is well-defined.
$0 \le X \le Y_\star$,
$\mathcal{F}_2(Y_\star) = Y_\star$
Lemma 4.3
F_2 is isotone on $[0, Y]$
$\mathcal{F}_2(X) \le \mathcal{F}_2(Y_\star) = Y_\star$.


Functional Iterations for QME : Convergence Analysis –Theorem 4.5, Theorem 4.6
24
Convergence Analysis
Theorem 4.5
F_2 is isotone and bounded above by a nonnegative fixed point
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in \mathbf{NA}(2)$, $Q(Y_\star) = 0$, $Y_\star \mathbf{1}_m > 0$
$X, Y \in [0, Y_\star]$
$X \le Y \Rightarrow \mathcal{F}_2(X) \le \mathcal{F}_2(Y) \le Y_\star$
Theorem 4.6
Monotone Convergence Theorem for $X_{k+1} = \mathcal{F}_2(X_k)$
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in \mathbf{NA}(2)$, $Q(Y_\star) = 0$, $Y_\star \mathbf{1}_m > 0$
$X_0 = 0$,
$X_{k+1} = \mathcal{F}_2(X_k)$
$X_0 \le X_1 \le X_2 \le \cdots$,
$\exists \lim_{k\to\infty} X_k := S$,
S is the EMN-solvent.
Sketch of the Proof of Theorem 4.6
$0 \le X_i \le X_{i+1} \le Y_\star$
$0 \le X_{i+1} = \mathcal{F}_2(X_i) \le \mathcal{F}_2(X_{i+1}) = X_{i+2} \le Y_\star$.


Functional Iterations for QME : Convergence Analysis –Lemma 4.9, Theorem 4.10
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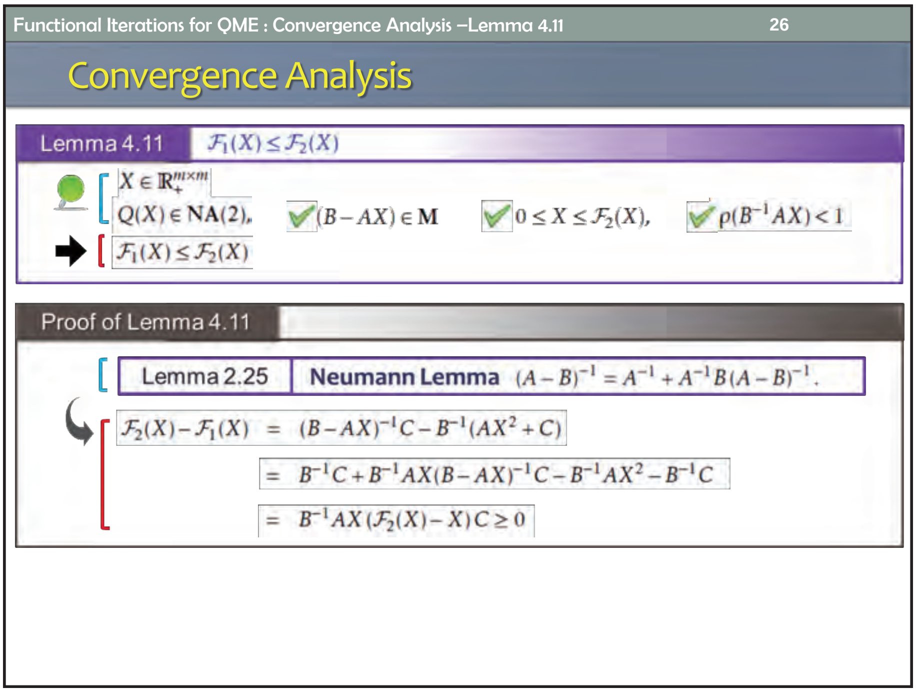
Convergence Analysis
Lemma 4.9
$\mathcal{F}_1$ is isotone on $[0,\infty)$.
$Y_\star \in \mathbb{R}_+^{m\times m}$, $Q(X)\in \mathbf{NA}(2)$, $Q(Y_\star)=0$, $Y_\star \mathbf{1}_m > 0$
$\mathcal{F}_1$ is isotone on $[0,\infty)$.
Theorem 4.10
Monotone Convergence Theorem for $Y_{k+1}=\mathcal{F}_1(Y_k)$
$Y_\star \in \mathbb{R}_+^{m\times m}$, $Q(X)\in \mathbf{NA}(2)$, $Q(Y_\star)=0$, $Y_\star \mathbf{1}_m > 0$
$Y_0 = 0$ $Y_{k+1}=\mathcal{F}_1(Y_k)$
$Y_0 \le Y_1 \le Y_2 \le \cdots$
$\exists \lim_{k\to\infty} Y_k := S$
S is the EMN-solvent.


Functional Iterations for QME : Convergence Analysis –Lemma 4.11
26
Convergence Analysis
Lemma 4.11
$\mathcal{F}_1(X) \le \mathcal{F}_2(X)$
$X \in \mathbb{R}_+^{m\times m}$
$Q(X)\in \mathbf{NA}(2)$, $(B-AX)\in \mathbf{M}$ $0 \le X \le \mathcal{F}_2(X)$, $\rho(B^{-1}AX)<1$
$\mathcal{F}_1(X) \le \mathcal{F}_2(X)$
Proof of Lemma 4.11
Lemma 2.25
Neumann Lemma $(A-B)^{-1}=A^{-1}+A^{-1}B(A-B)^{-1}$.
$\mathcal{F}_2(X)-\mathcal{F}_1(X) = (B-AX)^{-1}C - B^{-1}(AX^2+C)$
$= B^{-1}C + B^{-1}AX(B-AX)^{-1}C - B^{-1}AX^2 - B^{-1}C$
$= B^{-1}AX(\mathcal{F}_2(X)-X)C \ge 0$

Functional Iterations for QME : Convergence Analysis –Lemma 4.12 27

Convergence Analysis

Lemma 4.12 $Y_0 \le X_0 \Rightarrow G(Y_k) \le F(X_k)$

$F, G : \mathbf{D} \subset \mathbb{R}^{m\times m} \to \mathbb{R}^{m\times m}$

$\exists X_0, Y_0 \in \mathbf{D}$ s.t. $Y_0 \le X_0$, $X_{k+1} = F(X_k)$ and $Y_{k+1} = G(Y_k)$ are well-defined on $\mathbf{D}$.

$G(X_k) \le F(X_k), \quad \forall k \in \mathbb{N}_+$

G is isotone on $\mathbf{D}$

$Y_k \le X_k, \quad k = 0, 1, 2, \ldots.$

Proof of Lemma 4.12

$Y_i \le X_i$

✓ $G(Y_i) = Y_{i+1} \le G(X_i)$, ✓ $G(X_i) \le F(X_i) = X_{i+1}$.

$Y_{i+1} \le X_{i+1}$.

$Y_k \le X_k$ means that if two sequences have a same nonnegative limit, then X_k is more close to the limit than Y_k.

Functional Iterations for QME : Convergence Analysis –Theorem 4.13 28

Convergence Analysis

Theorem 4.13 $X_0 = Y_0 = 0 \Rightarrow \rho(\mathcal{F}_2(X_k) - S) \le \rho(\mathcal{F}_1(Y_k) - S)$

$Y_\star \in \mathbb{R}_+^{m\times m}$,

$Q(X) \in \mathbf{NA}(2)$, $Q(Y_\star) = 0$, $Y_\star \mathbf{1}_m > 0$ ✓ $\rho(B^{-1}AY_\star) < 1$

✓ $X_0 = Y_0 = 0$ ✓ $X_{k+1} = \mathcal{F}_2(X_k)$, $Y_{k+1} = \mathcal{F}_1(Y_k)$.

$\rho(X_k - S) \le \rho(Y_k - S), \qquad k = 0, 1, 2, \ldots$

S is the EMN-solvent

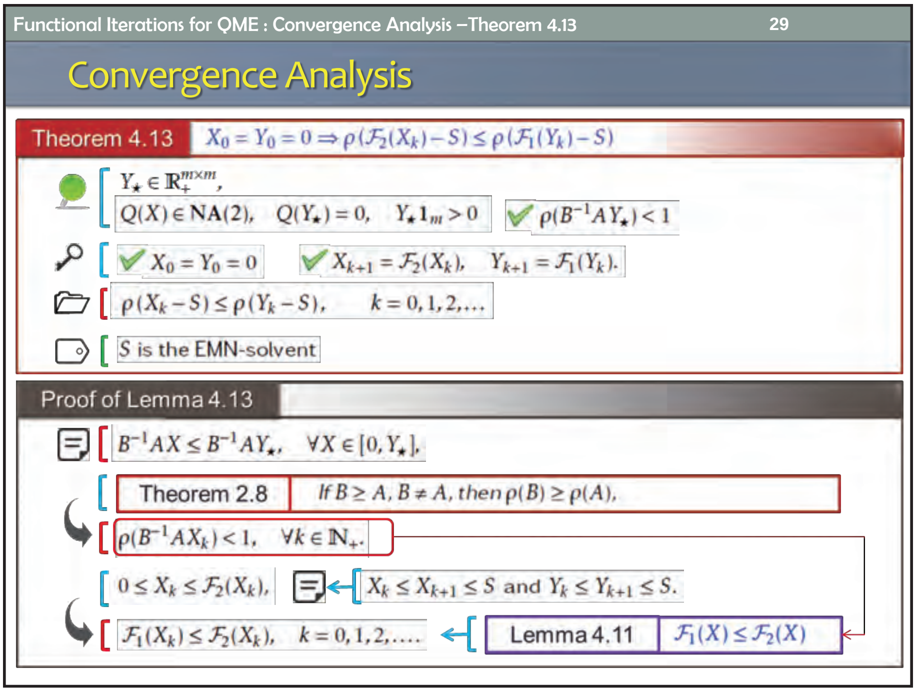
Proof of Lemma 4.13

Theorem 4.6	Monotone Convergence Theorem for $X_{k+1} = \mathcal{F}_2(X_k)$
Theorem 4.10	Monotone Convergence Theorem for $Y_{k+1} = \mathcal{F}_1(Y_k)$

$X_k \le X_{k+1} \le S$ and $Y_k \le Y_{k+1} \le S$.

$B^{-1} \ge 0$ ← Theorem 2.14: 1. $A \in \mathbf{M}$. 2. $A^{-1} \ge 0$

$B^{-1}AX \le B^{-1}AY_\star, \quad \forall X \in [0, Y_\star]$,


Functional Iterations for QME : Convergence Analysis –Theorem 4.13
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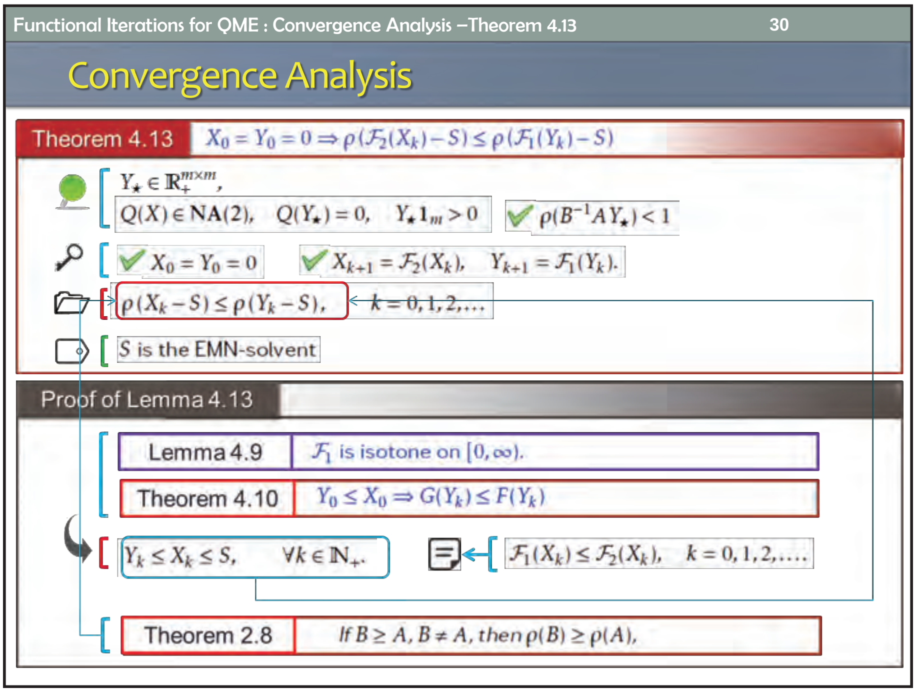
Convergence Analysis
Theorem 4.13
$X_0 = Y_0 = 0 \Rightarrow \rho(\mathcal{F}_2(X_k) - S) \le \rho(\mathcal{F}_1(Y_k) - S)$
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in \mathrm{NA}(2), \quad Q(Y_\star) = 0, \quad Y_\star 1_m > 0$
$\rho(B^{-1}AY_\star) < 1$
$X_0 = Y_0 = 0$
$X_{k+1} = \mathcal{F}_2(X_k), \quad Y_{k+1} = \mathcal{F}_1(Y_k).$
$\rho(X_k - S) \le \rho(Y_k - S), \quad k = 0, 1, 2, \ldots$
S is the EMN-solvent
Proof of Lemma 4.13
$B^{-1}AX \le B^{-1}AY_\star, \quad \forall X \in [0, Y_\star].$
Theorem 2.8
If $B \ge A, B \ne A$, then $\rho(B) \ge \rho(A).$
$\rho(B^{-1}AX_k) < 1, \quad \forall k \in \mathbb{N}_+.$
$0 \le X_k \le \mathcal{F}_2(X_k),$
$X_k \le X_{k+1} \le S$ and $Y_k \le Y_{k+1} \le S.$
$\mathcal{F}_1(X_k) \le \mathcal{F}_2(X_k), \quad k = 0, 1, 2, \ldots.$
Lemma 4.11
$\mathcal{F}_1(X) \le \mathcal{F}_2(X)$


Functional Iterations for QME : Convergence Analysis –Theorem 4.13
30
Convergence Analysis
Theorem 4.13
$X_0 = Y_0 = 0 \Rightarrow \rho(\mathcal{F}_2(X_k) - S) \le \rho(\mathcal{F}_1(Y_k) - S)$
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in \mathrm{NA}(2), \quad Q(Y_\star) = 0, \quad Y_\star 1_m > 0$
$\rho(B^{-1}AY_\star) < 1$
$X_0 = Y_0 = 0$
$X_{k+1} = \mathcal{F}_2(X_k), \quad Y_{k+1} = \mathcal{F}_1(Y_k).$
$\rho(X_k - S) \le \rho(Y_k - S), \quad k = 0, 1, 2, \ldots$
S is the EMN-solvent
Proof of Lemma 4.13
Lemma 4.9
$\mathcal{F}_1$ is isotone on $[0, \infty).$
Theorem 4.10
$Y_0 \le X_0 \Rightarrow G(Y_k) \le F(Y_k)$
$Y_k \le X_k \le S, \quad \forall k \in \mathbb{N}_+.$
$\mathcal{F}_1(X_k) \le \mathcal{F}_2(X_k), \quad k = 0, 1, 2, \ldots.$
Theorem 2.8
If $B \ge A, B \ne A$, then $\rho(B) \ge \rho(A).$

Functional Iterations for QME : Relaxed Functional Iteration - Introduce 31

Relaxed Functional Iteration

Generally, choosing a proper relaxation (especially over-relaxation) factor which accelerate the convergence speed is very difficult problem

because the problem much involves computing eigenvalues of a solution and estimating the asymtotic rate of convergence.

[Favati1998], [Young1971]

This seem to be a quite antinomy because the merit of functional iterations are on the simple implementation.

Our work is to propose a new relaxed algorithm of which relaxation factors are computed by only basic scalar-matrix operations and not to use any aspects related to eigenvalue problems.

Functional Iterations for QME : Relaxed Functional Iteration – Relaxed SAM 32

Relaxed Functional Iteration

Relaxed Iterative Method with $\Lambda[X]$

$$Z_{k+1} = \mathcal{R}[\Lambda](Z_k), \qquad k = 0, 1, 2, \ldots$$

$$\mathcal{R}[\Lambda](X) = \mathcal{F}_2(X) + \boxed{\Lambda[X]}(\mathcal{F}_2(X) - X)$$

Relaxation Operator

Ordinary Relaxed Iteration

$$X_{k+1} = (1 - \lambda_k)\mathcal{F}_2(X_k) + \lambda_k X_k.$$

Functional Iterations for QME : Relaxed Functional Iteration – Lemma 4.15 33

Relaxed Functional Iteration

Lemma 4.15 $\mathcal{F}_2(X+H) \geq \mathcal{F}_2(X) + (B-AX)^{-1}AH\mathcal{F}_2(X).$

$X \in \mathbb{R}_+^{m\times m}$

$Q(X) \in \mathbf{NA}(2),$ $(B-AX) \in \mathbf{M}$

$\rho((B-AX)^{-1}A\mathcal{F}_2(X)) < 1,$

$(B-AX)^{-1}AH\mathcal{F}_2(X) \geq 0, \quad \forall H \geq 0,$

$\mathcal{F}_2(X+H) \geq \mathcal{F}_2(X) + (B-AX)^{-1}AH\mathcal{F}_2(X), \quad \forall H \in [0, \mathcal{F}_2(X)].$

Proof of Lemma 4.15

Lemma 2.25 **Neumann Lemma** $(A-B)^{-1} = A^{-1} + A^{-1}B(A-B)^{-1}.$

$$\begin{aligned}
\mathcal{F}_2(X+H) &= ((B-AX)-AH)^{-1}C \\
&= (B-AX)^{-1}C \\
&\quad + \left(\sum_{k=1}^{\infty} ((B-AX)^{-1}AH)^k\right)\mathcal{F}_2(X) \\
&= \mathcal{F}_2(X) + (B-AX)^{-1}AH\mathcal{F}_2(X) + o(H) \\
&\geq \mathcal{F}_2(X) + (B-AX)^{-1}AH\mathcal{F}_2(X).
\end{aligned}$$

Functional Iterations for QME : Relaxed Functional Iteration – Lemma 4.16 34

Relaxed Functional Iteration

Lemma 4.16 $\mathcal{F}_2(X) \leq \mathcal{R}[\Lambda](X) \leq \mathcal{F}_2(\mathcal{F}_2(X))$

$X \in \mathbb{R}_+^{m\times m}$ $\quad Q \in \mathbf{NA}(2)$

✓ $(B-AX) \in \mathbf{M}$ ✓ $\rho((B-AX)^{-1}A\mathcal{F}_2(X)) < 1,$ ✓ $0 \leq X \leq \mathcal{F}_2(X)$

