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Abstract

The artificial compressible system gives a compressible approximation of the incompressible Navier-Stokes system. The latter system is obtained from the former one in the zero limit of the artificial Mach number ϵ which is a singular limit. The sets of stationary solutions of both systems coincide with each other. It is known that if a stationary solution of the incompressible system is asymptotically stable and the velocity field of the stationary solution satisfies an energy-type stability criterion, then it is also stable as a solution of the artificial compressible one for sufficiently small ϵ. In general, the range of ϵ shrinks when the spectrum of the linearized operator for the incompressible system approaches to the imaginary axis. This can happen when a stationary bifurcation occurs. It is proved that when a stationary bifurcation from a simple eigenvalue occurs, the range of ϵ can be taken uniformly near the bifurcation point to conclude the stability of the bifurcating solution as a solution of the artificial compressible system.

Keywords. Incompressible Navier-Stokes system, artificial compressible system, singular perturbation, stability, bifurcation.

1 Introduction

This paper studies the stability of stationary solutions of the artificial compressible system

$$\epsilon^2\partial_t p + \operatorname{div}\boldsymbol{v} = 0, \tag{1}$$

$$\partial_t\boldsymbol{v} - \nu\Delta\boldsymbol{v} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} + \nabla p = \boldsymbol{g}. \tag{2}$$

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on a bonded domain Ω of $\mathbb{R}^3$ with smooth boundary $\partial\Omega$. Here $\boldsymbol{v} = {}^\top(v^1(x,t), v^2(x,t), v^3(x,t))$ and $p = p(x,t)$ denote the unknown velocity field and pressure, respectively, at time $t > 0$ and position $x \in \Omega$; $\boldsymbol{g} = \boldsymbol{g}(x)$ is a given external force; and $\epsilon > 0$ is a small parameter, called the artificial Mach number.

The system of equations (1)–(2) is considered under the boundary condition

$$\boldsymbol{v}|_{\partial\Omega} = \boldsymbol{v}_*. \tag{3}$$

Here $\boldsymbol{v}_* = \boldsymbol{v}_*(x)$ is a given velocity field satisfying $\int_{\partial\Omega} \boldsymbol{v}_* \cdot \boldsymbol{n}\, dS = 0$, where $\boldsymbol{n}$ denotes the unit outward normal to $\partial\Omega$.

The system (1)–(2) was introduced by A. Chorin ([2, 3, 4]) in numerical computation to compute a stationary solution of the incompressible Navier-Stokes equations:

$$\operatorname{div} \boldsymbol{v} = 0, \tag{4}$$

$$\partial_t \boldsymbol{v} - \nu\Delta\boldsymbol{v} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} + \nabla p = \boldsymbol{g} \tag{5}$$

with the boundary condition (3). The idea of the method proposed by Chorin is as follows. Clearly, the set of stationary solutions of (1)–(2) coincides with that of (4)–(5). If solutions of the artificial compressible system (1)–(2) converge to a function $u_s = {}^\top(p_s, \boldsymbol{v}_s)$ as $t \to \infty$, then the limit u_s is a stationary solution of (1)–(2), and thus, u_s is a stationary solution of (4)–(5). By using this method, Chorin numerically obtained stationary cellular convection patterns of the Bénard convection problem described by the Oberbeck-Boussinesq equation.

A mathematical aspect of Chorin's method was studied in [11, 12]. Since the limit u_s in Chorin's method is a large time limit of solutions of (1)–(2), u_s is stable as a solution of (1)–(2). In [11], by considering the spectrum of the linearized operator, it was shown that if u_s is stable as a solution of (1)–(2), then it is also stable as a solution of (4)–(5). This means that stationary solutions obtained by Chorin's method represents observable flows in the real world.

In [11], a converse question was also considered. If stable stationary solutions of (4)–(5) are also stable as a solution of (1)–(2) when $0 < \epsilon \ll 1$, then one can conclude that (1)–(2) give a good approximation of (4)–(5). In [11], a sufficient condition for a stable stationary solution of (4)–(5) to be stable as a solution of (1)–(2) was obtained. The condition obtained in [11] was then improved in [12]. The result in [12] is stated as follows. We introduce the linearized operators around a stationary solution $u_s = {}^\top(p_s, \boldsymbol{v}_s)$ associated with the systems (1)–(2) and (4)–(5) with (3). Here and in what follows ${}^\top\cdot$ stands for the transposition. Let $\mathbb{L} : L^2_\sigma(\Omega) \to L^2_\sigma(\Omega)$ be the operator defined by

$$\mathbb{L} = -\nu\mathbb{P}\Delta + \mathbb{P}(\boldsymbol{v}_s \cdot \nabla + {}^\top(\nabla \boldsymbol{v}_s))$$

with domain $D(\mathbb{L}) = [H^2(\Omega) \cap H^1_0(\Omega)]^3 \cap L^2_\sigma(\Omega)$. Here $H^k(\Omega)$ denotes the k th order L^2-Sobolev space on Ω, $\mathbb{P}$ is the orthogonal projection, called the Helmholtz projection from $L^2(\Omega)^3$ to $L^2_\sigma(\Omega)$, and $L^2_\sigma(\Omega)$ denotes the set of all L^2-vector fields $\boldsymbol{w}$ on Ω satisfying $\operatorname{div} \boldsymbol{w} = 0$ and $\boldsymbol{w} \cdot \boldsymbol{n}|_{\partial\Omega} = 0$. We define the operator $L_\epsilon : H^1_*(\Omega) \times L^2(\Omega)^3 \to H^1_*(\Omega) \times L^2(\Omega)^3$, acting on $u = {}^\top(p, \boldsymbol{w})$, by

$$L_\epsilon = \begin{pmatrix} 0 & \frac{1}{\epsilon^2}\operatorname{div} \\ \nabla & -\nu\Delta + \boldsymbol{v}_s \cdot \nabla + {}^\top(\nabla \boldsymbol{v}_s) \end{pmatrix}$$

with domain $D(L_\epsilon) = H^1_*(\Omega) \times [H^2(\Omega) \cap H^1_0(\Omega)]^3$. Here $H^1_*(\Omega)$ denotes the set of all H^1 functions on Ω that have zero mean value over Ω.

It was shown in [12] that if $\rho(-\mathbb{L}) \supset \{\lambda \in \mathbb{C}; \operatorname{Re}\lambda \geq -b_0\}$ for some positive constant b_0, then there exist positive constants ϵ_0, κ_0 and b_1 such that $\rho(-L_\epsilon) \supset \{\lambda \in \mathbb{C}; \operatorname{Re}\lambda \geq -b_1\}$ for $0 < \epsilon \leq \epsilon_0$, provided that

$$\inf_{\boldsymbol{w} \in H^1_0(\Omega)^3, \boldsymbol{w} \neq \boldsymbol{0}} \frac{\operatorname{Re}(\mathbb{Q}\boldsymbol{w} \cdot \nabla \boldsymbol{v}_s, \mathbb{Q}\boldsymbol{w})_{L^2}}{\|\nabla \mathbb{Q}\boldsymbol{w}\|^2_{L^2}} \geq -\kappa_0. \tag{6}$$

Here $\mathbb{Q} = I - \mathbb{P}$ is the orthogonal projection from $L^2(\Omega)^3$ to the space $G^2(\Omega) = \{\nabla p; p \in H^1_*(\Omega)\}$ which is the orthogonal complement of $L^2_\sigma(\Omega)$. In general, ϵ_0 depends on b_0 and it may occur $\epsilon_0 \to 0$ as $b_0 \to 0$. So if b_0 approaches to zero, we have to take the range of ϵ smaller and smaller. This can happen when a stationary bifurcation occurs. Therefore, it is inconvenient to consider the stability of a bifurcating stationary solution near the bifurcation point; the range of ϵ shrinks when the bifurcation parameter approaches its critical value.

The aim of this paper is to investigate the spectrum of $-L_\epsilon$ near the origin when a stationary bifurcation occurs. We will show that the range of ϵ in the above mentioned result of [12] can be taken uniformly near the bifurcation point when the stability of a bifurcating solution from a simple eigenvalue is considered. This result can be applied to the Taylor and Bénard problems, i.e., a bifurcation of the Taylor vortex from the Couette flow and a bifurcation of spatially periodic convective patterns from the motionless state, respectively.

