Real Indeterminacy of Stationary Equilibria in Matching Models with Divisible Money

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Recently, real indeterminacy of stationary equilibria has been found in matching models with fiat money. In this paper, it is shown that real indeterminacy generically arises in most matching models with perfectly divisible money. In other words, the real indeterminacy follows from the condition for stationarity of money holdings, and surprisingly it has nothing to do with the other specifications, e.g., the bargaining procedures, of the models.

A sketch of our idea is as follows. Suppose the nominal stock of money is given. When the price level is lower, there is more liquidity in the economy, the trade is more frequent, and therefore the welfare level is higher. When the price level is higher, there is less liquidity in the economy, the trade is less frequent, and therefore the welfare level is lower. If we can find the corresponding equilibrium values of the other variables, such as the money holdings distribution and the value function, as the price level continuously varies, then the real indeterminacy follows. More precisely, if the number of variables is larger than that of equations, then by applying the implicit function theorem this property holds. In this paper, we show that the stationary condition of money holdings, common to all random matching models of money, has at least one more variable than the number of equations. Thus the stationary equilibria in such models are indeterminate.

More specifically, we consider the case of one fiat money. Suppose it is perfectly divisible and there is an upper bound of its holdings. We confine our attention to stationary equilibria in which, for some positive number $p$, all trades occur with its integer multiple amounts of money. We focus on stationary distributions on $\{0, \ldots, N\}$ expressed by $h = (h(0), \ldots, h(N))$, where $h(n)$ is the measure of the set of agents with $np$ amount of money, and $N < \infty$ is the upper bound. The condition for stationarity of money holdings is $O_n = I_n, n = 0, 1, \ldots, N$, and $\sum_{n=0}^{N} h(n) = 1$, where $O_n$ ($I_n$) is the outflow (inflow resp.) at $n$. Since $\sum_{n=0}^{N} O_n = \sum_{n=0}^{N} I_n$ always holds, then, at first glance, there seem to be $(N+1)$ independent equations. Thus it seems that the numbers of independent equations and variables, $h(n), n = 0, \ldots, N$, are the same. However, surprisingly it can be shown that one more equation is always redundant and that the system of equations has always at least one degree of freedom; namely, $\sum_{n=0}^{N} nO_n = \sum_{n=0}^{N} nI_n$ always holds. This fact is the key to the real indeterminacy of stationary equilibria.

We believe that the general results found in the present paper are worthy by themselves, but they also shed a new light on other aspects of monetary economics. In the literature, the welfare effect of monetary policy has often been discussed in matching models with money, and in most of these models money is indivisible and the stationary equilibria are determinate. Thus the standard comparative statics technique can be used, and the effects of the policies are determinate as well. However, if we assume the divisibility of money in these models, the stationary equilibria become indeterminate. Thus it is quite difficult to make accurate predictions of the effects of simple policies, such as the helicopter drop of money, using the comparative statics technique.