$0 \leq \Lambda[X](H) \leq (B-AX)^{-1}AH\mathcal{F}_2(X), \quad \forall H \geq 0,$

$\mathcal{F}_2(X) \leq \mathcal{R}[\Lambda](X) \leq \mathcal{F}_2(\mathcal{F}_2(X)).$

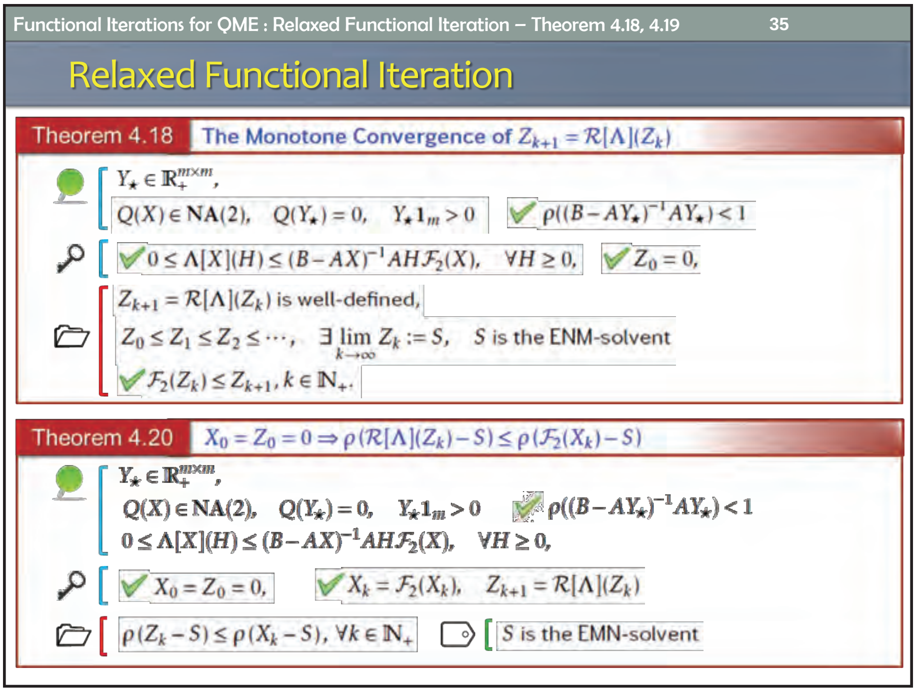
Theorem 4.17 $X \leq \mathcal{F}_2(X) \Rightarrow \mathcal{F}_2(X) \leq \mathcal{R}[\Lambda](X) \leq \mathcal{F}_2(\mathcal{F}_2(X)) \leq Y_\star$

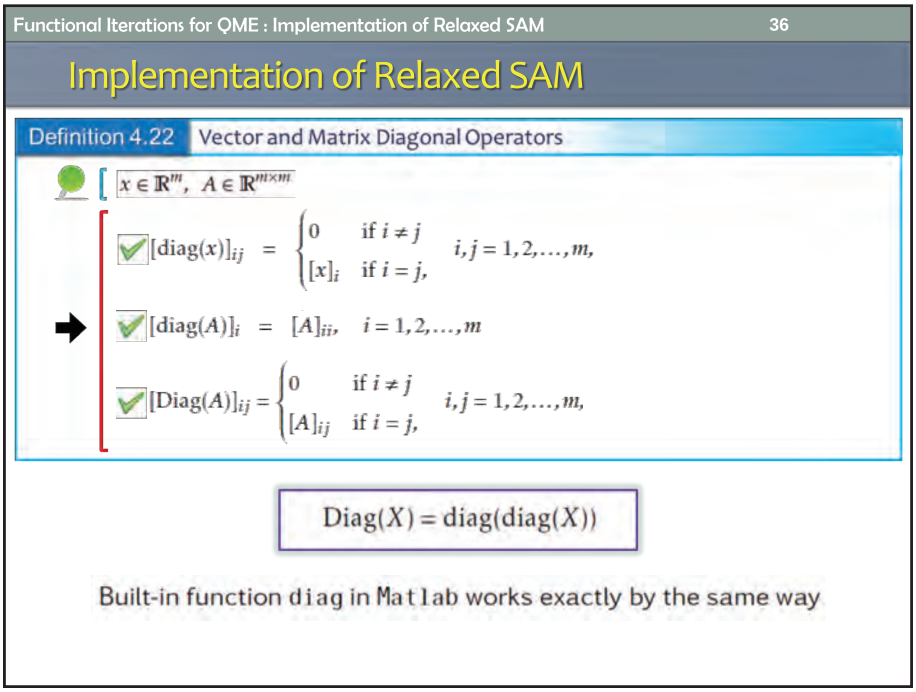
$Y_\star \in \mathbb{R}_+^{m\times m},$

$Q(X) \in \mathbf{NA}(2), \quad Q(Y_\star) = 0, \quad Y_\star \mathbf{1}_m > 0$

✓ $\rho((B-AY_\star)^{-1}AY_\star) < 1$ ✓ $0 \leq \Lambda[X](H) \leq (B-AX)^{-1}AH\mathcal{F}_2(X), \quad \forall H \geq 0,$

$X \leq \mathcal{F}_2(X) \Rightarrow \mathcal{F}_2(X) \leq \mathcal{R}[\Lambda](X) \leq \mathcal{F}_2(\mathcal{F}_2(X)) \leq Y_\star$


Functional Iterations for QME : Relaxed Functional Iteration – Theorem 4.18, 4.19
35
Relaxed Functional Iteration
Theorem 4.18
The Monotone Convergence of $Z_{k+1} = \mathcal{R}[\Lambda](Z_k)$
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in NA(2)$, $Q(Y_\star) = 0$, $Y_\star 1_m > 0$
$\rho((B - AY_\star)^{-1}AY_\star) < 1$
$0 \le \Lambda[X](H) \le (B - AX)^{-1}AH\mathcal{F}_2(X)$, $\forall H \ge 0$,
$Z_0 = 0$,
$Z_{k+1} = \mathcal{R}[\Lambda](Z_k)$ is well-defined,
$Z_0 \le Z_1 \le Z_2 \le \cdots$, $\exists \lim_{k\to\infty} Z_k := S$, S is the ENM-solvent
$\mathcal{F}_2(Z_k) \le Z_{k+1}, k \in \mathbb{N}_+$.
Theorem 4.20
$X_0 = Z_0 = 0 \Rightarrow \rho(\mathcal{R}[\Lambda](Z_k) - S) \le \rho(\mathcal{F}_2(X_k) - S)$
$Y_\star \in \mathbb{R}_+^{m\times m}$,
$Q(X) \in NA(2)$, $Q(Y_\star) = 0$, $Y_\star 1_m > 0$
$\rho((B - AY_\star)^{-1}AY_\star) < 1$
$0 \le \Lambda[X](H) \le (B - AX)^{-1}AH\mathcal{F}_2(X)$, $\forall H \ge 0$,
$X_0 = Z_0 = 0$,
$X_k = \mathcal{F}_2(X_k)$, $Z_{k+1} = \mathcal{R}[\Lambda](Z_k)$
$\rho(Z_k - S) \le \rho(X_k - S)$, $\forall k \in \mathbb{N}_+$
S is the EMN-solvent


Functional Iterations for QME : Implementation of Relaxed SAM
36
Implementation of Relaxed SAM
Definition 4.22
Vector and Matrix Diagonal Operators
$x \in \mathbb{R}^m$, $A \in \mathbb{R}^{m\times m}$
$[\mathrm{diag}(x)]_{ij} = \begin{cases} 0 & \text{if } i \ne j \\ [x]_i & \text{if } i = j, \end{cases}$ $i, j = 1, 2, \ldots, m$,
$[\mathrm{diag}(A)]_i = [A]_{ii}$, $i = 1, 2, \ldots, m$
$[\mathrm{Diag}(A)]_{ij} = \begin{cases} 0 & \text{if } i \ne j \\ [A]_{ij} & \text{if } i = j, \end{cases}$ $i, j = 1, 2, \ldots, m$,
$\mathrm{Diag}(X) = \mathrm{diag}(\mathrm{diag}(X))$
Built-in function diag in Matlab works exactly by the same way

Implementation of Relaxed SAM

$$\Lambda_1[X](H) := \mathrm{diag}((\mathbf{1}_m + d \circ \mathrm{diag}(X)) \circ d) \cdot H \cdot \mathrm{Diag}(\mathcal{F}_2(X))$$

$$\Lambda_2[X](H) := \mathrm{diag}(d) \cdot H \cdot \mathrm{Diag}(\mathcal{F}_2(X))$$

$d = \mathrm{diag}(B^{-1}A)$,

$E \circ F = [e_{ij} f_{ij}] \in \mathbb{R}^{m\times m}$ for $E = [e_{ij}] \in \mathbb{R}^{m\times m}$, $F = [f_{ij}] \in \mathbb{R}^{m\times m}$

Hadamard Product [Horn1994]

$$\begin{aligned}(B - AX)^{-1}A &\geq (I + B^{-1}AX)^{-1} B^{-1}A \\ &\geq (I + \mathrm{Diag}(B^{-1}A) \cdot \mathrm{Diag}(X)) \cdot \mathrm{Diag}(B^{-1}A) \\ &= \mathrm{diag}((\mathbf{1}_m + d \circ \mathrm{diag}(X)) \circ d)\end{aligned}$$

Numerical Experimentation

We have tested the iterative methods FIM, SAM, relaxed-SAM on the examples in [Elhafsi2007], [Latouche1993] and [Guo2003] by MATLAB.

IT : denote the number of iterations,

$$\mathrm{RES} := \|Q(\tilde{S})\|_\infty = \|A\tilde{S}^2 - B\tilde{S} + C\|_\infty$$

"ρ" to denote $\rho((B - A\tilde{S})^{-1}A\tilde{S})$.

an approximation of the solvent

The Stopping Criterion

$$\|X_k - X_{k-1}\|_\infty \leq 10^{-10}$$

Numerical Experimentation

Example 4.26 QME for the continuous Markov chain in [Elhafsi2007]

$$A = \begin{bmatrix} \mu & & & \\ & \mu & & \\ & & \ddots & \\ & & & \mu \end{bmatrix}, \quad C = \begin{bmatrix} \lambda & & & \\ & \lambda & & \\ & & \ddots & \\ & & & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} \lambda+\mu & -\mu & & \\ & \lambda+2\mu & \ddots & \\ & & \ddots & -\mu \\ & & & \lambda+2\mu \end{bmatrix}$$

$$0 < \lambda < 1, \quad \mu = 1-\lambda, \quad m = 24$$

Numerical Experimentation

	$\lambda = 0.9293$		$\lambda = 0.8407$		$\lambda = 0.8143$		$\lambda = 0.2543$	
	$\rho = 0.0781$		$\rho = 0.1895$		$\rho = 0.2281$		$\rho = 0.3410$	
Method	IT	RES	IT	RES	IT	RES	IT	RES
FIM	13	0.0638	22	0.1619	25	0.2822	38	0.3089
SAM without	11	0.0064	16	0.0688	18	0.0739	25	0.1489
SAM with Λ_1	9	0.0096	13	0.0063	14	0.0091	17	0.0966
SAM with Λ_2	9	0.0124	13	0.0195	14	0.0396	19	0.0287
SAM($\lambda = 0.1$)	12	0.0005	14	0.0008	17	0.0002	25	0.0007
SAM($\lambda = 0.2$)	17	0.0007	17	0.0007	17	0.0008	21	0.0003
SAM($\lambda = 0.3$)	22	0.0016	22	0.0015	22	0.0014	17	0.0011
SAM($\lambda = 0.4$)	28	0.0037	28	0.0032	28	0.0028	22	0.0012
SAM($\lambda = 0.5$)	37	0.0034	38	0.0054	36	0.0044	28	0.0013
SAM($\lambda = 0.6$)	49	0.0054	48	0.0056	48	0.0042	35	0.0024
SAM($\lambda = 0.7$)	69	0.0063	66	0.0081	66	0.0053	45	0.0027
SAM($\lambda = 0.8$)	106	0.0094	100	0.0082	98	0.0069	59	0.0037
SAM($\lambda = 0.9$)	209	0.0098	181	0.0096	173	0.0096	82	0.0038

Functional Iterations for QME : Numerical Experimentation – Example 4.26 41

Numerical Experimentation

	$\lambda = 0.3500$		$\lambda = 0.6160$		$\lambda = 0.5678$		$\lambda = 0.4733$	
	$\rho = 0.5384$		$\rho = 0.6233$		$\rho = 0.7611$		$\rho = 0.8986$	
Method	IT	RES	IT	RES	IT	RES	IT	RES
FIM	68	0.6390	88	0.7325	154	0.8036	381	0.9212
SAM without	41	0.3148	52	0.2750	88	0.3776	204	0.4356
SAM with Λ_1	26	0.0961	31	0.1777	51	0.1976	120	0.2811
SAM with Λ_2	29	0.1313	35	0.2483	58	0.2934	138	0.3483
SAM($\lambda = 0.1$)	43	0.0018	55	0.0020	92	0.0037	224	0.0039
SAM($\lambda = 0.2$)	38	0.0013	49	0.0017	84	0.0025	205	0.0034
SAM($\lambda = 0.3$)	33	0.0016	44	0.0013	76	0.0025	188	0.0033
SAM($\lambda = 0.4$)	29	0.0013	39	0.0015	69	0.0025	174	0.0030
SAM($\lambda = 0.5$)	29	0.0017	35	0.0012	64	0.0017	162	0.0025
SAM($\lambda = 0.6$)	38	0.0034	43	0.0048	58	0.0020	150	0.0027
SAM($\lambda = 0.7$)	47	0.0026	58	0.0040	56	0.0034	140	0.0025
SAM($\lambda = 0.8$)	62	0.0035	81	0.0053	77	0.0048	130	0.0022
SAM($\lambda = 0.9$)	86	0.0052	125	0.0089	116	0.0064	105	0.0021

Functional Iterations for QME : Numerical Experimentation – Example 4.27 42

Numerical Experimentation

Example 4.27 QME for the teletraffic system in [Latouch1997]

diagonal matrices

$A' = 192\rho_d I_{24}, \quad [C']_{ii} = 192(1-(i-1)/24),\ 1 \le i \le 24$

tridiagonal matrices

$[B']_{i,i+1} = ar(M-(i-1))/M,\ 1 \le i \le 23 \qquad [B']_{i,i-1} = (i-1)r,\ 2 \le i \le 24$

QME

$$AX^2 - X + C = 0$$

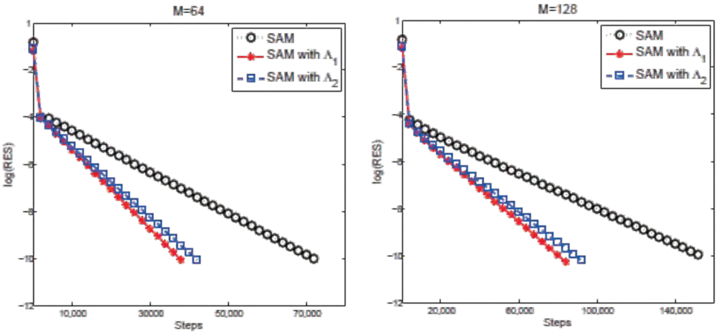
$$C = -B'^{-1}C', \quad A = -B'^{-1}A'$$

$$r = 1/300,\ a = 18.244,\ \rho_d = 0.28$$

Functional Iterations for QME : Numerical Experimentation – Example 4.27 43

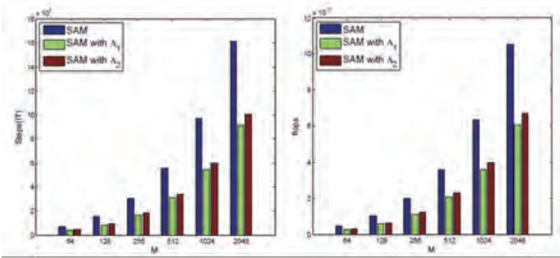
Numerical Experimentation

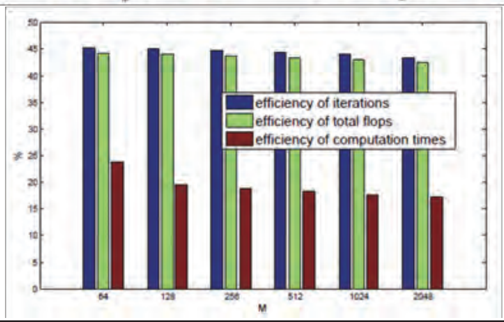
	SAM without		SAM with Λ_1		Efficiency	
M	IT(Steps)	Time (s.)	IT(Steps)	Time (s.)	IT(Steps)	Time
64	73,090	11.2472	40,020	8.5776	45%	24%
128	155,055	16.5822	85,271	13.3640	45%	19%
256	302,111	31.9773	166,890	25.9333	45%	19%
512	554,197	58.2796	307,945	47.6649	44%	18%
1024	969,440	101.8492	542,803	83.9380	44%	18%
2048	1,615,006	176.4935	913,167	146.0151	43%	17%



Functional Iterations for QME : Numerical Experimentation – Example 4.27 44

Numerical Experimentation





Functional Iterations for QME : Numerical Experimentation – Example 4.28 45

Numerical Experimentation

Example 4.28 QME from the Wiener-Hopf problems in [Guo2003]

$$Z^2 \mp VZ + Q = 0$$

V : a diagonal matrix which has positive *and* negative diagonal elements

$$-Q \in \mathbf{M}$$

$$V = \begin{bmatrix} aI_{10} & \\ & bI_{10} \end{bmatrix}, \quad Q = \begin{bmatrix} -1 & 1 & & \\ & -1 & \ddots & \\ & & \ddots & 1 \\ 1 & & & -1 \end{bmatrix} \in \mathbb{R}^{20\times 20},$$

(1) $a = 1, b = -1$ (2) $a = 2, b = -1$ (3) $a = 2, b = -0.1$ (4) $a = 1, b = -3.$

Functional Iterations for QME : Numerical Experimentation – Example 4.28 46

Numerical Experimentation

$$Z^2 - VZ + Q = 0$$

	(1)		(2)		(3)		(4)	
	$\rho = 0.7422$		$\rho = 0.6278$		$\rho = 0.5956$		$\rho = 0.6887$	
Method	IT	RES	IT	RES	IT	RES	IT	RES
FIM	379,797	2.2361	270	2.5040	112	2.160	155	1.1607
SAM without	309,085	1.6180	218	1.9836	82	1.7556	134	1.3991
SAM with Λ_1	263,400	1.1525	135	1.1317	52	0.9861	83	0.7064
SAM with Λ_2	264,188	1.1157	135	1.2848	53	0.8690	83	0.7224
SAM($\lambda = 0.1$)	failed		237	0.0195	88	0.0114	142	0.0116
SAM($\lambda = 0.2$)	failed		217	0.0169	79	0.0120	129	0.0107
SAM($\lambda = 0.3$)	failed		200	0.0489	72	0.0096	118	0.0098
SAM($\lambda = 0.4$)	failed		185	0.0138	66	0.0075	109	0.0079
SAM($\lambda = 0.5$)	failed		172	0.0127	61	0.0108	100	0.0091
SAM($\lambda = 0.6$)	failed		161	0.0110	86	0.0084	96	0.0080
SAM($\lambda = 0.7$)	failed		151	0.0098	130	0.0116	144	0.0088
SAM($\lambda = 0.8$)	failed		227	0.0109	226	0.0116	255	0.0098
SAM($\lambda = 0.9$)	failed		604	0.0121	715	0.0133	failed	

Numerical Experimentation

$$Z^2 + VZ + Q = 0$$

	(1)		(2)		(3)		(4)	
	$\rho = 0.7422$		$\rho = 0.5098$		$\rho = 0.4242$		$\rho = 0.6425$	
Method	IT	RES	IT	RES	IT	RES	IT	RES
FIM	379797	2.2236	215	1.8579	73	1.1726	248	3.1980
SAM without	309,086	1.6180	172	1.5222	57	0.9156	218	2.6760
SAM with Λ_1	263,400	1.1525	114	0.8165	43	0.4494	124	1.3505
SAM with Λ_2	264,187	1.1565	114	0.8590	43	0.4619	124	1.5293
SAM($\lambda = 0.1$)	failed		186	0.0136	59	0.0085	234	0.0240
SAM($\lambda = 0.2$)	failed		170	0.0117	53	0.0059	214	0.0211
SAM($\lambda = 0.3$)	failed		156	0.0109	48	0.0037	196	0.0217
SAM($\lambda = 0.4$)	failed		144	0.0102	52	0.0066	182	0.0176
SAM($\lambda = 0.5$)	failed		134	0.0086	71	0.0077	169	0.0165
SAM($\lambda = 0.6$)	failed		125	0.0077	111	0.0064	158	0.0142
SAM($\lambda = 0.7$)	failed		146	0.0072	212	0.0079	147	0.0157
SAM($\lambda = 0.8$)	failed		287	0.0092	1077	0.0098	204	0.0146
SAM($\lambda = 0.9$)	failed		failed		failed		495	0.0163

Thank you!