We briefly state the main result of this paper. The properties of the eigenvalues of the linearized operator around bifurcating solution near the bifurcation point is well known for the incompressible system (4)–(5). Let $\boldsymbol{v}_\eta$ be a basic stationary solution with a bifurcation parameter η, and let $\mathbb{L}_\eta$ denote the linearized operator around $\boldsymbol{v}_\eta$ for (4)–(5). We assume that $\rho(-\mathbb{L}_\eta) \supset \{\lambda \in \mathbb{C}; \operatorname{Re}\lambda \geq -\tilde{b}_0\} \backslash \{\lambda_\eta\}$ uniformly in $\eta \in [-\eta_0, \eta_0]$ for some positive constants $\tilde{b}_0$ and η_0. Here λ_η is a simple eigenvalue of $-\mathbb{L}_\eta$ and λ_η crosses the origin when η crosses 0. Then a stationary bifurcation occurs at $\eta = 0$ and there exists a nontrivial solution branch $\{\eta(\delta), \tilde{\boldsymbol{v}}(\delta)\}$ with $\{\eta(0), \tilde{\boldsymbol{v}}(0)\} = \{0, \boldsymbol{v}_0\}$, where $\eta(\delta)$ and $\tilde{\boldsymbol{v}}(\delta)$ are analytic in $\delta\, (0 < |\delta| \leq \delta_0)$. We denote by $\mathbb{L}(\delta)$ the linearized operator around $\tilde{\boldsymbol{v}}(\delta)$.

As for the spectrum of $-\mathbb{L}(\delta)$, it holds that

$$\rho(-\mathbb{L}(\delta)) \supset \{\lambda \in \mathbb{C}; \operatorname{Re}\lambda \geq -\frac{3}{4}\tilde{b}_0\} \backslash \{\lambda(\delta)\}$$

for $0 < |\delta| \leq \delta_0$. Here $\lambda(\delta)$ is analytic in δ and satisfies $\lambda(0) = 0$. Therefore, the stability of $\tilde{\boldsymbol{v}}(\delta)$ as a solution of (4)–(5) is determined by $\operatorname{sgn}(\lambda(\delta))$ the sign of $\lambda(\delta)$.

We denote by $L(\epsilon, \delta)$ the linearized operator around $\tilde{\boldsymbol{v}}(\delta)$ for the artificial compressible system (1)–(2). We will show, by a perturbation argument, that the spectrum of $-L(\epsilon, \delta)$ near the origin is given by a simple eigenvalue $\lambda(\epsilon, \delta)$ which satisfies

$$\lambda(\epsilon, \delta) = c_1(\epsilon^2, \delta)\lambda(\delta),$$

where $c_1(\epsilon^2, \delta)$ satisfies $c_1(\epsilon^2, \delta) \geq \frac{1}{2}$ uniformly for $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq \delta_0$. As a consequence, we have

$$\operatorname{sgn}(\lambda(\epsilon, \delta)) = \operatorname{sgn}(\lambda(\delta))$$

uniformly for $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq \delta_0$. This implies that if $\tilde{\boldsymbol{v}}(\delta)$ is unstable as a solution of (4)–(5), then it is also unstable as a solution of (1)–(2) for $0 < \epsilon \leq \epsilon_1$. Furthermore, if $\operatorname{sgn}(\lambda(\delta)) = -1$ and (6) is satisfied with $\boldsymbol{v}_s = \tilde{\boldsymbol{v}}(\delta)$ for $0 < |\delta| \leq \delta_0$, then $\tilde{\boldsymbol{v}}(\delta)$ is stable as a solution of (1)–(2) for $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq \delta_0$.

To prove our main result, we show that the spectrum of $-L(\epsilon, \delta)$ near the origin consists of a simple eigenvalue $\lambda(\epsilon, \delta)$ which is a small perturbation of the eigenvalue $\lambda(\delta)$ of $-\mathbb{L}(\delta)$ for $0 < \epsilon \ll 1$ and $0 < |\delta| \leq 1$. In fact, we prove that $\lambda(\epsilon, \delta)$ is analytic in ϵ^2 and δ. Once this is proved, by expanding $\lambda(\epsilon, \delta)$ in δ and using the structure of the nonlinearity of (4)–(5), one can show that if $\lambda(\delta) = \lambda_k \delta^k + \mathcal{O}(\delta^{k+1})$ ($\lambda_k \neq 0$) for some $k \in \mathbb{N}$, then $\lambda(\epsilon, \delta) = (1 + c_1(\epsilon^2))\lambda_k \delta^k + \Lambda_k(\epsilon, \delta)$ with $|c_1(\epsilon^2)| \leq \frac{1}{2}$ and $\Lambda_k(\epsilon, \delta) = \mathcal{O}(\delta^{k+1})$ uniformly for small ϵ and δ.

We close this section with related works. Témam studied the convergence of solutions as $\epsilon \to 0$ for the system (1)–(2) with the additional stabilizing nonlinear term $+\frac{1}{2}(\mathrm{div}\, \boldsymbol{v})\boldsymbol{v}$ on the left of (2). It was shown in [14, 15, 16] that there exists a weak solution ${}^\top(p_\epsilon, \boldsymbol{v}_\epsilon)$ for each $\epsilon > 0$ such that $\boldsymbol{v}_{\epsilon'} \to \boldsymbol{v}$ in $L^2(0, T; L^2(\Omega)^3)$ and $\nabla p_{\epsilon'} \to \nabla p$ weakly in $H^{-1}(\Omega \times (0, T))$ for all $T > 0$ along a sequence $\epsilon' \to 0$, where ${}^\top(p, \boldsymbol{v})$ is a weak solution of (1)–(2). Similar convergence results were obtained in the case of unbounded domains by Donatelli and Marcati by making use of the wave equation structure of the pressure and the dispersive estimates. See the papers of Donatelli [6, 7] and Donatelli and Marcati [8, 9] and references therein.

This paper is organized as follows. In section 2, we introduce notations used in this paper and state the main results about the stability of bifurcating solutions. Section 3 is devoted to the proofs of main results.

2 Main Results

We introduce notation used in this paper. For $1 \leq p \leq \infty$ we denote by $L^p(\Omega)$ the usual Lebesgue space over Ω and its norm is denoted by $\|\cdot\|_p$. The mth order L^2 Sobolev space over Ω is denoted by $H^m(\Omega)$, and its norm is denoted by $\|\cdot\|_{H^m}$. The inner product of $L^2(\Omega)$ is denoted by $(\cdot, \cdot)$, i.e.,

$$(f, g) = \int_\Omega f(x)\overline{g(x)}dx.$$

Here $\bar{z}$ denotes the complex conjugate of $z \in \mathbb{C}$. We also defined the weighted inner product $\langle\langle \cdot, \cdot \rangle\rangle_\epsilon$ by

$$\langle\langle u_1, u_2 \rangle\rangle_\epsilon = \epsilon^2(p_1, p_2) + (\boldsymbol{w}_1, \boldsymbol{w}_2)$$

for $u_j = {}^\top(p_j, \boldsymbol{w}_j)$, $j = 1, 2$.

We set

$$H_0^1(\Omega) = \text{the}\, H^1(\Omega)\text{-closure of } C_0^\infty(\Omega).$$

We define $L^2_*(\Omega)$ by

$$L^2_*(\Omega) = \{f \in L^2(\Omega);\ \int_\Omega f(x)dx = 0\}$$

and $H^m_*(\Omega)$, $m \in \mathbb{Z}$, $m \geq 0$, by

$$H^m_*(\Omega) = H^m(\Omega) \cap L^2_*(\Omega).$$

We set

$$L^2_\sigma(\Omega) = \{\boldsymbol{v} \in L^2(\Omega)^3;\ \operatorname{div}\boldsymbol{v} = 0 \text{ in } \Omega,\ \boldsymbol{v}\cdot\boldsymbol{n}|_{\partial\Omega} = 0\}.$$

It is known that $(L^2(\Omega))^3 = L^2_\sigma(\Omega) \oplus G^2(\Omega)$ where $G^2(\Omega) = \{\nabla p;\ p \in H^1_*(\Omega)\}$ is orthogonal complement of $L^2_\sigma(\Omega)$. The orthogonal projection $\mathbb{P}$ on $L^2_\sigma(\Omega)$ is called the Helmholtz projection. We set $\mathbb{Q} = I - \mathbb{P}$.

We denote the resolvent set of operator A by $\rho(A)$ and the spectrum of A by $\sigma(A)$. Let X and Y be Banach spaces. We denote by $B(X, Y)$ the set of bounded linear operators from X to Y.

Our interest of this paper is in the stability of a stationary solution bifurcating from a basic stationary flow. Let $\mathcal{R}$ be the Reynolds number and let $\boldsymbol{v}_\mathcal{R}$ be a basic stationary flow. We consider the following situation.

