The Littlewood-Richardson triangles and highest weight vectors

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Let $V_{n,p,q} = \mathrm{M}_{np}(\mathbb{C}) \oplus \mathrm{M}_{nq}(\mathbb{C})$ and let $\mathcal{P}(V_{n,p,q})$ be the algebra of polynomial functions on $V_{n,p,q}$. Let $\mathrm{GL}_n(\mathbb{C}) \times \mathrm{GL}_p(\mathbb{C}) \times \mathrm{GL}_q(\mathbb{C})$ act on $\mathcal{P}(V_{n,p,q})$ by

$$((g, h_1, h_2).f)(X, Y) = f(g^t X h_1, g^{-1} Y h_2)$$

and let $\mathcal{R}$ be the algebra generated by all the $\mathrm{GL}_n(\mathbb{C}) \times \mathrm{GL}_p(\mathbb{C}) \times \mathrm{GL}_q(\mathbb{C})$ highest weight vectors in $\mathcal{P}(V_{n,p,q})$. The algebra $\mathcal{R}$ is graded, and the dimension of each of its homogeneous components is equal to the multiplicity of an irreducible representation of GL_n in the tensor product of two irreducible representations of GL_n. This multiplicity is equal to the number of triangular arrays of integers of the form

$$
\begin{array}{ccccccccccc}
 & & & & & a_{00} & & & & & \\
 & & & & a_{01} & & a_{11} & & & & \\
 & & & a_{02} & & a_{12} & & a_{22} & & & \\
 & & a_{03} & & a_{13} & & a_{23} & & a_{33} & & \\
 & \vdots & & \cdots & & \cdots & & \cdots & & \ddots & \\
a_{0n} & & a_{1n} & & \cdots & & \cdots & & \cdots & & a_{nn}
\end{array}
$$

where the entries a_{ij} satisfies certain conditions. Such an array is called a *Littlewood-Richardson triangle*. We will construct a basis for $\mathcal{R}$ for which the elements of the basis are indexed by a set of Littlewood-Richardson triangles. This is joint work with Roger Howe and Yi Wang.