Computing resultant matrix of general multivariate polynomials and its determinant using Magma

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in cooperation with

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Hakata Workshop 2014, February 8th, 2014

Magma package is available on:
`http://imi.kyushu-u.ac.jp/~s-yokoyama/Resultant.html`

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First Remark 1

- This slides were used on February 8, 2014 at Hakata Workshop.
- Some slides were revised or deleted to be published as the proceedings.

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First Remark 2

- No topics on combinatorics
- No proofs
- No original mathematical ideas

Aim of this work: Improve the environment of Magma[1]

Important remark:
Our implementation is quite fast but cannot mark the world record[2].

- "Kimura's interpolation method" is the fastest implementation to compute resultant/discriminant with some great tools to treat matrices with symbolic elements.
- Magma's implementation of Kimura's interpolation is in progress.

[1] A computer algebra system, the University of Sydney.
[2] Compared with **Singular**, **Maple**+sdmp etc.

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Target problem

Compute the following determinant called **resultant efficiently**

$$Res_x(f,g) = \begin{vmatrix} a_m & a_{m-1} & \cdots & a_0 & & & \\ & a_m & a_{m-1} & \cdots & a_0 & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & & a_m & a_{m-1} & \cdots & a_0 \\ b_n & b_{n-1} & \cdots & \cdots & b_0 & & \\ & b_n & b_{n-1} & \cdots & \cdots & b_0 & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & b_n & b_{n-1} & \cdots & \cdots & b_0 \end{vmatrix}$$

A definition:
Determinant of the Sylvester matrix for given two polynomials
$f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_0$ and
$g(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_0$.

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Applications 1: for **Number** (or **Group**) **theory**

For given G and 2-dim'l (Galois) rep'n $\rho : G \to GL(V)$, identify where $Frob_p$ lands inside $C \subset G$ to compute $Tr(\rho(Frob_p)) = a_p$. (a_ps are from modular forms - analytic information)

Theorem (T. and V. Dokchitser, 2013)

Let $(a_i)_{1\leq i\leq \deg(K)}$ be the roots of P in K. For $C \subset Gal(K/\mathbb{Q})$ and some $h \in \mathbb{Q}[x]$, we define $\Gamma_C^h(X) := \prod_{\sigma\in C}\left(X - \sum_{1\leq i\leq \deg(K)} h(a_i)\sigma(a_i)\right)$. Then the following equivalence relation holds for almost all p:

$$Frob_p \in C \iff \Gamma_C^h\left(Tr_{\frac{\mathbb{F}_p[x]}{ch.pol(K)}/\mathbb{F}_p}(h(x)x^p)\right) \equiv 0 \bmod p.$$

"almost all" means: not dividing the denominators of the coefficients of P, its leading coefficient and $Res(\Gamma_C^h, \Gamma_{C'}^h)$ for $C \neq C' \subset G$.

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Applications 2: for **Computer algebra**

Quantifier Elimination (QE) can be used for industrial research:

- First-order formula can be translated into some algebraic constraints – without quantifiers, $\forall$ and $\exists$ for example. (A. Tarski: 1930, G. Collins: 1975)
- Toy example:

$$\exists x \in \mathbb{R}\ s.t.\ x^2 + bx + c = 0 \implies b^2 - 4c \geq 0$$

- Cost: **doubly exponential**
  $\implies$ virtual substitution, Sturm-Habicht sequence
- **Cylindrical Algebraic Decomposition** (CAD) is a key ingredient. Efficient computation of the resultant can improve the CAD algorithm.
- Interesting application of QE: use as a solver of high-school math problems – Todai Robot Project (NII, Fujitsu labs.)

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Applications 3: for **Computer graphics**

Can be used for **intersection algorithm** for planar parametric rational polynomial curves $\subset \mathbb{RP}^3$:

- Application for ray tracing and path tracing. (finding parametric patches)
- J. Kajiya (1982) produced an algorithm to compute some intermediate operation using efficient computation of the resultant of 4 variables[3].

[3] Coefficients become quite complicated!!

Benchmark

Definition

Discriminant of $f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_0$ is

$$Disc_x(f) = (-1)^{m(m-1)/2} \frac{1}{a_m} Res_x(f, f')$$

Benchmark: $m = 8$

Deg	#Terms	Built-in	Berkowitz	Minor+Opt.	× Speed
8	5247	15.460	0.281	0.047	328.9

Magma ver. 2.20-1 on Win7 64bit / Intel Core i7-2630QM 3.30GHz / 8GB Mem. / **cpusec.**

Built-in: Geddes-Czapor-Labahn's interpolation technique
Berkowitz: Berkowitz's algorithm (explained later)
Minor+Opt.: Minor expansion + optimization (explained later)

Why we choose Magma?

For the efficient implementation, the following CASs are recommended:

1. Mathematica
2. Maple
3. **Magma**
4. Singular
5. Sage
6. Professional CAS/subsystem（e.g. Risa/Asir, TRIP）.

The reason why we choose Magma:

- Contains high-speed algorithms on the computation of multivariate polynomials **except resultant**.
- Also contains great implementations for computing determinant of the matrix.

Speed-up/Step 1: Reduce the size of the matrix

(T. Seki)-Bézout matrix

ex.) For $f(x) = a_2x^2 + a_1x + a_0$, $g(x) = b_1x + b_0$, resultant matrix can be represented as

$$Res_x(f,g) = \begin{vmatrix} a_2 & a_1 & a_0 \\ b_1 & b_0 & 0 \\ 0 & b_1 & b_0 \end{vmatrix} \Longrightarrow Res_x(f,g) = \begin{vmatrix} b_0 & b_1 \\ -a_0b_1 & a_2b_0 - a_1b_1 \end{vmatrix}.$$

The size of $Res_x(f,g)$ can be $max(deg_x(f), deg_x(g))$. More precisely:

$$A = \begin{pmatrix} a_mb_0 - a_{m-n}b_n & \cdots & \cdots \\ \cdots & & \\ a_mb_{n-1} - a_{m-1}b_n & \cdots & \cdots \\ b_n & b_{n-1} & \cdots \\ & b_n & \cdots \\ & & \cdots \end{pmatrix}, v = \begin{pmatrix} x^{m-1} & \cdots & 1 \end{pmatrix}^T, Av = 0$$

Fact: $Res_x(f,g) = |A|$.

The size of each element can also be bounded!!

Speed-up/Step 2: Compute the determinant

We select (1) and (3) of the following:

1. **Minor expansion** No division, $O(2^n)$
2. **Fraction-free Gaussian elimination** Division required, $O(n^3)$
3. **Berkowitz's algorithm** No division, $O(n^4)$

(3)* Idea: $A_* = A$, $k = \dim(A_*) - 1$ and

$$A_* = \begin{pmatrix} A_k & S \\ R & a_{k+1,k+1} \end{pmatrix} \Longrightarrow T_{A_*} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ -a_{k+1,k+1} & 1 & \cdots & 0 \\ -RS & -a_{k+1,k+1} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ -RA_k^{k-2}S & -RA_k^{k-3}S & \cdots & 1 \\ -RA_k^{k-1}S & -RA_k^{k-2}S & \cdots & -a_{k+1,k+1} \end{pmatrix}$$

to compute eigenvectors. Repeat and terminate if $\dim(A_*) = 1$.

Works well for large matrix size but not faster than Kimura's interpolation technique.

* S. Berkowitz, *On computing the determinant in small parallel time using a small number of processors*, Inform. Process. Lett. **18** (1984), pp.147-150.

Speed-up/Step 3: Optimize well

ex.) Minor expansion:

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} \text{ can be expanded as}$$

$$\begin{aligned} & a_1(b_2(\underline{c_3d_4 - d_3c_4}) - c_2(\underline{b_3d_4 - d_3b_4}) + d_2(\underline{b_3c_4 - c_3b_4})) - b_1(a_2(\underline{c_3d_4 - d_3c_4}) \\ & -c_2(\underline{a_3d_4 - d_3a_4}) + d_2(\underline{a_3c_4 - c_3a_4})) + c_1(a_2(\underline{b_3d_4 - d_3b_4}) - b_2(\underline{a_3d_4 - d_3a_4}) \\ & +d_2(\underline{a_3b_4 - b_3a_4})) - d_1(a_2(\underline{b_3c_4 - c_3b_4}) - b_2(\underline{a_3c_4 - c_3a_4}) + c_2(\underline{a_3b_4 - b_3a_4})) \end{aligned}$$

Use the intermediate data repeatedly. Moreover,

- A problem from hash functions (from CAS impl. with Lisp language)
- + more techniques

Sample code

```
for K in [L..J+1 by -1] do
 for B2 in [B1..Q[K+1] by -1] do
  U:=U+CT[B2+1][V+1];
 B1:=Q[K+1]-2;
 V:=V-1;
```

Benchmark

Definition

Discriminant of $f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_0$ is

$$Disc_x(f) = (-1)^{m(m-1)/2} \frac{1}{a_m} Res_x(f, f')$$

Benchmark: $7 \leq m \leq 12$

Deg	#Terms	Built-in	Berkowitz	Minor+Opt.	× Speed
7	1103	2.387	0.047	0.026	91.8
8	5247	15.460	0.281	0.047	328.9
9	26059	204.174	3.120	0.390	523.5
10	133881	3201.349	46.207	3.994	801.5
11	706799	≥ 15hrs	907.372	48.064	≥ 1123.5
12	3815311	NT	NT	1004.287	—

Magma ver. 2.20-1 on Win7 64bit / Intel Core i7-2630QM 3.30GHz / 8GB Mem. / **cpusec.**

Benchmark: vs. Cayley

Special case: **Cayley's method**

- Only for computing the general formula of the discriminant.
- $Disc_x(f)$ of degree $m - 1$ is required to compute $Disc_x(f)$ of degree m.
- Faster than Minor+Opt.

Unfortunately, too slow compared with **Singular**.

Benchmark: vs. Cayley

Definition

Discriminant of $f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_0$ is

$$Disc_x(f) = (-1)^{m(m-1)/2} \frac{1}{a_m} Res_x(f, f')$$

Special case: Cayley's method **Benchmark:** $7 \le m \le 12$

Deg	#Terms	Built-in	Berkowitz	Minor+Opt.	vs. Cayley
7	1103	2.387	0.047	0.026	$\le$ 0.01
8	5247	15.460	0.281	0.047	0.031
9	26059	204.174	3.120	0.390	0.125
10	133881	3201.349	46.207	3.994	0.920
11	706799	$\ge$ 15hrs	907.372	48.064	8.237
12	3815311	NT	NT	1004.287	59.093

Magma ver. 2.20-1 on Win7 64bit / Intel Core i7-2630QM 3.30GHz / 8GB Mem. / **cpusec.**

Future Plan

In progress: Implement and optimize Kimura's interpolation method using Magma's user language.

Technical remark: **Intel AVX support**

Picture the future now Intel® AVX

Since June 26th, 2013 (v2.19-7) for Linux OS:

- **SIMD** (Single Instruction Multiple Data)
- **HTT** (Intel Hyper-Threading Technology)

However, current Magma's implementations (including our new package) are not optimized for AVX/HTT.

Moreover, HTT is not suitable for the research of symbolic computation (except some cases).

Software in Mathematics Demonstration Track

1

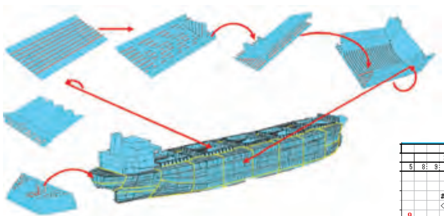
HEAPモデルによるプル型スケジューリング

HEAPモデルの概要と、スケジューリングプログラムの紹介

九州大学工学府海洋システム工学専攻　岩下 寛弥

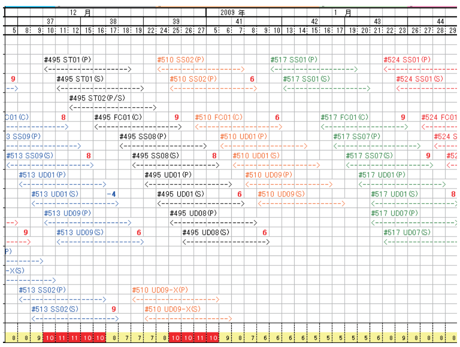
2

研究の背景：造船所の現状



- 船舶はブロック単位に分割して建造される。
- このブロックの製造工程計画を現在はEXCELにて、手作業で行っている。

工程計画を自動化したい



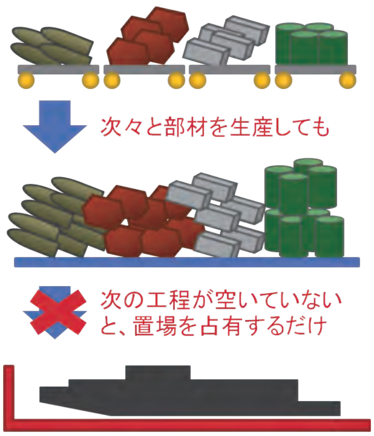
3

研究の背景 ： 造船所の現状

- 次のことを無視して、次々とブロックを製造している。
⇒プッシュ型スケジューリング
- 無駄な待機時間の発生
(アイドルタイム)
- ブロック置き場から溢れ、通路を塞ぐ等、生産効率の低下を招いている。

後の工程から考えて最適なタイミングで部品の投入、製造の開始を行いたい

プル型スケジューリングの適用



4

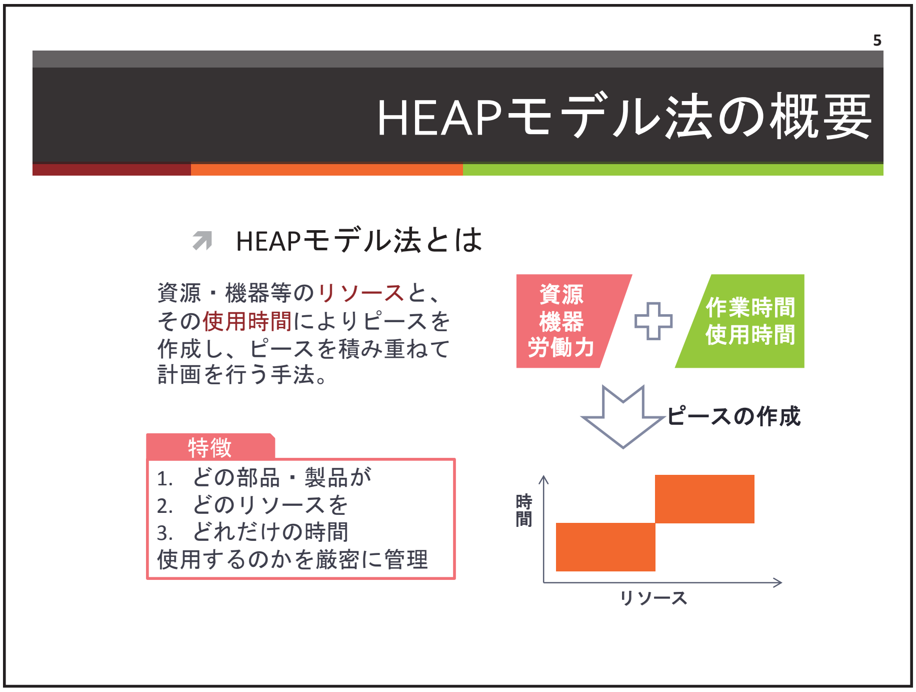
研究の目的

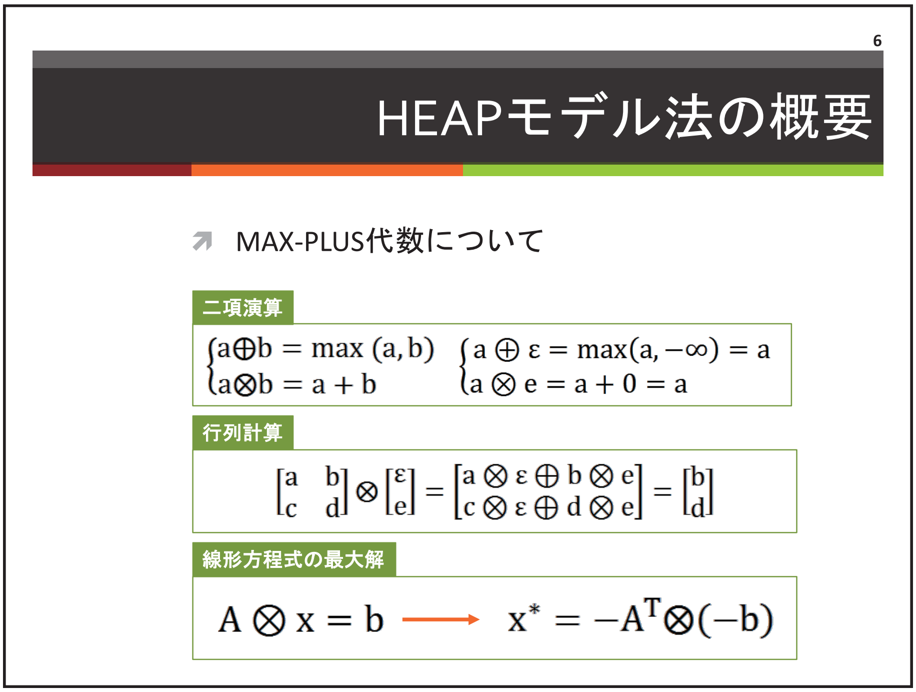
- プル型スケジューリングの適用
- 設備制約を守った工程計画
- 無駄な待機時間を最小化