(A0) There exists a positive number $\mathcal{R}_c$ such that if $\mathcal{R}$ is smaller than $\mathcal{R}_c$, then $\boldsymbol{v}_\mathcal{R}$ is stable; and if $\mathcal{R}$ is larger than $\mathcal{R}_c$, then $\boldsymbol{v}_\mathcal{R}$ is unstable and a stationary bifurcation occurs at $\mathcal{R} = \mathcal{R}_c$.

We thus introduce a bifurcation parameter $\eta = \mathcal{R} - \mathcal{R}_c$ and write $\boldsymbol{v}_\mathcal{R}$ as $\boldsymbol{v}_\eta$. The linearized operator $\mathbb{L}_\eta$ around $\boldsymbol{v}_\eta$ then takes the form,

$$\begin{aligned}\mathbb{L}_\eta &= -\mathbb{P}\Delta + (\mathcal{R}_c + \eta)\mathbb{P}(\boldsymbol{v}_\eta\cdot\nabla + ({}^\top\nabla\boldsymbol{v}_\eta)) \\ &= \mathbb{A} + (\mathcal{R}_c + \eta)\mathbb{P}\mathbb{M}[\boldsymbol{v}_\eta],\end{aligned}$$

with domain $D(\mathbb{L}_\eta) = D(\mathbb{A}) = [H^2(\Omega) \cap H^1_0(\Omega)]^3 \cap L^2_\sigma(\Omega)$, where

$$\mathbb{A} = -\mathbb{P}\Delta,\ \mathbb{M}[\boldsymbol{v}]\boldsymbol{w} = \boldsymbol{v}\cdot\nabla\boldsymbol{w} + \boldsymbol{w}\cdot\nabla\boldsymbol{v}.$$

We denote by $\mathbb{L}^*_\eta$ the adjoint operator of $\mathbb{L}_\eta$,

$$\mathbb{L}^*_\eta = \mathbb{A} + (\mathcal{R}_c + \eta)\mathbb{P}\mathbb{M}^*[\boldsymbol{v}_\eta]$$

with domain $D(\mathbb{L}_\eta) = D(\mathbb{A})$, where

$$\mathbb{M}^*[\boldsymbol{v}]\boldsymbol{w} = -\boldsymbol{v}\cdot\nabla\boldsymbol{w} + (\nabla\boldsymbol{v})\boldsymbol{w}.$$

We make the following assumptions.

(A1) $\boldsymbol{v}_\eta$ is a smooth stationary solution.

(A2) $\boldsymbol{v}_\eta$ is analytic in η in $(H^2 \cap H_0^1)(\Omega)^3$.

(A3) 0 is a simple eigenvalue of $-\mathbb{L}_0$ with $\mathrm{Ker}(\mathbb{L}_0) = \mathrm{span}\{\boldsymbol{w}_0\}$. The eigen-projection P_0 for the eigenvalue 0 is

$$P_0 \boldsymbol{w} = \langle \boldsymbol{w} \rangle \boldsymbol{w}_0.$$

Here and in what follows the symbol $\langle \boldsymbol{w} \rangle$ for $\boldsymbol{w} \in L^2(\Omega)^3$ is defined by

$$\langle \boldsymbol{w} \rangle = (\boldsymbol{w}, \boldsymbol{w}_0^*),$$

where $\boldsymbol{w}_0^*$ is the eigenfunction for the eigenvalue 0 of $\mathbb{L}_0^*$ satisfying $\langle \boldsymbol{w}_0 \rangle = 1$.

(A4) $\langle \mathbb{M}[\boldsymbol{v}_0]\boldsymbol{w}_0 + \mathcal{R}_c \mathbb{M}[\boldsymbol{v}_1]\boldsymbol{w}_0 \rangle \neq 0$, where $\boldsymbol{v}_1 = \partial_\eta \boldsymbol{v}_\eta|_{\eta=0}$.

(A5) There exists a positive constant $\widetilde{b_0} > 0$ such that

$$\{\lambda \in \mathbb{C};\, \mathrm{Re}\,\lambda \geq -\widetilde{b_0}\} \backslash \{0\} \subset \rho(-\mathbb{L}_0)$$

.

We are interested in a nontrivial solution branch $\{\eta, \boldsymbol{w}_\eta\}$, $\boldsymbol{w}_\eta \neq \mathbf{0}$, of

$$(\mathrm{NS})_\eta \qquad \mathbb{L}_\eta \boldsymbol{w}_\eta + (\mathcal{R}_c + \eta)\mathbb{P}\mathbb{N}(\boldsymbol{w}_\eta, \boldsymbol{w}_\eta) = 0$$

near $\{\eta, \boldsymbol{w}\} = \{0, 0\}$. Here $\mathbb{N}(\boldsymbol{w}_\eta, \boldsymbol{w}_\eta) = \boldsymbol{w}_\eta \cdot \nabla \boldsymbol{w}_\eta$. Note that $\boldsymbol{w}_\eta = \mathbf{0}$ is a solution of $(\mathrm{NS})_\eta$ for all η. Under (A1)–(A4) we have a nontrivial solution branch. In fact, by applying the standard bifurcation theory, one can prove the following proposition.

Proposition 2.1 *Assume (A1)–(A4). There exist a positive constant δ_0 and a solution branch $\{\eta(\delta),\, \boldsymbol{w}_\eta(\delta)\}$ of $(\mathrm{NS})_\eta$ with $\eta = \eta(\delta)$ of the form*

$$\eta(\delta) = \delta\sigma(\delta),$$

$$\boldsymbol{w}_\eta(\delta) = \delta(\boldsymbol{w}_0 + \delta \boldsymbol{w}_1(\delta)),$$

where $\sigma(\delta)$ is analytic in δ $(|\delta| \leq \delta_0)$, and $\boldsymbol{w}_1(\delta)$ is analytic in δ in $H^2(\Omega)$ $(|\delta| \leq \delta_0)$.

We next consider the stability of $\tilde{\boldsymbol{v}}(\delta) = \boldsymbol{v}_{\eta(\delta)} + \boldsymbol{w}_\eta(\delta)$. The linearized operator around $\tilde{\boldsymbol{v}}(\delta)$ is denoted by

$$\mathbb{L}(\delta) = -\mathbb{P}\Delta + (\mathcal{R}_c + \eta(\delta))\mathbb{M}[\tilde{\boldsymbol{v}}(\delta)].$$

As for the spectrum of $-\mathbb{L}(\delta)$, we have the following proposition.

Proposition 2.2 *Assume (A1)-(A5). There exists a positive number δ_0 such that*

$$\rho(-\mathbb{L}(\delta)) \supset \{\lambda \in \mathbb{C};\, \mathrm{Re}\,\lambda \geq -\frac{3}{4}\tilde{b}_0,\, |\lambda| > \frac{\tilde{b}_0}{4}\},$$

$$\sigma(-\mathbb{L}(\delta)) \cap \{\lambda \in \mathbb{C};\, |\lambda| \leq \frac{\widetilde{b_0}}{4}\} = \{\lambda(\delta)\},$$

for all $\delta \in (-\delta_0, \delta_0)$. Here $\lambda(\delta)$ is a simple eigenvalue given by

$$\lambda(\delta) = -\alpha(\delta)\delta\frac{d\eta}{d\delta}(\delta),$$

where $\alpha(\delta)$ is an analytic function of $\delta \in (-\delta_0, \delta_0)$ satisfying

$$\alpha(0) = -\langle \mathbb{M}[\boldsymbol{v}_0]\boldsymbol{w}_0 + \mathcal{R}_c\mathbb{M}[\boldsymbol{v}_1]\boldsymbol{w}_0\rangle (\neq 0).$$

Proposition 2.2 was obtained by Crandall-Rabinowitz [5] (See also [1, Theorem 27.2]).

Assuming (A0), we have $\alpha(0) > 0$. Therefore, concerning the stability of $\tilde{\boldsymbol{v}}(\delta)$, we have the following proposition.

Proposition 2.3 *Assume (A0)-(A5).*

(i) $\alpha(0) = -\langle \mathbb{M}[\boldsymbol{v}_0]\boldsymbol{w}_0 + \mathcal{R}_c\mathbb{M}[\boldsymbol{v}_1]\boldsymbol{w}_0\rangle > 0.$

(ii) $\lambda(\delta) = \lambda_k\delta^k + \mathcal{O}(\delta^{k+1})$ *if and only if* $\eta(\delta) = \eta_k\delta^k + \mathcal{O}(\delta^{k+1})$. *In this case, it follows that* $\lambda_k = -k\alpha(0)\eta_k$. *Therefore,* $\mathrm{sgn}(\lambda(\delta)) = -\mathrm{sgn}(\eta(\delta))$ *for* $0 < |\delta| \ll 1$.

