Self-organization in a foliated phase space

Zensho Yoshida

*University of Tokyo*

Self-organization of a structure is, at its surface, an antithesis of the entropy ansatz. However, disorder can still develop at micro scale while a structure emerges on some macro scale; it seems more common in various nonlinear systems that order and disorder are simultaneous, and such coexistence may be possible if the self-organization and the entropy principle work on different scales. The self-organization of a magnetospheric plasma vortex is an example.

A systematic formulation of a scale hierarchy is given by a "foliation" of the kinetic phase space in terms of actions (adiabatic invariants). Interpreting an action $\mu$ (the magnetic moment, for instance) as the number of quantized "quasi-particles," we can construct a grand-canonical distribution function $f_\alpha \propto e^{-\beta H - \alpha \mu}$ ($H$: Hamiltonian, $\beta$: inverse temperature, $\alpha$: chemical potential of the quasi-particle) that maximizes entropy on a macroscopic leaf where the microscopic action-angle variables are abstracted as quasi-particle numbers. While embedding the macroscopic leaf in the (laboratory-frame) total phase space, the transforming Jacobian weight forces an inhomogeneous density profile in the laboratory flat space.

An inhomogeneous magnetic field (typically a dipole) yields a strongly distorted invariant measure of the magnetized particles, creating a self-organized vortex with steep density gradient; this theory explains "inward diffusion" in magnetospheres, as well as their laboratory simulations [1].