以上の要望を満たす工程計画手法の開発

HEAPモデル法によるプル型スケジューリング

⇨ 提案された工程計画手法を造船所のデータへ適用するため、MATLABを用いたプログラムを開発した。


5
HEAPモデル法の概要
HEAPモデル法とは
資源・機器等のリソースと、その使用時間によりピースを作成し、ピースを積み重ねて計画を行う手法。
資源
機器
労働力
作業時間
使用時間
ピースの作成
時間
リソース
特徴
1. どの部品・製品が
2. どのリソースを
3. どれだけの時間
使用するのかを厳密に管理


6
HEAPモデル法の概要
MAX-PLUS代数について
二項演算
$\begin{cases} a \oplus b = \max(a, b) \\ a \otimes b = a + b \end{cases}$
$\begin{cases} a \oplus \varepsilon = \max(a, -\infty) = a \\ a \otimes e = a + 0 = a \end{cases}$
行列計算
$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \otimes \begin{bmatrix} \varepsilon \\ e \end{bmatrix} = \begin{bmatrix} a \otimes \varepsilon \oplus b \otimes e \\ c \otimes \varepsilon \oplus d \otimes e \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$
線形方程式の最大解
$A \otimes x = b \longrightarrow x^* = -A^T \otimes (-b)$

7

HEAP法の概要

HEAP法の計算方法

◆アッパーバウンド $u = [\varepsilon \quad 4 \quad 6 \quad 3]^T$

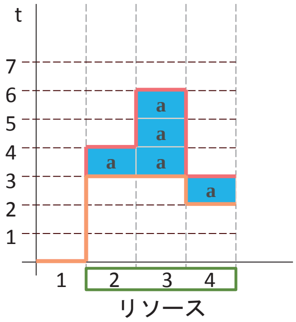
◆ロワーバウンド $l = [\varepsilon \quad 3 \quad 3 \quad 2]^T$

◆使用するリソース $R = [2 \quad 3 \quad 4]$

$$m_{ij}(\eta) = \begin{cases} u_i(\eta) - l_j(\eta) & \text{for } i,j \in R(\eta) \\ e & \text{for } i = j, j \notin R(\eta) \\ \varepsilon & \text{otherwise} \end{cases}$$

$$M(a) = \begin{bmatrix} e & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 & 2 \\ \varepsilon & 3 & 3 & 4 \\ \varepsilon & e & e & 1 \end{bmatrix}$$

ピースを表す
マトリックス



8

HEAP法の概要

HEAP法：プッシュ型スケジューリング

⇒ピースを積み上げる計画方法

全体のアッパーバウンド U

$U = [2 \quad 2 \quad 2 \quad 1]^T$

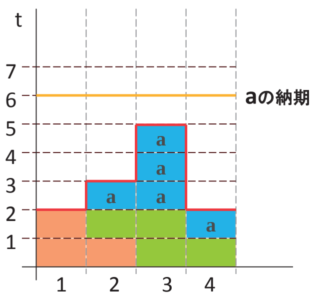
ピース a を積み上げる場合

$U_a = M(a) \otimes U$

$$= \begin{bmatrix} e & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 & 2 \\ \varepsilon & 3 & 3 & 4 \\ \varepsilon & e & e & 1 \end{bmatrix} \otimes \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \\ 2 \end{bmatrix}$$

新しい全体のアッパーバウンドが求まる

$U_a = [2 \quad 3 \quad 5 \quad 2]^T$



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HEAP法の概要

- HEAP法：プル型スケジューリング
  ⇒ピースを納期へ引っ張り上げる

先にあるピースと、aの納期を考慮した全体のロワーバウンドL

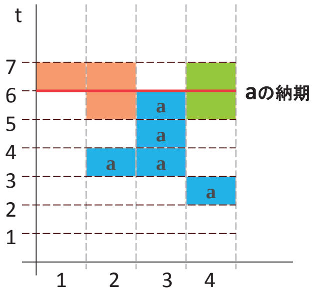
$$L = [6 \quad 5 \quad 6 \quad 5]^T$$

$$L_a = -M(a)^T \otimes (-L)$$

$$= -\begin{bmatrix} e & ε & ε & ε \\ ε & 1 & 1 & 2 \\ ε & 3 & 3 & 4 \\ ε & e & e & 1 \end{bmatrix}^T \otimes \left(-\begin{bmatrix} 6 \\ 5 \\ 6 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 3 \\ 3 \\ 2 \end{bmatrix}$$

新しい全体のロワーバウンドLaが求まる

$$L_a = [6 \quad 3 \quad 3 \quad 2]^T$$



10

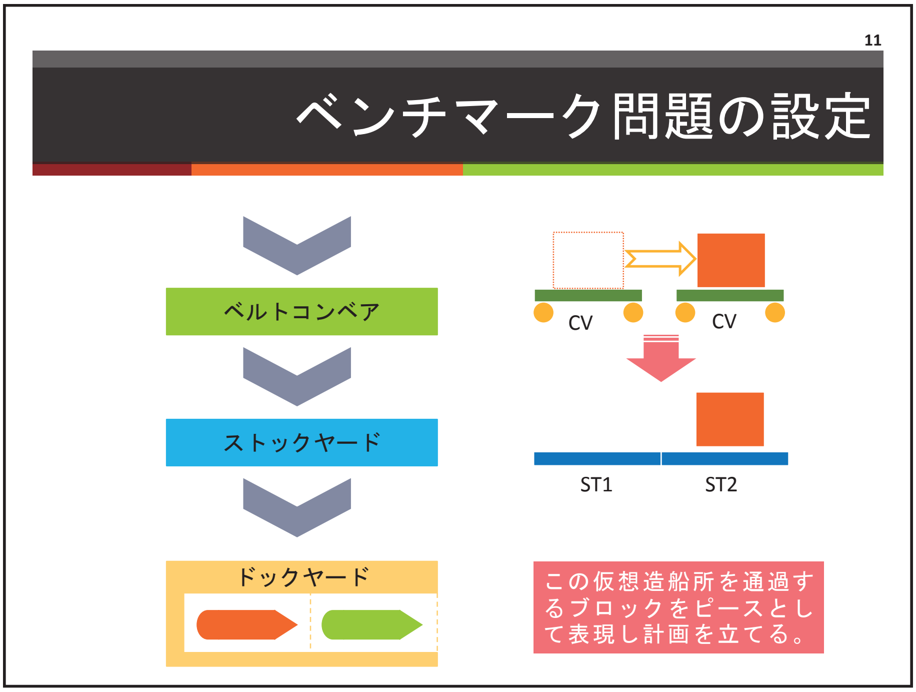
スケジューリングの最適化問題

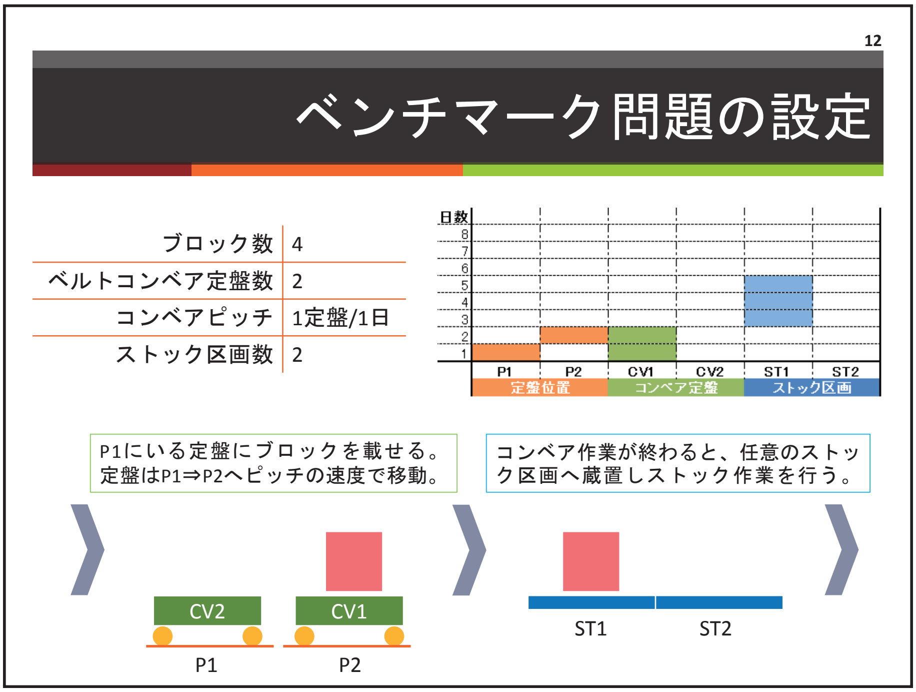
- 最小化する評価関数
  各ブロックのアイドル期間の総和
  アイドル期間＝納期日ー作業完了日

- 制約条件
  コンベア定盤の使用数が一定値を超えない
  ストック区画の使用数が一定値を超えない

- その他考慮すべきもの
  どのブロックからコンベアへ投入するか
  ⇒ブロック数Nのとき、投入順序N！通り

  ブロックをどのストック区画へ蔵置するか
  ⇒区画数Pのとき、P^N通り


11
ベンチマーク問題の設定
ベルトコンベア
ストックヤード
ドックヤード
CV
CV
ST1
ST2
この仮想造船所を通過するブロックをピースとして表現し計画を立てる。


12
ベンチマーク問題の設定
ブロック数 4
ベルトコンベア定盤数 2
コンベアピッチ 1定盤/1日
ストック区画数 2
日数
8
7
6
5
4
3
2
1
P1
P2
CV1
CV2
ST1
ST2
定盤位置
コンベア定盤
ストック区画
P1にいる定盤にブロックを載せる。定盤はP1⇒P2へピッチの速度で移動。
コンベア作業が終わると、任意のストック区画へ蔵置しストック作業を行う。
CV2
CV1
P1
P2
ST1
ST2

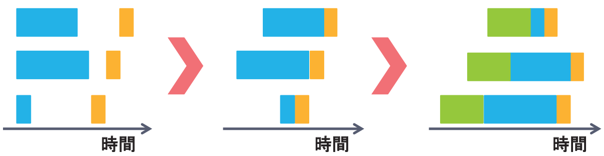
13

優先規則の設定

- どの順番でつくるのか？
  現場のやり方に似た優先規則の使用

「納期ーストック期間」の早いものから投入

納期　ストック期間　コンベア作業時間



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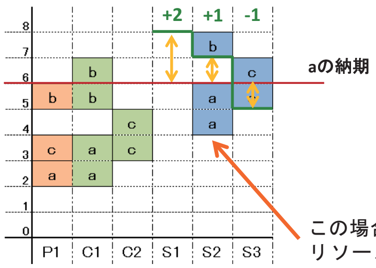
優先規則の設定

- どのストック区画に蔵置するのか？
  現在採用している配置案

隙間を埋める配置



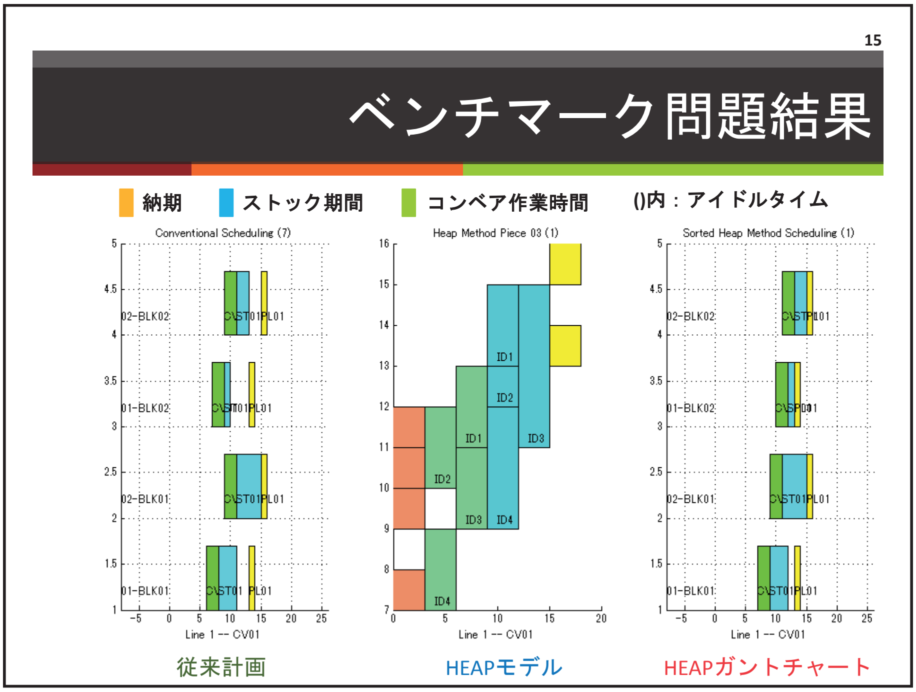
a をどこに置くか

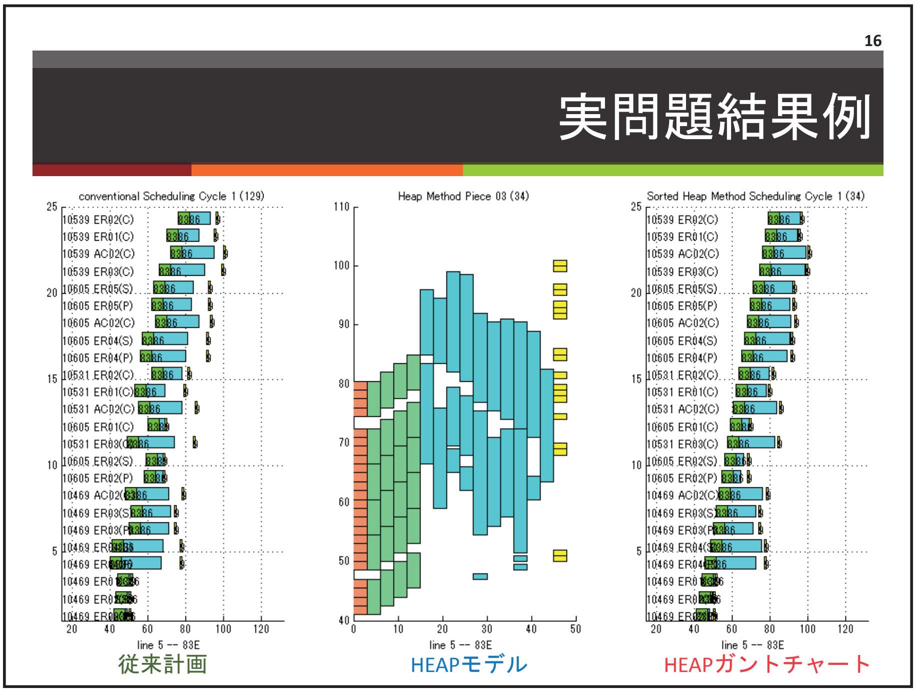
ブロックの納期(赤の線)からロワーバウンド(緑の線)までの距離を考える。

①距離が正且つ最小となるリソースを選択。

②もし距離が全て負の場合は、距離が最も0に近いリソースを選択。

この場合、aのブロックはストックのリソースとして、ST2を選択する。


15
ベンチマーク問題結果
納期
ストック期間
コンベア作業時間
()内：アイドルタイム
Conventional Scheduling (7)
Heap Method Piece 03 (1)
Sorted Heap Method Scheduling (1)
Line 1 -- CV01
従来計画
HEAPモデル
HEAPガントチャート


16
実問題結果例
conventional Scheduling Cycle 1 (129)
Heap Method Piece 03 (34)
Sorted Heap Method Scheduling Cycle 1 (34)
line 5 -- 83E
従来計画
HEAPモデル
HEAPガントチャート

構文解析に特化した翻訳ソフト

現在の機械翻訳は，SVO 言語である欧米の言語の間ではうまくいっているようですが，それらを日本語に訳すと文法的な誤りが目立つようになります．

そこで私は、構文解析の精度を上げる研究を行っています．**品詞**を特定し**語順**を変換し**文型**を表示し**文法解説**を付けるソフトの開発です．

今のところ登録している単語と文法事項に関しては間違えることはほとんどありません．文型表示や文法解説を付けているので，英語教育ソフトとしても使えると思います．

例

原文 If I had known his phone number, I would have called him.

文法 {If S had V-e …}, S would have V-e ….「もし S が…したなら、S は…しただろうに」

品詞 if{接} I{N} had{v-d} know-e{V-e} his{所} phone number{N} , I{N} would{v-d} have{v} call-e{V-e} him{N} .

文型 if Ｓ Ｖ Ｏ , Ｓ Ｖ Ｏ .

和訳 /もし{接}/ /私{N}は/ /彼の{所}電話番号{N}を/ /知っている-たなら{V-仮過完}/ /,/ /私{N}は/ /彼{N}/ /{に}電話をかける{V}-ただろうに{v-仮過完}/ /./

例 2　形から一つに絞れない場合は、複数表示します。

原文 The Prime Minister is to visit Beijing next month.

品詞 the{冠} Prime Minister{N} be to visit{V} Beijing{N} next month{副} .

文型 S be-to V O .