See Figure 1.

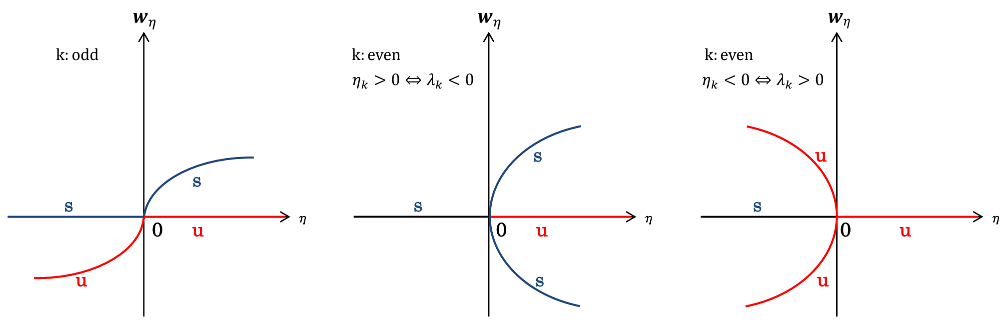

Figure 1: Bifurcation diagram of the incompressible system $(\mathrm{NS})_\eta$

We next consider relations between $\lambda^{(l)}$ and $\eta^{(l)}$.

Proposition 2.4 *The following (a)-(c) are equivalent:*

(a) $\lambda^{(l)}(0) = 0$ *for* $l = 1, \cdots, k$.

(b) $\eta^{(l)}(0) = 0$ *for* $l = 1, \cdots, k$.

(c) $\sigma^{(l-1)}(0) = 0$ *for* $l = 1, \cdots, k$.

Proof. We will prove the proposition by induction on k. Differentiating $\lambda(\delta)$ we have

$$\partial_\delta \lambda(\delta) = \dot{\alpha}(\delta)\delta\dot{\eta}(\delta) + \alpha(\delta)\dot{\eta}(\delta) + \alpha(\delta)\delta\ddot{\eta}(\delta),$$

and so

$$\partial_\delta \lambda(0) = \alpha(0)\dot{\eta}(0).$$

Therefore, we see that

$$\partial_\delta \lambda(0) = 0 \text{ if and only if } \sigma(0) = \dot{\eta}(0) = 0.$$

This shows that the proposition holds for $k = 1$. Let $k \geq 2$ and suppose that the proposition is true for $k - 1$. Then

$$\begin{aligned} \partial_\delta^k \lambda(\delta) &= \sum_{l=0}^{k} \begin{pmatrix} k \\ l \end{pmatrix} \{\delta\}^{(l)} \partial_\delta^{k-l}\{\alpha(\delta)\dot{\eta}(\delta)\} \\ &= \delta\partial_\delta^k\{\alpha(\delta)\dot{\eta}(\delta)\} + k\partial_\delta^{k-1}\{\alpha(\delta)\dot{\eta}(\delta)\}. \end{aligned} \tag{7}$$

Since,

$$\partial_\delta^{k-1}\{\alpha(\delta)\dot\eta(\delta)\} = \sum_{m=0}^{k-1} \begin{pmatrix} k-1 \\ m \end{pmatrix} \alpha^{(m)}(\delta)\dot\eta^{(k-1-m)}(\delta),$$

we see from (7) that

$$\lambda^{(k)}(0) = k\alpha(0)\dot\eta^{(k-1)}(0) = -k\alpha(0)\eta^{(k)}(0).$$

This shows that the proposition is true for k and the proof of the proposition is complete. □

We next consider the stability of the bifurcating solution $\tilde{\boldsymbol{v}}(\delta)$ as a solution of the artificial compressible system (4)–(5). We denote by $L(\epsilon,\delta)$ the linearized operator around $\tilde{\boldsymbol{v}}(\delta)$ which is an operator on $H^1_*(\Omega)\times L^2(\Omega)^3$ given by

$$L(\epsilon,\delta) = \begin{pmatrix} 0 & \frac{1}{\epsilon^2}\mathrm{div} \\ \nabla & -\Delta + (\mathcal{R}_c + \eta(\delta))\mathbb{M}[\tilde v(\delta)] \end{pmatrix}$$

with domain $D(L(\epsilon,\delta)) = D := H^1_*(\Omega)\times[H^2(\Omega)\cap H^1_0(\Omega)]^3$. We also introduce $\mathbb{K}(\delta)$ and $K(\delta)$ defined by

$$\mathbb{K}(\delta) = (\mathcal{R}_c + \eta(\delta))\mathbb{M}[\tilde{\boldsymbol{v}}(\delta)] - \mathcal{R}_c\mathbb{M}[\boldsymbol{v}_0],$$

$$K(\delta) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{K}(\delta) \end{pmatrix}.$$

It follows from Proposition 2.1 that $\mathbb{M}(\delta)$ and $M(\delta)$ can be expanded as

$$\mathbb{K}(\delta) = \sum_{k=1}^{\infty} \delta^k \mathbb{K}_k,$$

$$K(\delta) = \sum_{k=1}^{\infty} \delta^k K_k, \quad K_k = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{K}_k \end{pmatrix}.$$

Here $\mathbb{K}_k$ satisfies the estimate

$$\|\mathbb{K}_k \boldsymbol{w}\|_2 \le c_k \|\boldsymbol{w}\|_{H^1} \tag{8}$$

uniformly for $\boldsymbol{w} \in H^1(\Omega)$ with positive constant c_k satisfying $\sum_{k=1}^\infty c_k\delta^k < \infty$ for $|\delta| \le \delta_1$.

We now state the result on the spectrum of $-L(\epsilon,\delta)$ near the origin.

Theorem 2.1 *Let* $\lambda(\delta) = \lambda_k\delta^k + \mathcal{O}(\delta^{k+1})$ *with* $\lambda_k \neq 0$ *for some* $k \geq 1$*. Then there exist positive constants* $\delta_1 = \delta_1(\tilde{b}_0, \boldsymbol{v}_0)$ *and* $\epsilon_1 = \epsilon_1(\tilde{b}_0, \boldsymbol{v}_0)$ *such that*

$$\sigma(-L(\epsilon, \delta)) \cap \{\lambda \in \mathbb{C};\ |\lambda| \leq \frac{\tilde{b}_0}{4}\} = \{\lambda(\epsilon, \delta)\},$$

$$\lambda(\epsilon, \delta) = \delta^k((1 + c_1(\epsilon^2))\lambda_k + \Lambda_k(\epsilon, \delta))$$

with some $\Lambda_k(\epsilon, \delta) = \mathcal{O}(\delta)$ *uniformly for* $0 < \epsilon \leq \epsilon_1$, $0 < |\delta| \leq \delta_1$*. Here* $c_1(\epsilon^2)$ *satisfies* $|c_1(\epsilon^2)| \leq \frac{1}{2}$ *for* $0 < \epsilon \leq \epsilon_1$*.*

Combining Theorem 2.1 and the argument of the proof of [12, Theorem 2.1], we have the following result on the stability of the bifurcating solution $\tilde{\boldsymbol{v}}(\delta)$ as a solution of the artificial compressible system (4)–(5).

Theorem 2.2 *Assume that (A0)-(A5). Then there exist positive constants* $\epsilon_1 = \epsilon_1(\widetilde{b_0}, \boldsymbol{v}_0)$ *and* $\delta_1 = \delta_1(\widetilde{b_0}, \boldsymbol{v}_0)$ *such that the following assertions hold true for* $0 < |\delta| \leq \delta_1$*.*

(i) If $\tilde{\boldsymbol{v}}(\delta)$ *is unstable as a solution of* (1)–(2) *then so is* $\tilde{\boldsymbol{v}}(\delta)$ *as a solution of* (4)–(5) *for* $0 < \epsilon \leq \epsilon_1$*.*

(ii) Let $\tilde{\boldsymbol{v}}(\delta)$ *be stable as a solution of* (1)–(2)*. Then there exist positive constants* $\epsilon_2 = \epsilon_2(\tilde{b}_0, \boldsymbol{v}_0)$ *and* κ *such that if*

$$\inf_{\boldsymbol{w} \in H_0^1(\Omega)^3, \boldsymbol{w} \neq \boldsymbol{0}} \frac{\mathrm{Re}\,(\mathbb{Q}\boldsymbol{w} \cdot \nabla\tilde{\boldsymbol{v}}(\delta), \mathbb{Q}\boldsymbol{w})}{\|\nabla\mathbb{Q}\boldsymbol{w}\|^2} \geq -\kappa, \tag{9}$$

then $\tilde{\boldsymbol{v}}(\delta)$ *is stable as a solution of* (4)–(5) *for* $0 < \epsilon \leq \epsilon_2$*.*

Similarly to the proof of Theorems 2.1 and 2.2, it is possible to prove the stability and instability of the basic flow $\boldsymbol{v}_\eta$. In fact, one can show that the spectrum of the linearized operator $\mathbb{L}_\eta$ satisfies

$$\sigma(-\mathbb{L}_\eta) = \{\lambda \in \mathbb{C};\ \mathrm{Re}\,\lambda \geq -\frac{3}{4}\tilde{b}_0\} \cup \{\lambda_\eta\},\ \eta \in [-\eta_0, \eta_0]$$

for some positive constant η_0. Here λ_η is a simple eigenvalue of $-\mathbb{L}_\eta$ and satisfies

$$\lambda_\eta = \alpha(0)\eta + \mathcal{O}(\eta^2).$$

Let $L_{\epsilon,\eta}$ be the linearized operator around $u_\eta = {}^\top(p_\eta, \boldsymbol{v}_\eta)$ of the artificial compressible system. Here p_η is the pressure corresponding to $\boldsymbol{v}_\eta$. As in the proof of Theorem 2.1, we have the following result.