和訳 /その{冠}総理大臣{N}は/ /来月{副}/ /北京{N}を/ /訪れる{V}ことになっている/ /./

和訳 /その{冠}総理大臣{N}は/ /来月{副}/ /北京{N}を/ /訪れる{V}-なければならない/ /./

和訳 /その{冠}総理大臣{N}は/ /来月{副}/ /北京{N}を/ /訪れる{V}ことができる/ /./

文型 S Vi [to V O .

和訳 /その{冠}総理大臣{N}は/ /来月{副}/ /北京{N}を/ /訪れる{V}こと である{V}/ /./

moses、nltk などのツールは使わず、すべて自分でプログラムしています.

以上の文章で使われている単語・文法事項のみで構成されている文であれば解析できるはずです.

一文ずつ入力してください. ピリオドで終えてください.

TRDRDに基づくサッカーの分析プログラム

Software in Mathematics Demonstration Track
in Hakata Workshop 2014
2/8(Sat) 博多リファレンスビル

愛媛大学 理工学研究科
H.24年度修了 岩浅 真秀人

愛媛大学 理学部
H.24年度卒業 浪花 竜平

愛媛大学 理工学研究科
大塚 寛

動 機

愛媛FCの元GKコーチ/愛大・教育学生支援機構の山中氏からのデータの提供と要望

サッカー試合中の**選手とボールの<u>時間ごと</u> (5fps)** の位置データ (2次元)

抽出

ディフェンスに関する情報
(想定する利用者: 指導者)

Ex. ディフェンスライン, スペース,
バイタルエリア

なわばり

研究の背景

計算幾何　Voronoi Region　距離に基づく領域(近い)

ボロノイ図

$$V(p_i) = \bigcap_{j \neq i} \{x \in \mathbb{R}^2 \mid d(p_i, x) < d(p_j, x)\}$$

↓

サッカーへ適用　Dominant Region　到達時間に基づく領域(早い)

優勢領域図

$$Dom(p_i) = \bigcap_{j \neq i} \{x \in \mathbb{R}^2 \mid t(p_i, x) < t(p_j, x)\}$$

$t = t(p_i, x)$　として、運動方程式

$$|x - p_i| = \left| V_{\max}\left(t - \frac{1 - e^{-\alpha t}}{\alpha}\right)\mathbf{e} + \left(\frac{1 - e^{-\alpha t}}{\alpha}\right) v_i \right|$$

を t について解く

p_i：母点 = 選手の位置
v_i：選手の速度(方向含む)

時間制限付き優勢領域図 (TRDRD)

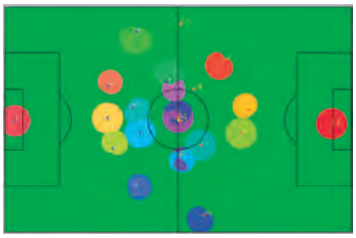
選手 p_i の時間制限付き優勢領域

(Time Restricted Dominant Region, TRDR)

$$Dom(p_i, k) = \bigcap_{j \neq i} \{x \in \mathbb{R}^2 \mid t(p_i, x) < t(p_j, x), t(p_i, x) < k\}$$

時間制限付き優勢領域図
(TRDR Diagram, TRDRD ($k = 1.2$))

守備範囲に制限を設ける
⇒ 実際の選手の支配領域を表現



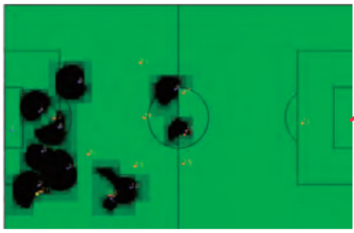
2種類のプログラム + 映像で比較

Ball Centric Simulation
(Voronoi図と双対)

TRDRD Simulation
ー 攻撃権とパスの表示含む

TRDRDから得られる情報

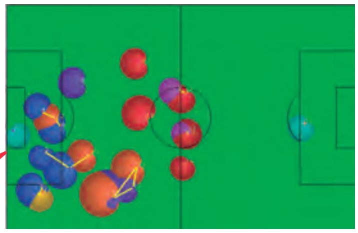
四分木による領域探索



守備側選手

全選手

隣接(近接性)グラフ



スペース ≒
守備側選手のTRDRがないエリア

四分木のノードの深さにより明度を変化

選手のグループ (マーク) ≒
選手同士の隣接関係

ドロネーグラフに代表される近接性

TRDRDの有効性を示すために、数値的なデータとの比較が必要

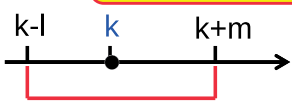
攻撃権とパスの抽出

各時点ごとのフィジカルな情報

- ボールの状態 — ボールの移動に基づいて分類
  - ボールの速度とボールの角度の変化
- ボールの所属 — ボールの支配に基づいて分類
  - ボールの近傍にいる選手の人数 (未定含む)

ボール中心/
TRDRD 各々で決定

連続した期間でのチームの情報



- チームの攻撃権 — ボールの所属に基づいて (未定では時間を下って) 分類
- 1攻撃権の中の(成功した)パスの抽出

ボール中心 /
TRDRDによらない

2種類のプログラム + 映像で比較　映像

Ball Centric PassWork	サンフレッチェ広島 vs.愛媛FC
TRDRD PassWork	サンフレッチェ広島 vs.横浜マリノス

A Java-based simulation-oriented design for robust financial portfolios
Omar Rifki

Abstract. Two Java-based applications are presented. The first one (`jPortRob`) is about the financial model of portfolio optimization, which is known to be extremely sensitive to small perturbations in input data, especially asset expected returns (means). Through a simulation-oriented approach evaluating portfolios' performance through multiple scenarios of assets means, slightly and randomly derived from a nominal case, the aim is to select robust portfolios to errors affecting the means. The source of the considered portfolios is diverse, namely the evolutionary algorithms plus quadratic programming procedure. On the other hand, the second application (`GetAssetsDataSet`) is an automatized downloader for market asset prices. Based on Selenium, which is a software framework for automate web browsers usually used for testing purposes of web applications, the application allows the user, for input parameters as the beginning and end date of data, the frequency of data plus asset names, to download multiple financial prices in a short time.

Keyword: financial engineering, portfolio optimization, robustness assessment, and simulation model

Omar Rifki, Economic engineering department of Kyushu University, 3ec12015g@s.kyushu-u.ac.jp

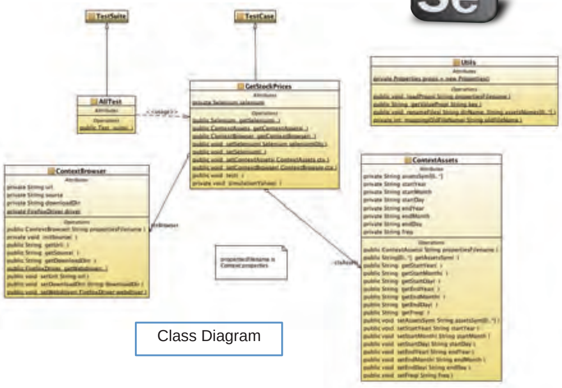
'GetAssetsDataSet'
An automatized download of financial asset prices data through Selenium

Selenium is a suite of open-source automated tools for functional testing, performing scenarios of user interaction with a web application. On one hand, it allows to confirm the functionality of the web application to be tested, and on the other hand, it tests its compatibility with heterogeneous client environments (different browsers and OS which may used by the client).

Method. Our use of Selenium framework does not fall within testing purposes. It is, however, used for performing actuals operations through the browser. By operations, we mean:
1. Opening the browser
2. Opening a target website URL (URL for downloading)
3. Miming factual operations that a user may perform to download data, as entering input parameters, clicking on buttons.
4. Closing the browser

Advantage. The main advantage of automating this execution, through Selenium, is the **gain of time**. For instance, we could download 23 years (from 1st Jan 1989 to 1st Jan 2013) of daily stock prices of the Nikkei 225 index in around 2 minutes CPU time. Beyond downloading, any action, involving web browser, as Mozilla Firefox or Google Chrome, that could be done within a computer-human interaction context can be supervised via the Selenium Client API (Application Programming Interface).

Junit. Because Selenium client API methods are oriented to be used within the **JUnit framework**, our code implementation is merely a suite of test methods of the JUnit framework (the project class diagram is in the below figure). Noting that JUnit is the standard unit-testing framework for the Java language programming.



Class Diagram

Java Platform: JDK 1.7
Pseudorandom number generator: Mersenne twister

'jPortRob'
A simulation-oriented design for robust financial portfolios

Model. To evaluate the sensitivity of the POP problem, defined in the bellow box, against errors in asset means estimates, an input of the problem, we perform the following simulation,

	w_{I_0}	...	w_{I_r}	$w_{I_{avg}}$	w_{avg}	w_{ga1}	w_{ga2}	w_{ga3}
(I_0) Nominal case	$EU_{I_0}(w_{I_0})$	...	$EU_{I_0}(w_{I_r})$	$EU_{I_0}(w_{I_{avg}})$	$EU_{I_0}(w_{avg})$	$EU_{I_0}(w_{ga1})$	$EU_{I_0}(w_{ga2})$	$EU_{I_0}(w_{ga3})$
(I_1) Asset Scenario 1	$EU_{I_1}(w_{I_0})$	...	$EU_{I_1}(w_{I_r})$	$EU_{I_1}(w_{I_{avg}})$	$EU_{I_1}(w_{avg})$	$EU_{I_1}(w_{ga1})$	$EU_{I_1}(w_{ga2})$	$EU_{I_1}(w_{ga3})$
...	...	...	...	...	...	...	...	...
(I_r) Asset Scenario r	$EU_{I_r}(w_{I_0})$	...	$EU_{I_r}(w_{I_r})$	$EU_{I_r}(w_{I_{avg}})$	$EU_{I_r}(w_{avg})$	$EU_{I_r}(w_{ga1})$	$EU_{I_r}(w_{ga2})$	$EU_{I_r}(w_{ga3})$
% of being 1^{st} ranked solution	$1/(1+r)\%$	...	$1/(1+r)\%$	0%	0%	0%	0%	0%
% of being 2^{nd} ranked solution	-	...	-	-	-	-	-	-
The mean of $(EU_{I_i}(w_s)/EU_{I_i}^{max})$	-	...	-	-	-	-	-	-

Such that, each scenario $(I_j) = (u')$ represents a perturbed case from the nominal scenario $(I_0) = (u)$ according to asset means:

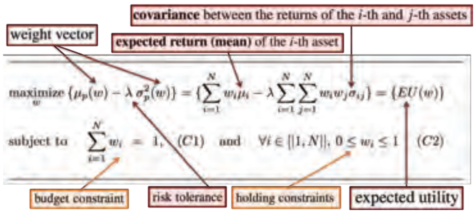
$$\mu_i' = \mu_i\,(1 + \varsigma\, X_i), \quad X_i \sim \mathcal{N}(0,1) \quad \forall i \in [|1, N|],$$

with ς the magnitude of the applied normally distributed noises, e.g. 0.05. The compared portfolios are generated from Genetic Algorithm (GA), noted as w_{ga} ,plus the optimal portfolios $w_{I_0}, w_{I_1}, \ldots, w_{I_r}$, which are respectively computed for $I_0, I_1, \ldots, I_r$.

Concept. The idea is to evaluate the portfolio's performance (Expected Utility EU) through multiple scenarios of asset means, slightly and randomly derived from a nominal case. Three robustness measures are defined to assess the portfolio's behaviors. Two are raking-based. Actually, for each scenario, we can rank portfolios according to their EU. This ranking may differ for different scenarios (since the input's model change). Hence, it is possible to count the number of times a solution is coming up first in all the rankings, which is our first measure, and second in all the rankings, which is our second measure. The third measure is defined for each portfolio by taking the mean over all the scenarios of the ratio $EU_{I_i}(w_s)/EU_{I_i}^{max}$ such that $EU_{I_i}^{max}$ is the maximum utility achieved in the scenario I_i .

Implementation. Our java program calls the **ECJ framework** (Java Evolutionary Computation) for GA simulations and **ILOG CPLEX 12.5** for Quadratic Programming solving. The genetic parameters are choosing through a *parameter tuning*.

Portfolio Optimization Problem (POP) is a financial problem aiming to find an optimal allocation (the vector w) of financial capital among a set of available assets. It is based on two criteria, minimizing the risk while maximizing the expected return of the investment [1]. The standard general formulation with N risky assets can be as follows,



$$\underset{w}{\text{maximize}}\ (\mu_p(w) - \lambda\,\sigma_p^2(w)) = (\sum_{i=1}^{N} w_i\mu_i - \lambda \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}) = (EU(w))$$

$$\text{subject to} \quad \sum_{i=1}^{N} w_i = 1, \ (C1) \quad \text{and} \quad \forall i \in [|1,N|],\ 0 \le w_i \le 1 \quad (C2)$$

Reference
[1] H. Markowitz, "Portfolio selection," The journal of finance, vol. 7, no. 1, pp. 77–91, 1952.

ラプラシアン固有マップ法における評価方法及びその応用

谷口研究室 D0941 吉野 聖人

はじめに

データ圧縮はデータ活用の上で非常に重要である。

<u>従来手法</u>

- (カーネル)主成分分析
- ラプラシアン固有マップ法
- ISOMAP
- LLE
- SDE

<u>本研究の成果</u>

- ラプラシアン固有マップ法の改善

本研究の流れ

1. はじめに
2. ラプラシアン固有マップ法
   - 手法の概要
   - 問題点
3. 評価方法の提案
   - 寄与率の定義
   - 有用性の確認
4. 評価方法の応用例
   - 応用例1 ユークリッド空間上の Spectral Based Interpolation
   - 応用例2 類似度が与えられた集合上の Spectral Based Interpolation
5. まとめ

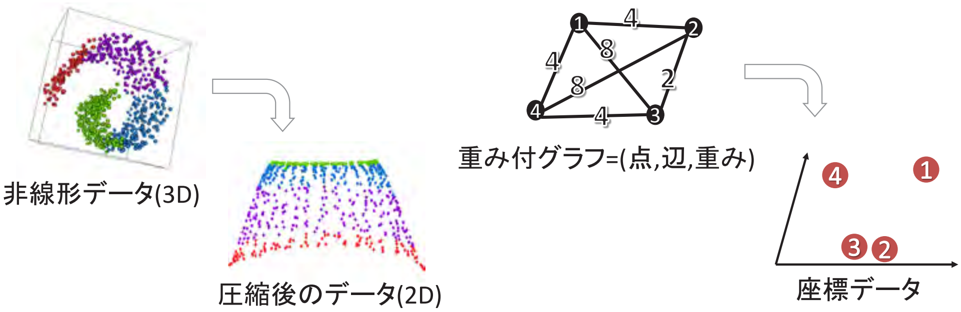
2.ラプラシアン固有マップ法
（Laplacian Eigenmaps : LE法）

- 手法の概要
- 問題点

２－１　手法の概要

分類：重み付グラフの空間埋め込み手法

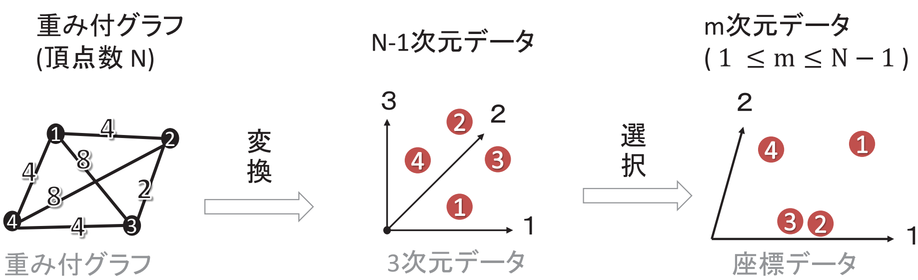
特徴：高次元中の非線形な図形を抽出可能
　　　構造が複雑な対象にも適用可能



２－１　手法の概要②

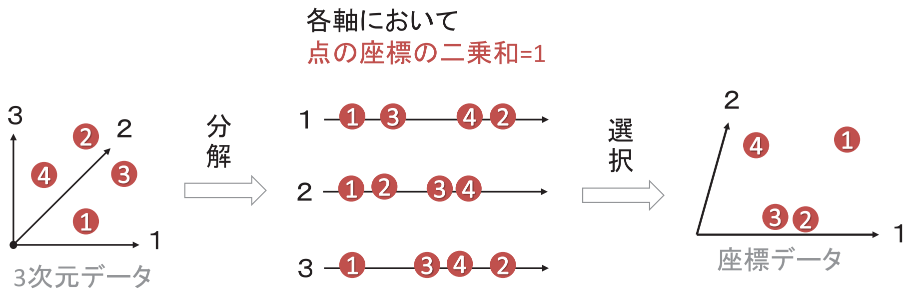
グラフ埋め込みの流れ

1. グラフ (頂点：Nコ) を N-1次元空間で表す.
2. グラフの情報を表している，座標軸のみを取り出す.



2−2 問題点

問題点：高次元データへ変換したとき、各軸は正規化されている。
すなわち、ノイズが拡大し、グラフの情報が縮小されてしまう。



3.評価方法の提案

- 寄与率の定義
- 有用性の確認

3－1　寄与率の定義

概要：各軸に定義され、その軸の重要度の数値化.
　　　各軸に各寄与率を乗ずれば、縮尺を修正可能.

定義：第k軸の寄与率 $c_{k+1} = \dfrac{\lambda}{\lambda_{k+1}}$

ただし，グラフの固有値：$\lambda_1 \leq \cdots \leq \lambda_N$

λ = 定数 $\left(= \sum_{k=1}^{N-1} \lambda_k^{-1}\right)$

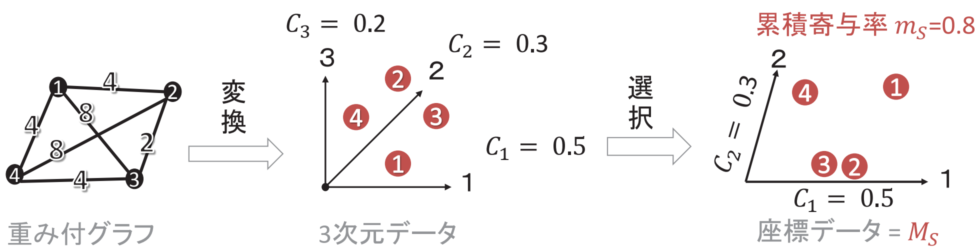
性質：寄与率の総和は1

3－1　累積寄与率の定義

概要：あるグラフの埋め込みを評価

定義：軸s_i (s_i = 選んだ軸) からなる座標M_S

の累積寄与率 $m_s = \sum_{s_i \in S} c_{s_i}$



３−２　有用性の確認

具体的例で示す.