Theorem 2.3 *There exist positive constants* $\tilde{\eta}_1 = \tilde{\eta}_1(\tilde{b}_0, \boldsymbol{v}_0)$ *and* $\epsilon_3 = \epsilon_3(\tilde{b}_0, \boldsymbol{v}_0)$ *such that*

$$\sigma(-L_{\epsilon,\eta}) \cap \{\lambda \in \mathbb{C};\, |\lambda| \leq \frac{\tilde{b}_0}{4}\} = \{\lambda_{\epsilon,\eta}\}$$

$$\lambda_{\epsilon,\eta} = \eta(c_1(\epsilon^2)\alpha(0) + \Lambda_{\epsilon,\eta})$$

with some $\Lambda_{\epsilon,\eta} = \mathcal{O}(\eta)$ *uniformly for* $0 < \epsilon \leq \epsilon_3$ *and* $0 < |\eta| \leq \tilde{\eta}_1$.

Combining Theorems 2.1 and 2.3, we can see that the same exchange of stability as in the case of (1)–(2) holds for the case of (4)–(5) uniformly for small ϵ. For definiteness, we consider the case where k is even and η_k is positive in Proposition 2.3 (ii). In this case we have the following result.

Theorem 2.4 *Let* k *be even and* η_k *be positive in Proposition 2.3 (ii). Then there exist positive constants* ϵ_4 *and* δ_2 *such that*

(i) The basic flow $\boldsymbol{v}_{\eta(\delta)}$ *is unstable for* $0 < |\delta| \leq \delta_2$ *and* $0 < \epsilon \leq \epsilon_4$.

(ii) There exist positive constants ϵ_5, δ_3, $\tilde{\eta}_2$ *and* $\tilde{\kappa}$ *such that if*

$$\inf_{\boldsymbol{w}\in H_0^1(\Omega)^3, \boldsymbol{w}\neq \boldsymbol{0}} \frac{\mathrm{Re}\,(\mathbb{Q}\boldsymbol{w}\cdot\nabla\boldsymbol{v}_0, \mathbb{Q}\boldsymbol{w})}{\|\nabla\mathbb{Q}\boldsymbol{w}\|^2} \geq -\tilde{\kappa},$$

then $\boldsymbol{v}_\eta$ *is stable for* $-\tilde{\eta}_2 \leq \eta < 0$ *and* $0 < \epsilon \leq \epsilon_5$ *and* $\tilde{\boldsymbol{v}}(\delta)$ *is stable for* $0 < |\delta| \leq \delta_3$ *and* $0 < \epsilon \leq \epsilon_5$.

The other cases where k is odd or η_k is negative, one can obtain similar results. See Figure 2.

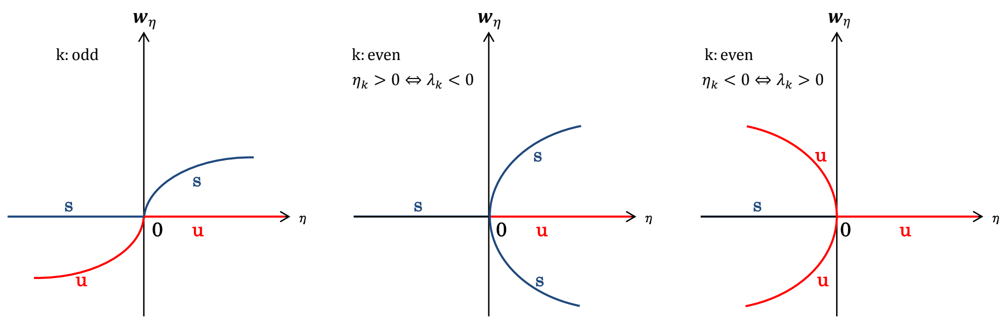

Figure 2: Bifurcating diagram of the artificial compressible system

Remark 2.1 *Theorem 2.4 is applicable to the Taylor and Bénard problems, i.e., a bifurcation of the Taylor vortex from the Couette flow and a bifurcation of spatially periodic convective patterns from the motionless state, respectively.*

3 Proofs of Theorems 2.1 and 2.2

To prove Theorem 2.1 we introduce the operator $\mathcal{L}_{\epsilon,\lambda}$ on $H^1_*(\Omega) \times L^2(\Omega)^3$ defined by

$$D(\mathcal{L}_{\epsilon,\lambda}) = D,$$

$$\mathcal{L}_{\epsilon,\lambda} = \begin{pmatrix} 0 & \frac{1}{\epsilon^2}\mathrm{div} \\ \nabla & \lambda - \Delta + \mathcal{R}_c\mathbb{M}[\boldsymbol{v}_0] \end{pmatrix}.$$

We also introduce the operator $\boldsymbol{A}_\lambda(\delta)$:

$$\boldsymbol{A}_\lambda(\delta)\boldsymbol{w} = -\Delta\boldsymbol{w} + \mathcal{R}_c\mathbb{M}[\boldsymbol{v}_0]\boldsymbol{w} + \mathbb{K}(\delta)\boldsymbol{w}.$$

We give an expression for $\mathcal{L}_{\epsilon,\lambda}^{-1}$ in terms of $(\lambda+\mathbb{L}_0)^{-1}$. To do so, we prepare the following lemma.

Lemma 3.1 *([11, Lemma 4.2]) There exists a bounded linear operator* $\boldsymbol{V}$: $H^1_*(\Omega) \to [H^2(\Omega) \cap H^1_0(\Omega)]^3$ *such that*

$$\mathrm{div}\,\boldsymbol{V}f = f, \quad \|\boldsymbol{V}f\|_{H^2} \le C\|f\|_{H^1}$$

for $f \in H^1_*(\Omega)$.

This lemma follows from the solvability for the nonhomogeneous Stokes problem $\mathrm{div}\,\boldsymbol{v} = f$, $-\Delta\boldsymbol{v} + \nabla p = \boldsymbol{0}$ under the boundary conditions $v|_{\partial\Omega} = \boldsymbol{0}$. See, e.g., [10].

The following expression and estimate for $\mathcal{L}_{\epsilon,\lambda}^{-1}$ were obtained in [11].

Lemma 3.2 *([11, Prop.6.2]) Let* $\lambda \in \Sigma_0 := \{\lambda \in \mathbb{C}; \mathrm{Re}\,\lambda \ge -\tilde{b}_0\}\backslash\{0\}$. *Then*

$$\mathcal{L}_{\epsilon,\lambda}^{-1}F = \begin{pmatrix} p_\lambda[\boldsymbol{g} - \epsilon^2\boldsymbol{A}_\lambda\boldsymbol{V}f] \\ (\lambda + \mathbb{L}_0)^{-1}\mathbb{P}[\boldsymbol{g} - \epsilon^2\boldsymbol{A}_\lambda\boldsymbol{V}f] + \epsilon^2\boldsymbol{V}f \end{pmatrix},$$

$$\|\mathcal{L}_{\epsilon,\lambda}^{-1}F\|_{H^1\times H^2} \le C(\frac{1}{|\lambda|} + 1)\{\epsilon^2\|f\|_{H^1} + \|\boldsymbol{g}\|_2\}$$

for $F = {}^\top(f, \boldsymbol{g}) \in H^1_*(\Omega) \times L^2(\Omega)^3$*. Here* $p_\lambda = p_\lambda[\boldsymbol{g}] \in H^1_*(\Omega)$ *is the unique function satisfying*

$$\lambda \boldsymbol{w} - \Delta \boldsymbol{w} + \mathcal{R}_c \mathbb{M}[\boldsymbol{v}_0]\boldsymbol{w} + \nabla p_\lambda = \boldsymbol{g}$$

with $\boldsymbol{w} = (\lambda + \mathbb{L}_0)^{-1}\mathbb{P}\boldsymbol{g}$*; and* C *is a positive constant independent of* $\lambda \in \Sigma_0$ *and* F*.*

We first show that the spectrum of $-L(\epsilon, \delta)$ near the origin consists of a simple eigenvalue which is analytic in ϵ^2 and δ.