対象	確認内容
単純な重み付グラフ	寄与率（縮尺変更）の効果
高次元の図形	累積寄与率の評価
複雑な重み付グラフ	

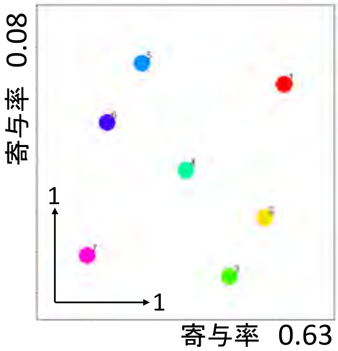
３−２　有用性の確認

①単純な重み付グラフ

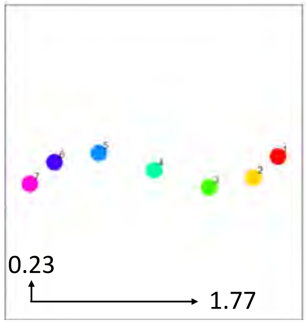
重み付グラフ （隣との重み=1）

3－2 有用性の確認

①単純な重み付グラフ



ラプラシアン固有マップ法

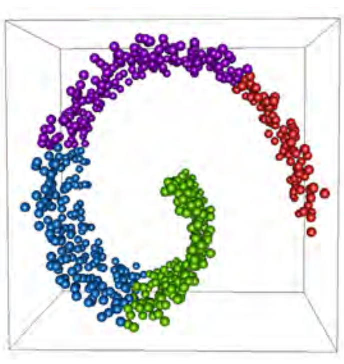


改善ラプラシアン固有マップ法

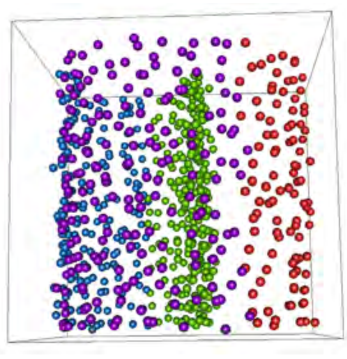
寄与率による縮尺改善の効果

3－2 有用性の確認

②高次元の図形



側面図

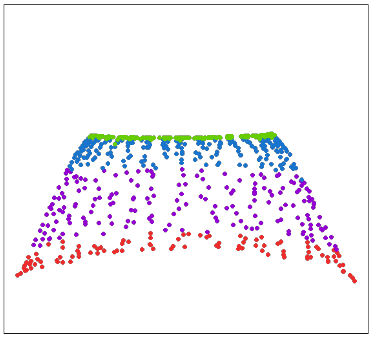


上面図

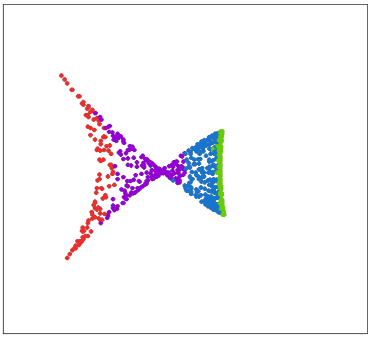
3−2 有用性の確認

②高次元の図形

累積寄与率0.14



累積寄与率0.09

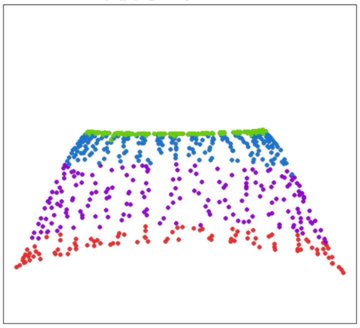


累積寄与率で相対的に評価

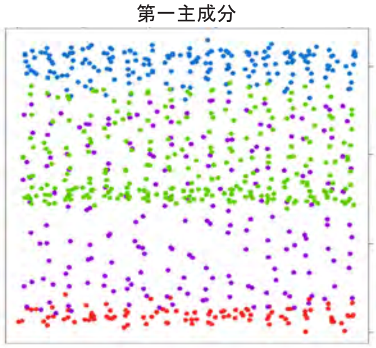
3−2 有用性の確認

②高次元の図形

累積寄与率0.14



改善ラプラシアン固有マップ法

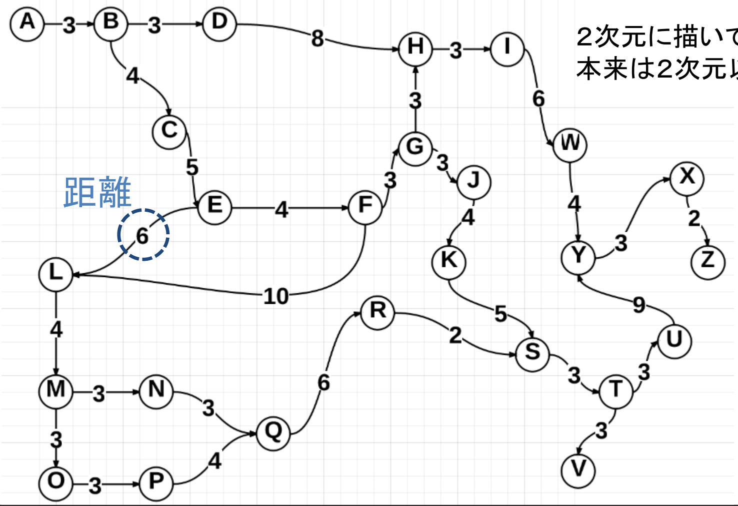


主成分分析

主成分分析との比較

３－２　有用性の確認

③複雑な重み付グラフ

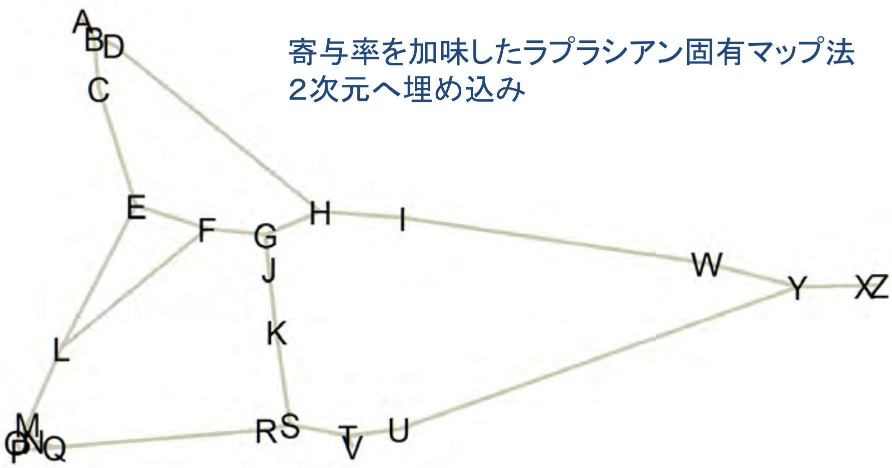


2次元に描いているが
本来は2次元以上

３－２　有用性の確認

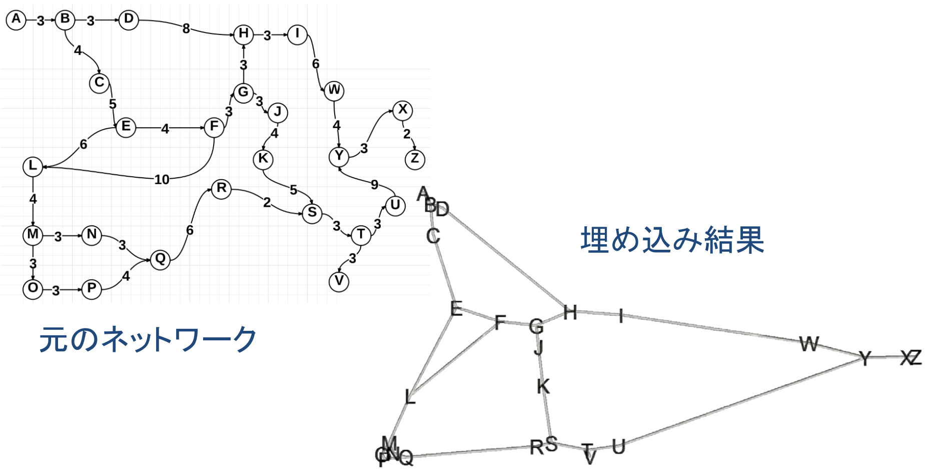
③複雑な重み付グラフ

寄与率を加味したラプラシアン固有マップ法
２次元へ埋め込み



３－２ 有用性の確認

③複雑な重み付グラフ



4.応用例

- 応用例1 ユークリッド空間上のSBI
- 応用例2 類似度が与えられた集合上のSBI

※SBI = Spectral Based Interpolation

４−１ユークリッド空間上の SBI

- Spectral Based Interpolationとは（高橋ら[3]によって考案）
  グラフの固有値，固有ベクトルを用いた，補間手法.
  集団を意識した，動きを実現可能

- 補間とは
  初期状態と結果から、その間の状態を推測すること.

Source Formation　　　Target Formation

参考文献: [3]Shigeo Takahashi, Kenichi Yoshida, Taesoo Kwon, Kang Hoon Lee, Jehee Lee and Sung, Yong Shin, Spectral-Based Group Formation Control , *Computer Graphics Forum*, Vol.28, pp.639{648, 2009

４−１ユークリッド空間上の SBI

<u>従来の方法</u>

・**rule-based approach**
・**force-field-based approach**

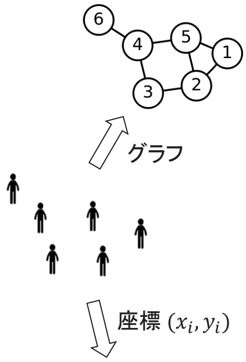
多くのパラメータを適切に定め、グループを意識した補間を実現する。

<u>新しい方法</u>

・ **Spectral-Based Interpolation(SBI)**

スペクトルを用いることで、容易にグループを意識した補間を実現する。

４－１ SBIの原理　補完する量の抽出



グラフラプラシアン

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

正規直交
固有ベクトル

$$\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4, \boldsymbol{e}_5, \boldsymbol{e}_6$$

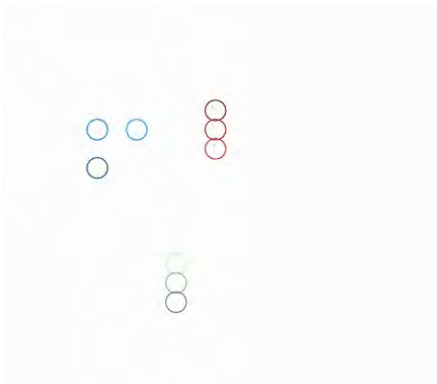
座標変換

補間する

$$\boldsymbol{x} \coloneqq (x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6) = \alpha_1 \boldsymbol{e}_1 + \alpha_2 \boldsymbol{e}_2 + \cdots + \alpha_6 \boldsymbol{e}_6$$

$$\boldsymbol{y} \coloneqq (y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6) = \beta_1 \boldsymbol{e}_1 + \beta_2 \boldsymbol{e}_2 + \cdots + \beta_6 \boldsymbol{e}_6$$

４－１　例1 クラスタの保存



応用例2
類似度の定義された集合上のSBI

以降
類似度の定義された集合 ⇒ Weighted Set

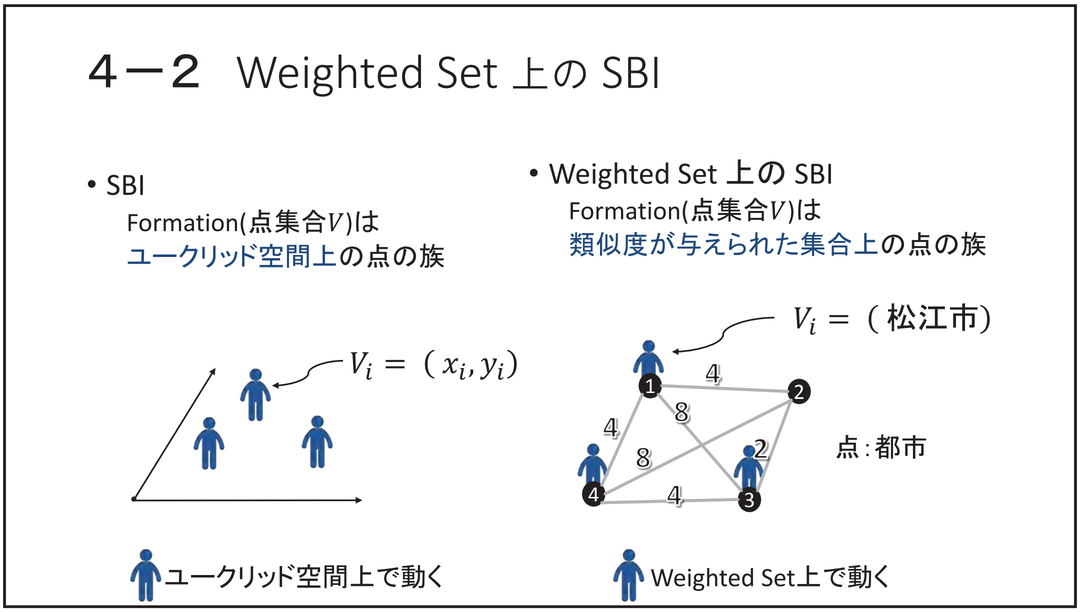
4－2 類似度の定義された集合とは

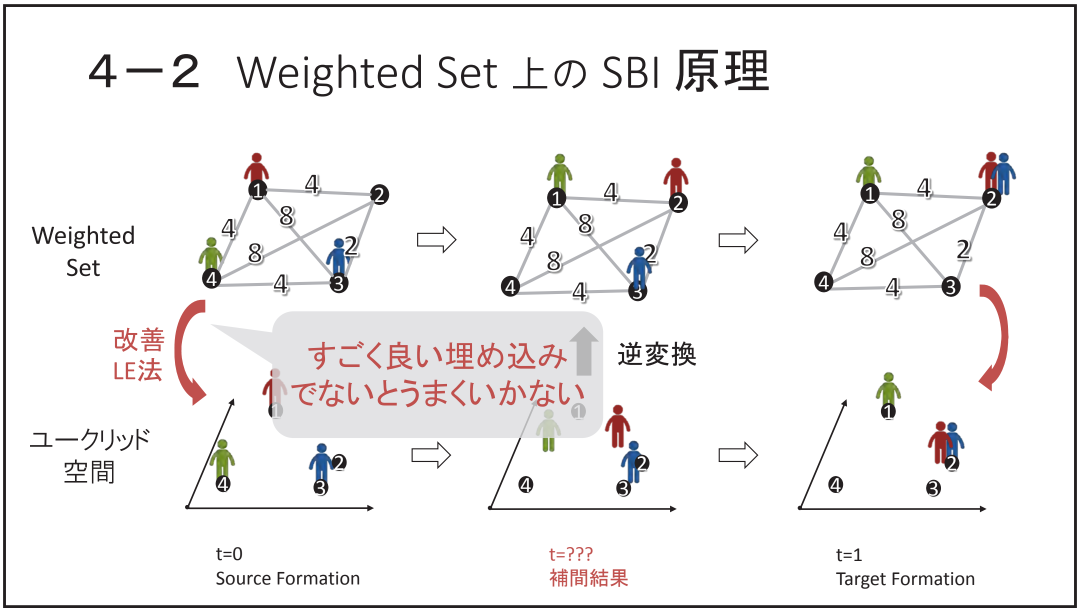
- 定義

Weighted Set とは次をみたす組$W = (V, w)$である.

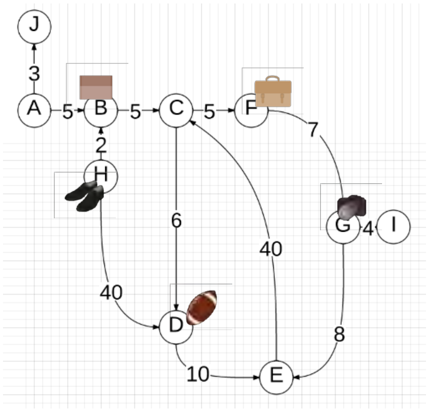
- Vは集合
- wは$w: V \times V \to R$である写像

集合に重みがついたもの！成立条件が緩い！


4－2 Weighted Set 上の SBI
• SBI
Formation(点集合V)は
ユークリッド空間上の点の族
• Weighted Set 上の SBI
Formation(点集合V)は
類似度が与えられた集合上の点の族
V_i ＝（x_i, y_i）
V_i ＝（松江市）
点：都市
ユークリッド空間上で動く
Weighted Set上で動く

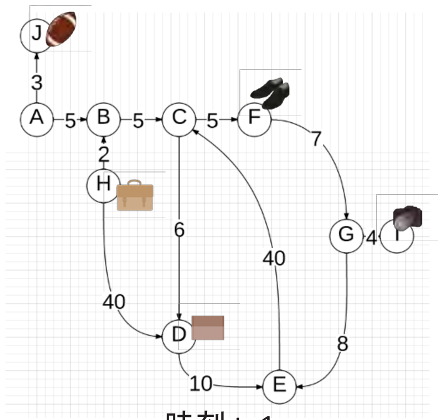

4－2 Weighted Set 上の SBI 原理
Weighted Set
改善
LE法
すごく良い埋め込み
でないとうまくいかない
逆変換
ユークリッド空間
t=0
Source Formation
t=???
補間結果
t=1
Target Formation

４－２　Weighted Set 上の SBIの例



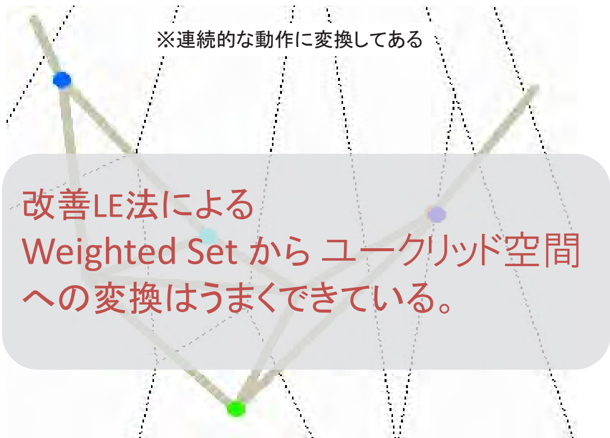
時刻 t=0

⇨



時刻 t=1

４－２　Weighted Set 上の SBIの例



実際にやってみた。

４－２　Weighted Set 上の SBIの例2

Linear Interpolation　　　　　　**SBI**

※連続的な動作に変換してある　　※連続的な動作に変換してある

実際にやってみた。

４－２　改良前のLE法を使った場合

※連続的な動作に変換してある

実際にやってみた。

４－２ Weighted Set 上の SBIのまとめ

結論

- 新LEの良い埋め込みにより，簡易的な拡張は可能
- SBIの（つられる）動きは反映されている

５ まとめ

提案により

- 重み付グラフから得られる、N-1次元空間における各軸の評価が可能になった。

成果

- あるグラフの埋め込みを相対的に評価できる
- グラフの埋め込みの縮尺を改善できる

今後の課題

- 埋め込みが有用となる，累積寄与率の閾値の設定

Coq Modules for Automata and Sticker Systems

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Shuichi Inokuchi, Faculty of Mathematics, Kyushu University
Yoshihiro Mizoguchi, Institute of Mathematics for Industry, Kyushu University

Abstract

We implemented objects and functions appeared in the theory of automata and formal language by Coq proof-assistant. And we gave formal proofs about properties of them by Coq. Though the alphabet used in language is finite, but the language has infinite elements and enumeration program stop never. The property of language, which includes infinite elements, is proved recursively by using ssreflect (a Small Scale Reflection Extension for the Coq system). We also implemented the modules for Sticker Systems.

Păun and Rozenberg introduced a concrete method to transform an automaton to a Sticker System in 1998. One of our aims is to present formal proofs of the correctness of their transformation. To prove above, we used our implementation of automata and sticker systems. In this demonstration, we show some formal proofs for simple propositions about automata and sticker systems by Coq and some examples of auto-verification about them.

Introduction

Many Problems formalized with Coq proof-assistant:
· Four-color Theorem . . . G.Gonthier 2008,
· Kepler Conjecture . . . T.C.Hales 2005,
· Shannon Theorem . . . R.Affeldt et.al. 2012.

We are trying to formalize the transformation to check the correctness by using computer.

Coq Proof Assistant

· formal proof management system
· by using Coq, it enables to write machine-checkable proof.
– Ssreflect (a Small Scale Reflection Extension for the Coq) –

Traditional Computing VS DNA Computing

Leonard Adleman presented better answer about TSP(Travelling Salesman Problem) by using DNA Computing in 1994.
DNA Computing uses new operation such as *cut*, *paste*, *adjoining*, *insert*, and *delete*, instead of *and*, *or*, *not*, and arithmetic operators that used in traditional computing.

$FA \ni M$ $\quad L(FA) = RL \xrightarrow{CoqFunction} SS \ni \gamma(M)$
$\cap \qquad\qquad \cap$
$LG \ni G$ $\quad L(LG) = LL \xrightarrow{CoqFunction} SS \ni \gamma(G)$
$\cap \qquad\qquad \cap$
CFG $\quad L(CFG) = CFL$

FA:Finite Automaton,LG:Linear Language,CFG:Context Free Grammar
RL:Regular Language,LL:Linear Language,CFL:Context Free Language
SS:Sticker System.

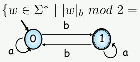
Automata in Coq

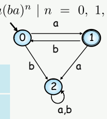
We denote an automaton "`m = (q, s, d, s0, f)`", where `q`:States, `s`:Symbols, `d`:States → Symbol → States, `s0`:State and `f`:States.

```
Definition Automaton:=
          (States*Symbols*(State->Symbol->State)*State*States).
```

Examples

$L(M_1) = \{w \in \Sigma^* \mid |w|_b \ mod\ 2 = 1\}$ $\qquad L(M_2) = \{\, a(ba)^n \mid n = 0, 1, 2, \ldots\}$





```
Definition m1: Automaton:=
([:: 0;1], [:: "a"%char;"b"%char],m1_d,0,[:: 1]).
Definition m2: Automaton:=
([:: 0;1;2], [:: "a"%char;"b"%char], m2_d, 0, [:: 1]).
```

Machine Checkable Proofs

Lemma 1 *For all $n \in \mathbb{N}$, a word "b^{2n+1}" is accepted by $L(M_1)$.*

$$\text{"}b^{2n+1}\text{"} \in L(M_1)$$

```
Fixpoint b2np1 (n:nat) : SymbolString :=
match n with
|0 => "b"%string
|S p => "bb"%string ++ (b2np1 p)
end.

Lemma b2np1Am1 : forall n :nat, accept m1 (b2np1 n).
Proof.
by elim;simpl.
Qed.
```

Lemma 2 *For all $n \in \mathbb{N}$, a word "$a(ba)^n$" is accepted by $L(M_2)$.*

$$\text{"}a(ba)^n\text{"} \in L(M_2)$$

```
Fixpoint aban (n:nat) : SymbolString :=
match n with
|0 => "a"%string
|S p => (aban p) ++ "ba"%string
end.

Lemma abanAm2 : forall n : nat, accept m2 (aban n).
Proof.
elim;rewrite //.
simpl;move => n.
rewrite !inE;move /eqP => H;apply /eqP.
rewrite dstarLemma.
rewrite H //=.
Qed.
```

Sticker System

$(l, r, x, y) \in \Sigma^* \times \Sigma^* \times N \times N$ is a **Domino** over (Σ, ρ), if following conditions holds :

if $x \geq 0$ then $(l[i + x], r[i]) \in \rho, 1 \leq i \leq min(|r| - x, |l|)$,
if $y \geq 0$ then $(l[x], r[i + y]) \in \rho, 1 \leq i \leq min(|r| - y, |l|)$,
$x \times y = 0$.

D: a set of all Dominos

The sticker operator $\mu : D \times D \to D \cup \{\perp\}$ is defined as follows:

$$\mu((l_1, r_1, x_1, y_1), (l_2, r_2, x_2, y_2)) = \begin{cases} (l_1 l_2, r_1 r_2, x_1, y_1) \text{ (if } (*)), \\ \perp \text{ (otherwise)}. \end{cases}$$

$(*)(l_1 l_2, , r_1 r_2, x_1, y_1) \in D$ and $x_1 + y_2 + |r_1| = x_2 + y_1 + |l_1|$

We denote sticker system "`gamma = (s, rho, a, r)`", where `s`:Symbols, `rho`:Symbol*Symbol, `a`:list of Domino,`r`:list of Domino*Domino.
We define two relations $\Rightarrow$ and $\Rightarrow^*$ for sticker:

$x \Rightarrow y \overset{def}{\iff} \exists (u, v) \in R, y = \mu(u, \mu(x, u))$,
Let $\Rightarrow^*$ be the reflective and transitive closure of $\Rightarrow$.

We also define WK as follows:
$WK = \{(l, r, 0, 0) \in D \,|\, |l| = |r|\}$.

```
Definition Domino:= (SymbolString*SymbolString*nat*nat).
Definition mu (rho:Rho) (d1:Domino) (d2:Domino):list Domino.
Definition Sticker := (list Symbol*Rho*list Domino*list (Domino*Domino)).
```

Our Improved Theorem

Let $M = (Q, \Sigma, \delta, q_0, F_M)$ be an Automaton. Construct the sticker system $\gamma_M = (Q, \rho, A, R)$, where:

$$\begin{aligned}
\rho &= \{(a, a) | a \in \Sigma\}, \ A = A_1 \cup A_2, \ R = D \cup F, \ k = |Q| - 1, \\
A_1 &= \{(x, x, 0, 0) | x \in L(M), |x| \leq k + 2\}, \\
A_2 &= \bigcup_{i=1}^{k+1} \{(xu, x, 0, 0) | x \in \Sigma^{k+2-i}, u \in \Sigma^i, \delta^*(0, xu) = i - 1\}, \\
D &= \bigcup_{i=1}^{k+1} \bigcup_{j=1}^{k+1} \{((\lambda, \lambda, 0, 0), (xu, vx, 0, |v|)) | x \in \Sigma^{k+2-i}, u \in \Sigma^i, v \in \Sigma^j, \delta^*(j - 1, xu) = i - 1\}, and \\
F &= \bigcup_{i=1}^{k+2} \bigcup_{j=1}^{k+1} \{((\lambda, \lambda, 0, 0), (x, vx, 0, |v|)) | v \in \Sigma^j, x \in \Sigma^i, \delta^*(j - 1, x) \in F_M\}.
\end{aligned}$$

Then $L(M) = L_\gamma(\gamma_M)$.

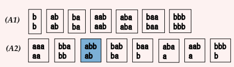
Our Verification

In the automaton M_1 (given in Example),
a word $abbaba$ have three alphabets b, so $abbaba \in L(M_1)$.
Let $\gamma_{M_1} = (Q, \rho, A, R)$ be a sticker system constructed from M_1 with limit length of x is $k + 1$ in F, and $\gamma_{M_1'} = (Q, \rho, A, R')$ with limit length of x is $k - 1$,then

$\gamma_{M_1} = (Q, \rho, A, R) \ : \ (aba, baba, 0, 1) \in R \ \Rightarrow \ abbaba \in L(\gamma_{M_1})$
$\gamma_{M_1'} = (Q, \rho, A, R') \ : \ (aba, baba, 0, 1) \notin R' \ \Rightarrow \ abbaba \notin L(\gamma_{M_1'})$

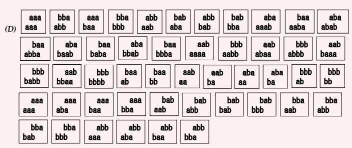
So we corrected limit length of x in F from $k - 1$ to $k + 1$.

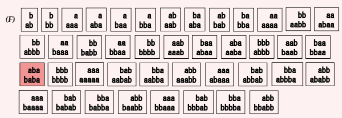
Translation Example from Automaton M_1 to Sticker System



$$\begin{aligned}
\gamma_{M_1} &= (\Sigma, \rho, A, R), \\
\rho &= \{(a, a), (b, b)\}, \ A = A_1 \cup A_2, \ R = D \cup F.
\end{aligned}$$

abb/ab + aba/baba = abbaba/abbaba $\in WK$





Conclusion

We introduced Coq modules for automata and sticker systems. Notions of accepted languages are implemented formally using Coq proof-assistant with ssreflect (a small scale reflection extension).We can make formal proofs for properties of automata and sticker systems. We introduced some examples of formal proofs.

Our future work includes to make a general formal proof of the equivalence theorem of finite automata and sticker systems.

Acknowledgement

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The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions

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February 8th, 2014

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- Preliminary
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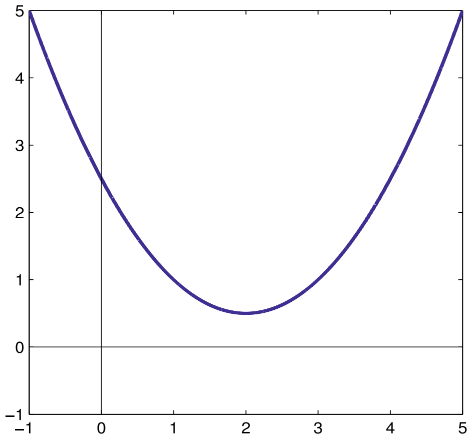
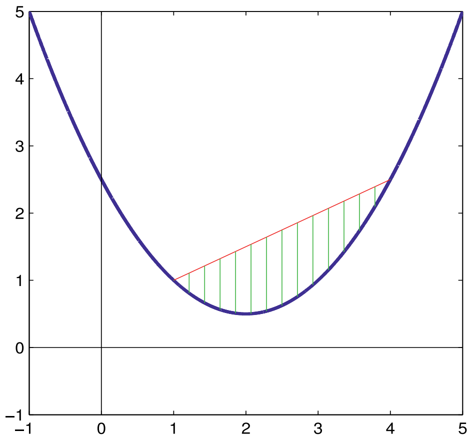
The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Introduction

A Convex Function



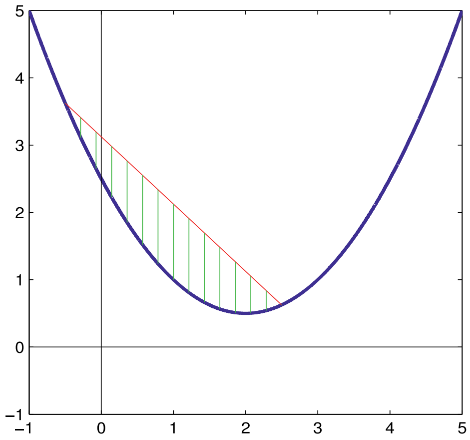
The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Introduction

A Convex Function



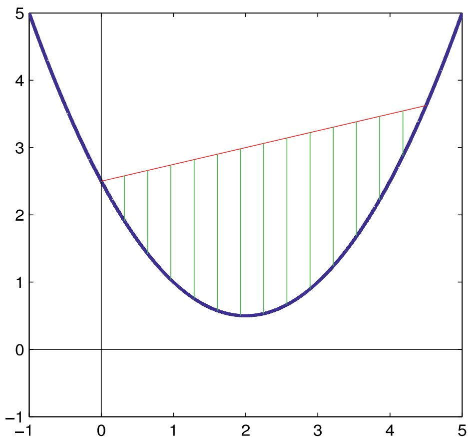
The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Introduction

A Convex Function



The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Introduction

A Convex Function



Positive Matrix

Definition

Let $A = [a_{ij}] \in \mathbb{R}^{m\times n}$ and $B = [b_{ij}] \in \mathbb{R}^{m\times n}$.

1. A : a positive matrix (nonnegative matrix) if

   $a_{ij} > 0 (a_{ij} \geq 0), \quad \text{for all } 1 \leq i \leq m,\ 1 \leq j \leq n.$

2. $A > B (A \geq B)$

   if $A - B$ is a positive matrix (nonnegative matrix).

We say that A and B are comparable if $A \geq B$ or $A \leq B$.

M-matrix I

Definition

Let $A \in \mathbb{R}^{n\times n}$. A is a Z-matrix

if all its off-diagonal elements are nonpositive.

Any Z-matrix A can be written as $sI_n - B$ with $B \geq 0$.

Definition

A matrix $A \in \mathbb{R}^{n\times n}$ is an M-matrix

if $A = rI_n - B$ for $B \geq 0$ and r with $r \geq \rho(B)$;

A : a singular M-matrix if $r = \rho(B)$

A : a nonsingular M-matrix if $r > \rho(B)$.

M-matrix II

Theorem

For a Z-matrix A, the following are equivalent:

1. A is a (nonsingular) M-matrix.
2. A^{-1} is nonnegative.
3. $Av > 0$ for some vector $v > 0$.
4. All eigenvalues of A have positive real parts.

The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Preliminary
M-matrix

M-matrix III

Theorem

Let $A \in \mathbb{R}^{n \times n}$ be an M-matrix. Then,

1. $Av \geq 0$ implies $v \geq 0$,
2. If B is a Z-matrix and $B \geq A$, then B is an M-matrix.

Kronecker Product

For $A \in \mathbb{C}^{m\times n}$ and $B \in \mathbb{C}^{p\times q}$, the Kronecker Product is the matrix $(A \otimes B) \in \mathbb{C}^{mp\times nq}$ defined by

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{22}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}, \text{ where } A = [a_{ij}].$$

Vec Operator

The Vec Operator of a matrix A is the column vector

$$\text{vec}(A) = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_n \end{bmatrix}, \text{ where } A = \left[\begin{array}{c|c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{array}\right].$$

Clearly, $A \leq B$ if and only if $\text{vec}(A) \leq \text{vec}(B)$.

Kronecker Product and Vec Operator

Theorem[R.A.Horn, C.R.Johnson]

let $A \in \mathbb{C}^{m \times n}$, $B \in \mathbb{C}^{p \times q}$, and $C \in \mathbb{C}^{m \times q}$ be given and let $X \in \mathbb{C}^{n \times p}$ be unknown. The matrix equation

$$AXB = C$$

is equivalent to the system of $q \times m$ equations in $n \times p$ unknowns given by

$$(B^T \otimes A)\mathrm{vec}(X) = \mathrm{vec}(C)$$

that is, $\mathrm{vec}(AXB) = (B^T \otimes A)\mathrm{vec}(X)$.

Fréchet Derivative

Definition

Let $G,\ F : \mathbb{C}^{m \times n} \to \mathbb{C}^{p \times q}$.

If $G(X+H) - G(X) = G'(X)H + F(H)$,

where $\|F(H)\|/\|H\| \to 0$ as $\|H\| \to 0$,

G : Fréchet-differentiable at X

$G'_X(H)$: the Fréchet-derivative in the direction H.

Example. For $Q(X) = AX^2 + BX + C$,

$$Q'_X(H) = AHX + (AX + B)H.$$

Definition of CPD-condition

Definition

Let $F : \mathfrak{U} \subset \mathbb{R}^{m\times n} \to \mathbb{R}^{p\times q}$ be a Fréchet differentiable matrix function. For all comparable X, Y and $H \geq 0$, F has the elementwise convex condition(C-condition) on a convex set $\mathfrak{C} \subset \mathfrak{U}$ if

$$F'_X(H) \leq F'_Y(H) \tag{1}$$

F has the positive derivative condition(PD-condition) on $\mathfrak{N} \subset \mathfrak{U}$ if

$$0 \leq F'_X(H). \tag{2}$$

Definition of Order-Convex

Definition [Ortega, 2000]

Let $f : D \subset \mathbb{R}^n \to \mathbb{R}^m$ be a function. Then, f is order-convex(o-convex) on a convex subset $D_0 \subset D$ if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) \tag{3}$$

where $x, y \in D_0$ are comparable and $t \in (0, 1)$.

The Differentiable Order-Convex Vector Functions

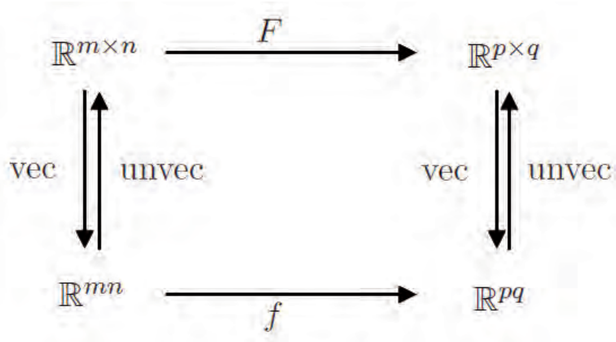
Theorem [Ortega, 2000]

Let $f : D \subset \mathbb{R}^n \to \mathbb{R}^m$ be Fréchet differentiable on the convex set $D_0 \subset D$. Then, the following statements are equivalent.

1. f is order-convex on D_0;
2. $f(y) - f(x) \geq f'_x(y - x)$, for comparable $x, y \in D_0$;
3. $[f'_y - f'_x](y - x) \geq 0$, for comparable $x, y \in D_0$.