Lemma 3.3 *Let* $\lambda(\delta) = \lambda_k \delta^k + \mathcal{O}(\delta^{k+1})$ *with* $\lambda_k \neq 0$ *for some* $k \geq 1$*. Then there exist positive constants* $\delta_1 = \delta_1(\tilde{b}_0, \boldsymbol{v}_0)$ *and* $\epsilon_1 = \epsilon_1(\tilde{b}_0, \boldsymbol{v}_0)$ *such that*

$$\sigma(-L(\epsilon, \delta)) \cap \{\lambda \in \mathbb{C};\, |\lambda| \leq \frac{\tilde{b}_0}{4}\} = \{\lambda(\epsilon, \delta)\}$$

for $0 < \epsilon \leq \epsilon_1$ *and* $0 < |\delta| < \delta_1$*. Here* $\lambda(\epsilon, \delta)$ *is a simple eigenvalue of* $-L(\epsilon, \delta)$ *and is analytic in* ϵ^2 *and* δ*.*

Proof. By Lemma 3.2, if $|\lambda| = r_0$, $0 < r_0 \leq \frac{3}{4}\tilde{b}_0$, then

$$\|\mathcal{L}^{-1}_{\epsilon,\lambda} F\|_{H^1 \times H^2} \leq C \frac{1}{r_0}\{\epsilon^2 \|f\|_{H^1} + \|\boldsymbol{g}\|_2\} \tag{10}$$

for $F = {}^\top(f, \boldsymbol{g}) \in H^1_*(\Omega) \times L^2(\Omega)^3$. We set

$$J = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

It then follows from (10) and (8) that

$$\|\lambda \mathcal{L}^{-1}_{\epsilon,\lambda} J F\|_{H^1 \times H^2} \leq C\epsilon^2 \|f\|_{H^1}, \tag{11}$$

$$\|\mathcal{L}^{-1}_{\epsilon,\lambda} K(\delta) F\|_{H^1 \times H^2} \leq C \frac{\delta}{r_0} \|\boldsymbol{g}\|_{H^1} \leq C \frac{\delta}{r_0} \|\boldsymbol{g}\|_{H^2}. \tag{12}$$

We see from (11) and (12) that there exist positive constants ϵ_0 and δ_0 which depend only on $\tilde{b}_0$ and $\boldsymbol{v}_0$ such that if $0 \leq \epsilon \leq \epsilon_0$ and $0 \leq |\delta| \leq \delta_0$, then

$$\|(\lambda \mathcal{L}^{-1}_{\epsilon,\lambda} J + \mathcal{L}^{-1}_{\epsilon,\lambda} K(\delta)) F\|_{H^1 \times H^2} \leq \frac{1}{2} \|F\|_{H^1 \times H^2}$$

for all $F \in D(L(\epsilon,\delta))$ uniformly for λ with $\frac{\tilde{b}_0}{4} \le |\lambda| \le \frac{3}{4}\tilde{b}_0$. This implies that there exists a bounded inverse $(I + \lambda \mathcal{L}_{\epsilon,\lambda}^{-1} J + \mathcal{L}_{\epsilon,\lambda}^{-1} K(\delta))^{-1}$ on $D(L(\epsilon,\delta))$ for $0 < \epsilon \le \epsilon_0$, $0 < |\delta| \le \delta_0$ and $\frac{\tilde{b}_0}{4} \le |\lambda| \le \frac{3}{4}\tilde{b}_0$, with estimate

$$\|(I + \lambda \mathcal{L}_{\epsilon,\lambda}^{-1} J + \mathcal{L}_{\epsilon,\lambda}^{-1} K(\delta))^{-1} F\|_{H^1 \times H^2} \le 2\|F\|_{H^1 \times H^2}.$$

It then follows that $\lambda + L(\epsilon,\delta)$ has the inverse

$$(\lambda + L(\epsilon,\delta))^{-1} = (I + \lambda \mathcal{L}_{\epsilon,\lambda}^{-1} J + \mathcal{L}_{\epsilon,\lambda}^{-1} K(\delta))^{-1} \mathcal{L}_{\epsilon,\lambda}^{-1} \tag{13}$$

which is bounded on $H^1_*(\Omega) \times L^2(\Omega)^3$ and is also bounded from $H^1_*(\Omega) \times L^2(\Omega)^3$ to $D(L(\epsilon,\delta))$. Furthermore, $(\lambda + L(\epsilon,\delta))^{-1}$ is given by the Neumann series:

$$(\lambda + L(\epsilon,\delta))^{-1} = \sum_{N=0}^{\infty} (-1)^N (\lambda \mathcal{L}_{\epsilon,\lambda}^{-1} J + \mathcal{L}_{\epsilon,\lambda}^{-1} K(\delta))^N \mathcal{L}_{\epsilon,\lambda}^{-1};$$

and hence, by Lemma 3.2, it is analytic in ϵ^2, δ and λ. We set

$$\begin{aligned} P(\epsilon,\delta) &= \frac{1}{2\pi i} \int_{|\lambda| = \frac{\tilde{b}_0}{2}} (\lambda + L(\epsilon,\delta))^{-1}\, d\lambda \qquad (14) \\ &= \sum_{N=0}^{\infty} \frac{(-1)^N}{2\pi i} \int_{|\lambda| = \frac{\tilde{b}_0}{2}} (\lambda \mathcal{L}_{\epsilon,\lambda}^{-1} J + \mathcal{L}_{\epsilon,\lambda}^{-1} K(\delta))^N \mathcal{L}_{\epsilon,\lambda}^{-1} d\, \lambda. \end{aligned}$$

Then, by [11, Theorem 5.2 (ii)], $P(\epsilon,\delta)$ is the total eigenprojection to the eigenvalues lying inside of $\{\lambda;\, |\lambda| = \frac{\tilde{b}_0}{2}\}$ and it is analytic in ϵ^2 and δ in $B(X,Y)$ with $X = H^1_*(\Omega) \times L^2(\Omega)^3$ and $Y = H^1_*(\Omega) \times H^2(\Omega)^3$. If $\delta = 0$, then $P(\epsilon,0)$ is an eigenprojection for the eigenvalue 0 of $-L(\epsilon,0)$. Since 0 is a simple eigenvalue of $-\mathbb{L}(0)$, we see from [11, Proposition 4.3] that 0 is a simple eigenvalue of $-L(\epsilon,0)$ and the eigenprojection $P(\epsilon,0)$ for 0 is given by

$$P(\epsilon,0)u = \langle\langle u, u_0^* \rangle\rangle_\epsilon u_{0,\epsilon} \quad (u \in H^1_*(\Omega) \times L^2(\Omega)^3).$$

Here

$$u_{0,\epsilon} = c_0(\epsilon) \begin{pmatrix} p_0 \\ \boldsymbol{w}_0 \end{pmatrix},\ c_0(\epsilon^2) = (1 + \epsilon^2 (p_0, p_0^*))^{-1},$$

$$u_0^* = \begin{pmatrix} p_0^* \\ \boldsymbol{w}_0^* \end{pmatrix},$$

where p_0 and p_0^* are the pressures corresponding to $\boldsymbol{w}_0$ and $\boldsymbol{w}_0^*$, respectively, i.e., $L(\epsilon,0)u_0 = L^*(\epsilon,0)u_0^* = 0$. Note that u_0^* is independent of ϵ

and $\langle\langle u_{0,\epsilon}, u_0^*\rangle\rangle_\epsilon = 1$. Since $P(\epsilon,\delta)$ is continuous in ϵ^2 and δ, we find from the perturbation theory that $\dim R(P(\epsilon,\delta)) = \dim (P(\epsilon,0)) = 1$, and hence,

$$\sigma(-L(\epsilon,\delta)) \cap \{\lambda;\ |\lambda| < \frac{\tilde{b}_0}{2}\} = \{\lambda(\epsilon,\delta)\},$$

where $\lambda(\epsilon,\delta)$ is a simple eigenvalue of $-L(\epsilon,\delta)$.