Consider $\mathbb{R}^{m\times n}$. For $X, Y \in \mathbb{R}^{m\times n}$, $\mathrm{tr}(X^T Y)$ is a inner product and $\mathrm{tr}(X^T Y) = \mathrm{vec}(X)^T \mathrm{vec}(Y)$.
So, the vec operator is an isomorphism preserving

1. the inner product,
2. the norm induced by the inner product, and
3. the partial order relation.



$$f = \mathrm{vec} \circ F \circ \mathrm{unvec}$$

If F is Fréchet differentiable at X, then

$$\begin{aligned}
&(\mathrm{vec} \circ F'_X \circ \mathrm{unvec})(h) \\
&\quad = (\mathrm{vec} \circ F'_X)(H) \\
&\quad = (\mathrm{vec} \circ F)(X + H) - (\mathrm{vec} \circ F)(X) + \mathrm{vec}(O(H)) \\
&\quad = (f \circ \mathrm{vec})(X + H) - (f \circ \mathrm{vec})(X) + \mathrm{vec}(O(H)) \\
&\quad = f(x + h) - f(x) + o(h) \\
&\quad = f'_x(h)
\end{aligned}$$

where $x = \mathrm{vec}(X)$, $h = \mathrm{vec}(H)$, and $\|O(H)\|/\|H\| \to 0$ as $\|H\| \to 0$. Therefore, $f'_x = \mathrm{vec} \circ F'_X \circ \mathrm{unvec}$.
From now, for convenience, we write $\mathcal{F}_X$ instead of $f'_{\mathrm{vec}(X)}$.

Order-Convex of Matrix Functions

Definition

Let $F : \mathfrak{D} \subset \mathbb{R}^{m\times n} \to \mathbb{R}^{p\times q}$ be a function. Then, F is order-convex(o-convex) on a convex subset $\mathfrak{D}_0 \subset \mathfrak{D}$ if

$$F(tX + (1-t)Y) \leq tF(X) + (1-t)F(Y) \tag{4}$$

where $X, Y \in \mathfrak{D}_0$ are comparable and $t \in (0, 1)$.

The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Order-Convex Functions
Matrix Functions

Order-Convex of Matrix Functions

Theorem

Let $F : \mathfrak{D} \subset \mathbb{R}^{m\times n} \to \mathbb{R}^{p\times q}$ be Fréchet differentiable on the convex set $\mathfrak{D}_0 \subset \mathfrak{D}$. Then, the following statements are equivalent.

1. F is order-convex on $\mathfrak{D}_0$;
2. $F(Y) - F(X) \geq F'_X(Y - X)$, for comparable $X, Y \in \mathfrak{D}_0$;
3. $[F'_Y - F'_X](Y - X) \geq 0$, for comparable $X, Y \in \mathfrak{D}_0$.

Theorem A

Let S be a fixed point of $F : \mathfrak{D} \subset \mathbb{R}^{m\times n} \to \mathbb{R}^{m\times n}$ that has CPD-condition. Suppose the fixed point iteration $\{X_i\} \to S$ satisfying the following conditions.

1. $X_0 \leq X_1 \leq X_2 \leq \cdots \leq S$,
2. $S - X_0 > 0$,
3. $\mathcal{F}_{X_0}\mathbf{1}_{mn\times 1} > 0$.

Then,

$$\limsup_{i\to\infty} \sqrt[i]{\|X_i - S\|_F} = \rho(\mathcal{F}_S). \quad (5)$$

The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Order-Convex Functions
Matrix Functions

Linear Convergence Theorem [Ortega, 2000]

Suppose that $f : D \subset \mathbb{R}^n \to \mathbb{R}^n$ has a fixed point $x^* \in \text{int}(D)$ and is Fréchet differentiable at x^*. If $\rho(f'_{x^*}) < 1$, then

$$\limsup_{i\to\infty} \sqrt[i]{\|x_i - x^*\|} = \rho(f'_{x^*})$$

for any norm $\|\cdot\|$.

By this theorem, we can prove that $\leq$ side, easily.

($\geq$)
Since F has CPD-condition,

$$0 \leq \mathcal{F}_{X_0} \leq \mathcal{F}_{X_1} \leq \mathcal{F}_{X_2} \leq \cdots \leq \mathcal{F}_S.$$

Note that $\lim_{i\to\infty} \rho(\mathcal{F}_{X_i}) = \rho(\mathcal{F}_S)$. So, for any $\epsilon > 0$, $\exists l \in \mathbb{N}$ s.t.

$$\rho(\mathcal{F}_{X_l}) \geq \rho(\mathcal{F}_S) - \epsilon.$$

$$\begin{aligned}
\mathrm{vec}(S - X_i) &= \mathrm{vec}(F(S) - F(X_{i-1})) \\
&\geq \mathrm{vec}(F'_{X_{i-1}}(S - X_{i-1})) \\
&= \mathcal{F}_{X_{i-1}} \cdot \mathrm{vec}(S - X_{i-1}) \\
&\vdots \\
&\geq \mathcal{F}_{X_{i-1}} \mathcal{F}_{X_{i-2}} \cdots \mathcal{F}_{X_0} \cdot \mathrm{vec}(S - X_0),
\end{aligned}$$

The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Order-Convex Functions
Matrix Functions

$$\begin{aligned}
\Rightarrow \|X_i - S\|_F &= \|\mathrm{vec}(X_i - S)\|_2 \\
&\geq \|\mathcal{F}_{X_{i-1}} \mathcal{F}_{X_{i-2}} \cdots \mathcal{F}_{X_0} \cdot \mathrm{vec}(S - X_0)\|_2 \\
&\geq \|\mathcal{F}_{X_l}^{i-l} \mathcal{F}_{X_0}^{l} \cdot \mathrm{vec}(S - X_0)\|_2
\end{aligned}$$

Since $\mathcal{F}_{X_0}^{l} \cdot \mathrm{vec}(S - X_0) > 0$, $\exists c_1 > 0$ and $\exists v \geq 0 (\in \mathbb{R}^{mn})$ s.t.

$$\begin{aligned}
\|\mathcal{F}_{X_l}^{i-l} \mathcal{F}_{X_0}^{l} \cdot \mathrm{vec}(S - X_0)\|_2 &\geq \|\mathcal{F}_{X_l}^{i-l} c v\|_2 \\
&= c\|\mathcal{F}_{X_l}^{i-l} v\|_2 \\
&= c\|\mathcal{F}_{X_l}^{i-l}\|_2.
\end{aligned}$$

Therefore,

$$\begin{aligned} \limsup_{i\to\infty} \sqrt[i]{\|X_i - S\|_F} &\geq \limsup_{i\to\infty} \sqrt[i]{c\|\mathcal{F}_{X_l}^{i-l}\|_2} \\ &\geq \limsup_{i\to\infty} \sqrt[i]{\|\mathcal{F}_{X_l}^{i-l}\|_2} \\ &= \rho(\mathcal{F}_{X_l}) \geq \rho(\mathcal{F}_S) - \epsilon. \end{aligned}$$

Since $\epsilon > 0$ is arbitrary, we have

$$\limsup_{i\to\infty} \sqrt[i]{\|X_i - S\|_F} = \rho(\mathcal{F}_S)$$

Conclusion : $0 \leq \rho(\mathcal{F}_S) \leq 1$

Consider a quadratic matrix equation

$$Q(X) = AX^2 - BX + C$$

and the fixed point iteration for

$$G(X) = B^{-1}(AX^2 + C)$$

where $A, C \geq 0$ and B is a nonsingular M-matrix. If $B - (A + C)$ is a nonsingular or singular irreducible M-matrix, then $\{X_i\}$ with $X_0 = 0$ converges to the elementwise minimal nonnegative solvent S.

For the Newton iteration $X_{i+1} = X_i - (Q'_{X_i})^{-1}(Q(X_i))$, we obtain $-Q'_X(H) = AHX + (AX - B)H$.
So, we can simplify each step of the Newton iteration like the following.

$$\begin{cases} AH_iX_i + (AX_i - B)H_i = -Q(X_i), \\ X_{i+1} = X_i + H_i. \end{cases}$$

Furthermore,

$$AX_{i+1}X_i + (AX_i - B)X_{i+1} = AX_i^2 - C.$$

If G satisfies the conditions of Theorem A,

$$-\mathcal{Q}_S = I_n \otimes B - (S^T \otimes A + I_n \otimes AS)$$

is an M-matrix.

$$\left(\begin{array}{lcl} (\because)\;\; (I_n \otimes B^{-1})(-\mathcal{Q}_S) & = & I_{n^2} - (I_n \otimes B^{-1})(S^T \otimes A + I_n \otimes AS) \\ & = & I_{n^2} - (S^T \otimes B^{-1}A + I_n \otimes B^{-1}AS) \\ & = & I_{n^2} - \mathcal{G}_S. \end{array} \right.$$

$\Rightarrow -\mathcal{Q}_X \geq -\mathcal{Q}_S$ for all $0 \leq X \leq S$.

Claim. $X_i \leq S$.

(i) $X_0 = 0 \leq S$.

(ii) If $X_i \leq S$, then

$$\begin{aligned} -Q'_{X_i}(S - X_{i+1}) &= -A(S - X_{i+1})X_i - (AX_i - B)(S - X_{i+1}) \\ &\geq AS^2 + AX_i^2 - ASX_i - AX_iS \\ &= A(S - X_i) \geq 0 \end{aligned}$$

Since $-\mathcal{Q}_{X_i}$ is an M-matrix, $S - X_{i+1} \geq 0$.

The Monotone Convergence of Newton's Method for Differentiable Convex Matrix Functions
Quadratic Matrix Equations

Claim. $X_i \leq X_{i+1}$.

(i) $X_0 = 0 \leq -(Q'_{X_0})^{-1}(C) = X_1$.

(ii) If $X_i \leq X_{i+1}$, then

$$\begin{aligned} &-Q'_{X_{i+1}}(X_{i+2} - X_{i+1}) \\ &\quad = Q(X_{i+1}) \\ &\quad = AX_{i+1}^2 - BX_{i+1}^2 + C \\ &\quad = AX_{i+1}^2 - AX_{i+1}X_i - AX_iX_{i+1} + AX_i^2 \\ &\quad = A(X_{i+1} - AX_i)^2 \geq 0 \end{aligned}$$

Since $-\mathcal{Q}_{X_{i+1}}$ is an M-matrix, $X_{i+2} \geq X_{i+1}$.

Future Work

1. To prove monotone convergence for other equations that have CPD-condition.
2. To generalize the proof using CPD-condition.

Thank you!

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COE Lecture Note	Mitsuhiro T. NAKAO Kazuhiro YOKOYAMA	Computer Assisted Proofs - Numeric and Symbolic Approaches - 199pages	August 22, 2006
COE Lecture Note	M.J.Shai HARAN	Arithmetical Investigations - Representation theory, Orthogonal polynomials and Quantum interpolations- 174pages	August 22, 2006
COE Lecture Note Vol.3	Michal BENES Masato KIMURA Tatsuyuki NAKAKI	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2005 155pages	October 13, 2006
COE Lecture Note Vol.4	宮田　健治	辺要素有限要素法による磁界解析 - 機能数理学特別講義 21pages	May 15, 2007
COE Lecture Note Vol.5	Francois APERY	Univariate Elimination Subresultants - Bezout formula, Laurent series and vanishing conditions - 89pages	September 25, 2007
COE Lecture Note Vol.6	Michal BENES Masato KIMURA Tatsuyuki NAKAKI	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2006 209pages	October 12, 2007
COE Lecture Note Vol.7	若山　正人 中尾　充宏	九州大学産業技術数理研究センター キックオフミーティング 138pages	October 15, 2007
COE Lecture Note Vol.8	Alberto PARMEGGIANI	Introduction to the Spectral Theory of Non-Commutative Harmonic Oscillators 233pages	January 31, 2008
COE Lecture Note Vol.9	Michael I.TRIBELSKY	Introduction to Mathematical modeling 23pages	February 15, 2008
COE Lecture Note Vol.10	Jacques FARAUT	Infinite Dimensional Spherical Analysis 74pages	March 14, 2008
COE Lecture Note Vol.11	Gerrit van DIJK	Gelfand Pairs And Beyond 60pages	August 25, 2008
COE Lecture Note Vol.12	Faculty of Mathematics, Kyushu University	Consortium “MATH for INDUSTRY” First Forum 87pages	September 16, 2008
COE Lecture Note Vol.13	九州大学大学院 数理学研究院	プロシーディング「損保数理に現れる確率モデル」 — 日新火災・九州大学 共同研究 2008 年 11 月 研究会 — 82pages	February 6, 2009

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COE Lecture Note Vol.14	Michal Beneš, Tohru Tsujikawa Shigetoshi Yazaki	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2008 77pages	February 12, 2009
COE Lecture Note Vol.15	Faculty of Mathematics, Kyushu University	International Workshop on Verified Computations and Related Topics 129pages	February 23, 2009
COE Lecture Note Vol.16	Alexander Samokhin	Volume Integral Equation Method in Problems of Mathematical Physics 50pages	February 24, 2009
COE Lecture Note Vol.17	矢嶋　　徹 及川　正行 梶原　健司 辻　　英一 福本　康秀	非線形波動の数理と物理　66pages	February 27, 2009
COE Lecture Note Vol.18	Tim Hoffmann	Discrete Differential Geometry of Curves and Surfaces　75pages	April 21, 2009
COE Lecture Note Vol.19	Ichiro Suzuki	The Pattern Formation Problem for Autonomous Mobile Robots —Special Lecture in Functional Mathematics—　23pages	April 30, 2009
COE Lecture Note Vol.20	Yasuhide Fukumoto Yasunori Maekawa	Math-for-Industry Tutorial: Spectral theories of non-Hermitian operators and their application　184pages	June 19, 2009
COE Lecture Note Vol.21	Faculty of Mathematics, Kyushu University	Forum "Math-for-Industry" Casimir Force, Casimir Operators and the Riemann Hypothesis 95pages	November 9, 2009
COE Lecture Note Vol.22	Masakazu Suzuki Hoon Hong Hirokazu Anai Chee Yap Yousuke Sato Hiroshi Yoshida	The Joint Conference of ASCM 2009 and MACIS 2009: Asian Symposium on Computer Mathematics Mathematical Aspects of Computer and Information Sciences　436pages	December 14, 2009
COE Lecture Note Vol.23	荒川　恒男 金子　昌信	多重ゼータ値入門　111pages	February 15, 2010
COE Lecture Note Vol.24	Fulton B.Gonzalez	Notes on Integral Geometry and Harmonic Analysis　125pages	March 12, 2010
COE Lecture Note Vol.25	Wayne Rossman	Discrete Constant Mean Curvature Surfaces via Conserved Quantities 130pages	May 31, 2010
COE Lecture Note Vol.26	Mihai Ciucu	Perfect Matchings and Applications　66pages	July 2, 2010

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COE Lecture Note Vol.27	九州大学大学院 数理学研究院	Forum "Math-for-Industry" and Study Group Workshop Information security, visualization, and inverse problems, on the basis of optimization techniques　100pages	October 21, 2010
COE Lecture Note Vol.28	ANDREAS LANGER	MODULAR FORMS, ELLIPTIC AND MODULAR CURVES LECTURES AT KYUSHU UNIVERSITY 2010　62pages	November 26, 2010
COE Lecture Note Vol.29	木田　雅成 原田　昌晃 横山　俊一	Magma で広がる数学の世界　157pages	December 27, 2010
COE Lecture Note Vol.30	原　　　隆 松井　　卓 廣島　文生	Mathematical Quantum Field Theory and Renormalization Theory 201pages	January 31, 2011
COE Lecture Note Vol.31	若山　正人 福本　康秀 高木　　剛 山本　昌宏	Study Group Workshop 2010 Lecture & Report　128pages	February 8, 2011
COE Lecture Note Vol.32	Institute of Mathematics for Industry, Kyushu University	Forum "Math-for-Industry" 2011 "TSUNAMI-Mathematical Modelling" Using Mathematics for Natural Disaster Prediction, Recovery and Provision for the Future　90pages	September 30, 2011
COE Lecture Note Vol.33	若山　正人 福本　康秀 高木　　剛 山本　昌宏	Study Group Workshop 2011 Lecture & Report　140pages	October 27, 2011
COE Lecture Note Vol.34	Adrian Muntean Vladimír Chalupecký	Homogenization Method and Multiscale Modeling　72pages	October 28, 2011
COE Lecture Note Vol.35	横山　俊一 夫　　紀恵 林　　卓也	計算機代数システムの進展　210pages	November 30, 2011
COE Lecture Note Vol.36	Michal Beneš Masato Kimura Shigetoshi Yazaki	Proceedings of Czech-Japanese Seminar in Applied Mathematics 2010 107pages	January 27, 2012
COE Lecture Note Vol.37	若山　正人 高木　　剛 Kirill Morozov 平岡　裕章 木村　正人 白井　朋之 西井　龍映 栄　伸一郎 穴井　宏和 福本　康秀	平成 23 年度 数学・数理科学と諸科学・産業との連携研究ワークショップ　拡がっていく数学　～期待される "見えない力" ～ 154pages	February 20, 2012

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COE Lecture Note Vol.49	照井　　章 小原　功任 濱田　龍義 横山　俊一 穴井　宏和 横田　博史	マス・フォア・インダストリ研究所　共同利用研究集会 II 数式処理研究と産学連携の新たな発展　137pages	August 9, 2013
MI Lecture Note Vol.50	Ken Anjyo Hiroyuki Ochiai Yoshinori Dobashi Yoshihiro Mizoguchi Shizuo Kaji	Symposium MEIS2013: Mathematical Progress in Expressive Image Synthesis　154pages	October 21, 2013
MI Lecture Note Vol.51	Institute of Mathematics for Industry, Kyushu University	Forum “Math-for-Industry” 2013 “The Impact of Applications on Mathematics”　97pages	October 30, 2013
MI Lecture Note Vol.52	佐伯　　修 岡田　勘三 髙木　　剛 若山　正人 山本　昌宏	Study Group Workshop 2013 Abstract, Lecture ＆ Report 142pages	November 15, 2013
MI Lecture Note Vol.53	四方　義啓 櫻井　幸一 安田　貴徳 Xavier Dahan	平成25年度　九州大学マス・フォア・インダストリ研究所 共同利用研究集会　安全・安心社会基盤構築のための代数構造 ～サイバー社会の信頼性確保のための数理学～　158pages	December 26, 2013
MI Lecture Note Vol.54	Takashi Takiguchi	Inverse problems for practice, the present and the future　93pages	January 30, 2014
MI Lecture Note Vol.55	栄　伸一郎 溝口　佳寛 脇　　隼人 渋田　敬史	Study Group Workshop 2013 数学協働プログラム Lecture & Report 98pages	February 10, 2014

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