We now show that $\lambda(\epsilon,\delta)$ is analytic in ϵ^2 and δ. Let

$$u(\epsilon,\delta) = \begin{pmatrix} p(\epsilon,\delta) \\ \boldsymbol{w}(\epsilon,\delta) \end{pmatrix} = P(\epsilon,\delta)u_{0,\epsilon}.$$

Then $u(\epsilon,\delta)$ is analytic in ϵ^2 and δ in $H^1_*(\Omega) \times H^2(\Omega)^3$ and

$$-L(\epsilon,\delta)u(\epsilon,\delta) = \lambda(\epsilon,\delta)u(\epsilon,\delta). \tag{15}$$

Taking $\langle\langle\cdot,\cdot\rangle\rangle_\epsilon$ of (15) with u_0^* we have

$$\lambda(\epsilon,\delta)\langle\langle u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon = -\langle\langle L(\epsilon,\delta)u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon.$$

Clearly, $\langle\langle u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon$ is analytic in ϵ^2 and δ. We also have

$$\begin{aligned} &\langle\langle L(\epsilon,\delta)u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon \\ &\quad = \epsilon^2(\frac{1}{\epsilon^2}\operatorname{div}\boldsymbol{w}(\epsilon,\delta), p_0^*) + (\boldsymbol{A}_0(\delta)\boldsymbol{w}(\epsilon,\delta) + \nabla p(\epsilon,\delta), \boldsymbol{w}_0^*) \\ &\quad = (\operatorname{div}\boldsymbol{w}(\epsilon,\delta), p_0^*) + (\boldsymbol{A}_0(\delta)\boldsymbol{w}(\epsilon,\delta), \boldsymbol{w}_0^*). \end{aligned}$$

This shows that $\langle\langle L(\epsilon,\delta)u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon$ is analytic in ϵ^2 and δ. Furthermore, since $\langle\langle u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon|_{\epsilon=\delta=0} = \langle \boldsymbol{w}_0\rangle = 1$, we see that $\langle\langle u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon \neq 0$ for sufficiently small ϵ and δ. Therefore, we have

$$\lambda(\epsilon,\delta) = -\frac{\langle\langle L(\epsilon,\delta)u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon}{\langle\langle u(\epsilon,\delta), u_0^*\rangle\rangle_\epsilon}, \tag{16}$$

and hence, conclude that $\lambda(\epsilon,\delta)$ is analytic in ϵ^2 and δ. □

From the proof of Lemma 3.3 one can see that $\lambda(\epsilon,\delta)$ is given by a small perturbation of $\lambda(\delta)$. In fact, we have the following relation.

Lemma 3.4 *It holds that* $\lambda(0,\delta) = \lambda(\delta)$.

Proof. We see from [11, Proposition 5.1(ii)] that

$$P(\epsilon,\delta)F \to P(0,\delta)F \text{ in } H^1_*(\Omega)\times H^2(\Omega)^3 \text{ as } \epsilon\to 0. \tag{17}$$

Here

$$P(0,\delta)F = \begin{pmatrix} \Pi(\delta)\boldsymbol{g} \\ \mathbb{P}(\delta)\mathbb{P}\boldsymbol{g} \end{pmatrix} = \begin{pmatrix} \frac{1}{2\pi i}\int_{|\lambda|=\frac{\tilde{b}_0}{2}} p_\lambda[\boldsymbol{g}]\,d\lambda \\ \frac{1}{2\pi i}\int_{|\lambda|=\frac{\tilde{b}_0}{2}} (\lambda+\mathbb{L}(\delta))^{-1}\mathbb{P}\boldsymbol{g}\,d\lambda \end{pmatrix}.$$

Since $\mathbb{P}(\delta)$ is the total eigenprojection for the eigenvalues of $-\mathbb{L}(\delta)$ lying in $|\lambda| = \frac{\tilde{b}_0}{2}$, $\mathbb{P}(\delta)$ is the eigenprojection for the eigenvalue $\lambda(\delta)$. Therefore,

$$u(\epsilon,\delta)\to u(0,\delta) = \begin{pmatrix} p(0,\delta) \\ \boldsymbol{w}(0,\delta)\end{pmatrix} \text{ in } H^1_*(\Omega)\times H^2(\Omega)^3 \text{ as } \epsilon\to 0,$$

where $\boldsymbol{w}(0,\delta)$ is an eigenfunction of $-\mathbb{L}(\delta)$ and $p(0,\delta)$ is the corresponding pressure. (In [11], the convergence in (17) is written in $H^1_*(\Omega)\times L^2(\Omega)^3$; but it is easily verified that the convergence in (17) also holds in $H^1_*(\Omega)\times H^2(\Omega)^3$. In fact, it follows from (12), (13) and (14).) In (16) we let $\epsilon\to 0$. Then

$$\begin{aligned}\lambda(0,\delta) &= -\frac{(\operatorname{div}\boldsymbol{w}(0,\delta),p_0^*) + (\boldsymbol{A}_0(\delta)\boldsymbol{w}(0,\delta)+\nabla p(0,\delta),\boldsymbol{w}_0^*)}{(\boldsymbol{w}(0,\delta),\boldsymbol{w}_0^*)} \\ &= -\frac{(\mathbb{L}(\delta)\boldsymbol{w}(0,\delta),\boldsymbol{w}_0^*)}{(\boldsymbol{w}(0,\delta),\boldsymbol{w}_0^*)} = \frac{\lambda(\delta)(\boldsymbol{w}(0,\delta),\boldsymbol{w}_0^*)}{(\boldsymbol{w}(0,\delta),\boldsymbol{w}_0^*)} = \lambda(\delta).\end{aligned}$$

This completes the proof. □

To complete the proof of Theorem 2.1, we prepare the following lemma. Let X_0 and X_1 be defined by

$$X_0 = P_0L^2_\sigma(\Omega) \text{ and } X_1 = (I-P_0)L^2_\sigma(\Omega).$$

Then

$$L^2_\sigma(\Omega) = X_0\oplus X_1.$$

Furthermore, since 0 is a simple eigenvalue of $\mathbb{L}_0 = \mathbb{L}(0)$, the restriction of $\mathbb{L}_0$ to $X_1\cap D(\mathbb{L}_0)$ has a bounded inverse on X_1. In other words, let $\boldsymbol{g}\in L^2_\sigma(\Omega)$, then $\mathbb{L}(0)\boldsymbol{w} = \boldsymbol{g}$ is solvable if and only if $\langle\boldsymbol{g}\rangle = 0$. In this case, there exists a unique solution $\boldsymbol{w} = (\mathbb{L}_0|_{X_1})^{-1}\boldsymbol{g}\in X_1\cap D(\mathbb{L}_0)$ of $\mathbb{L}_0\boldsymbol{w} = \boldsymbol{g}$ satisfying $\langle\boldsymbol{w}\rangle = 0$.

Lemma 3.5 *Let $\lambda(\delta)$ be expanded as $\lambda(\delta) = \sum_{l=0}^{\infty} \lambda_l \delta^l$ and let $k \in \mathbb{N}$. If $\lambda_l = 0$ for $l = 0, \cdots, k-1$, then*

$$\sum_{m=0}^{l-1} \langle \mathbb{K}_{m+1} \boldsymbol{W}_{l-m-1} \rangle = 0 \textit{ for } l = 1, \cdots, k-1$$

and

$$\lambda_k = -\sum_{m=0}^{k-1} \langle \mathbb{K}_{m+1} \boldsymbol{W}_{k-m-1} \rangle.$$

Here $\boldsymbol{W}_l$ $(l = 0, 1, \cdots, k)$ are inductively given by

$$\boldsymbol{W}_0 = \boldsymbol{w}_0,$$

$$\boldsymbol{W}_l = -(\mathbb{L}_0|_{X_1})^{-1} \sum_{m=0}^{l-1} \mathbb{K}_{m+1} \boldsymbol{W}_{l-m-1} \ (l = 1, \cdots, k-1)$$

and

$$\boldsymbol{W}_k = -\lambda_k (\mathbb{L}_0|_{X_1})^{-1} \boldsymbol{w}_0 + \sum_{m=0}^{k-1} \mathbb{K}_{m+1} \boldsymbol{W}_{k-m-1}.$$

The proof of Lemma 3.5 will be given in the end of this section.

We expand $\lambda(\epsilon, \delta)$ in δ in the following way.

$$\lambda(\epsilon, \delta) = \sum_{k=0}^{\infty} \delta^k \lambda_k(\epsilon, 0).$$

We then have the following relation.

Lemma 3.6 *If $\lambda_l = 0$ for $l = 0, \cdots, k-1$, then*

$$\lambda_k(\epsilon, 0) = c_0(\epsilon^2) \lambda_k(0, 0) = c_0(\epsilon^2) \lambda_k,$$

and furthermore, if $\lambda_k = 0$, then

$$u_k(\epsilon, 0) = c_0(\epsilon^2) U_k.$$

Here $U_k = {}^\top(p_k, \boldsymbol{W}_k) \in D$ is the unique solution of

$$L(1, 0) U_k = -\begin{pmatrix} 0 \\ \lambda_k \boldsymbol{w}_0 + \sum_{m=0}^{k-1} \mathbb{K}_{m+1} \boldsymbol{W}_{k-m-1} \end{pmatrix},$$

where $\boldsymbol{W}_l$ $(l = 0, 1, \cdots, k)$ are the function given in Lemma 3.5.

Proof. By Lemma 3.4, we have

$$\lambda(0,\delta) = \sum_{k=0}^{\infty} \delta^k \lambda_k(0,0) = \sum_{k=0}^{\infty} \delta^k \lambda_k.$$

Therefore, by assumption, $\lambda_l(0,0) = 0$ for $l = 0, \cdots, k-1$, and we also have $\lambda_k(0,0) = \lambda_k$. Since $u(\epsilon,\delta)$ is analytic in ϵ^2 and δ, we have

$$u(\epsilon,\delta) = \sum_{k=0}^{\infty} \delta^k u_k(\epsilon,0).$$

We recall that $L(\epsilon,\delta)$ is written as

$$L(\epsilon,\delta) = L(\epsilon,0) + K(\delta) = L(\epsilon,0) + \sum_{k=1}^{\infty} \delta^k K_k,$$

where $K(\delta) = \sum_{k=1}^{\infty} \delta^k K_k = \sum_{k=1}^{\infty} \delta^k \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{K}_k \end{pmatrix}$. Since

$$(\lambda(\epsilon,\delta) + L(\epsilon,\delta))u(\epsilon,\delta) = 0,$$

we have

$$\sum_{m,l=0}^{\infty} \delta^{m+l} \lambda_m(\epsilon,0)u_l(\epsilon,0) + \sum_{l=0}^{\infty} \delta^l L(\epsilon,0)u_l(\epsilon,0) + \sum_{m,l=0}^{\infty} \delta^{m+1+l} K_{m+1}u_l(\epsilon,0) = 0.$$

We equate the coefficient of each power of δ to 0. As for the coefficient of δ^0, we have

$$\lambda_0(\epsilon,0)u_0(\epsilon,0) + L(\epsilon,0)u_0(\epsilon,0) = 0,$$

and so

$$\lambda_0(\epsilon,0) = 0,\ u_0(\epsilon,0) = P(\epsilon,0)u_{0,\epsilon} = u_{0,\epsilon} = c_0(\epsilon^2)u_0.$$

From the coefficient of δ^1, we see that

$$(\lambda_0(\epsilon,0) + L(\epsilon,0))u_1(\epsilon,0) + \lambda_1(\epsilon,0)u_0(\epsilon,0) + K_1 u_0(\epsilon,0) = 0,$$

and hence,

$$\lambda_1(\epsilon,0) = -\langle\langle K_1 u_0(\epsilon,0), u_0^*\rangle\rangle_\epsilon = -c_0(\epsilon^2)\langle \mathbb{K}_1 \boldsymbol{W}_0 \rangle.$$

This, together with Lemma 3.5, implies that $\lambda_1(\epsilon,0) = c_0(\epsilon^2)\lambda_1$. We thus find that the lemma holds for $k=1$.

Suppose that lemma holds for $k=m$. Then $\lambda_l(\epsilon,0) = c_0(\epsilon^2)\lambda_l(0,0) = 0$ and $u_l(\epsilon,0) = c_0(\epsilon^2)U_l(0,0)$ for $l = 0,\cdots,m$. On the other hand, as for the coefficient of δ^{m+1}, we have

$$(\lambda_0(\epsilon,0)+L(\epsilon,0))u_{m+1}(\epsilon,0)+\sum_{l=1}^{m+1}\lambda_l(\epsilon,0)u_{m+1-l}(\epsilon,0)+\sum_{l=0}^{m}K_{l+1}u_{m-l}(\epsilon,0) = 0,$$

and hence,

$$\begin{aligned}\lambda_{m+1}(\epsilon,0) &= -\sum_{l=0}^{m}\langle\langle K_{l+1}u_{m-l}(\epsilon,0), u_0^*\rangle\rangle_\epsilon \\ &= -c_0(\epsilon^2)\sum_{l=0}^{m}\langle\mathbb{K}_{l+1}\boldsymbol{W}_{m-l}\rangle.\end{aligned}$$

Therefore, by Lemma 3.5, we have

$$\lambda_{m+1}(\epsilon,0) = c_0(\epsilon^2)\lambda_{m+1}.$$

Furthermore, if $\lambda_{m+1} = 0$, then

$$\begin{aligned}u_{m+1}(\epsilon,0) &= -L(\epsilon,0)^{-1}\sum_{l=0}^{m}K_{l+1}u_{m-l} \\ &= -L(1,0)^{-1}\sum_{l=0}^{m}K_{l+1}u_{m-l} \\ &= c_0(\epsilon^2)U_{m+1}.\end{aligned}$$

Here we used the fact that $L(\epsilon,0)^{-1}F = L(1,0)^{-1}F$ for all F of the form $F = {}^\top(0,\boldsymbol{g})$. This shows that the lemma holds for $k = m+1$ and the proof of the lemma is complete. □

We are now in a position to prove Theorem 2.1

Proof of Theorem 2.1. By the assumption of Theorem 2.1, we have $\lambda_l = 0$ for $l = 0,1,\cdots,k-1$. Therefore, we see from Lemma 3.6 that $\lambda_l(\epsilon,0) = 0$

for $l = 1, \cdots, k-1$ and $\lambda_k(\epsilon, 0) = c_0(\epsilon)\lambda_k$. It then follows that

$$\begin{aligned}\lambda(\epsilon,\delta) &= \delta^k c_0(\epsilon)\lambda_k(0,0) + \sum_{l=k+1}^{\infty} \delta^l \lambda_l(\epsilon,0) \\ &= \delta^k \left(c_0(\epsilon)\lambda_k + \sum_{l=1}^{\infty} \delta^l \lambda_{k+l}(\epsilon,0) \right).\end{aligned}$$

Since $c_0(\epsilon^2) - 1 = \mathcal{O}(\epsilon^2)$, setting $c_1(\epsilon^2) = c_0(\epsilon^2) - 1$, we have the desired result. This completes the proof. □

We finally give a proof of Theorem 2.2.

Proof of Theorem 2.2. Since $\lambda(\delta)$ is analytic in δ, we see that $\lambda(\delta)$ has the form $\lambda(\delta) = \lambda_k\delta^k + \mathcal{O}(\delta^{k+1})$ with $\lambda_k \neq 0$ for some $k \in \mathbb{N}$. By Proposition 2.2, the stability of $\tilde{\boldsymbol{v}}(\delta)$ as a solution of (1)–(2) is determined by $\mathrm{sgn}\lambda(\delta)$. We see that there exist a positive constant δ_0 such that $\mathrm{sgn}(\lambda(\delta)) = \mathrm{sgn}(\lambda_k\delta^k)$ for $|\delta| \leq \delta_0$. On the other hand, by Theorem 2.1, we find that the spectrum near the origin consists of a simple eigenvalue $\lambda(\epsilon, \delta)$:

$$\lambda(\epsilon,\delta) = \delta^k((1 + c_1(\epsilon^2))\lambda_k + \Lambda_k(\epsilon,\delta)).$$

Since $|c_1(\epsilon^2)| \leq \frac{1}{2}$ for $0 < \epsilon \leq \epsilon_1$, we see that there exists a positive constant δ_1 such that

$$\mathrm{sgn}(\lambda(\epsilon,\delta)) = \mathrm{sgn}(\lambda_k\delta^k) \tag{18}$$

for $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq \delta_1$.

If $\tilde{\boldsymbol{v}}(\delta)$ is unstable as a solution of (1)–(2), then $\mathrm{sgn}(\lambda(\delta)) = \mathrm{sgn}(\lambda_k\delta^k) = 1$, and hence, by (18), $\mathrm{sgn}(\lambda(\delta)) = 1$ for $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq |\delta_1|$. This shows (i).

As for (ii), we see from the proof of Theorem 2.1 and the arguments in [11, Section 6] that $\sigma(-L(\epsilon,\delta)) \cap \{\lambda;\, \mathrm{Re}\,\lambda \geq -\frac{3}{4}\tilde{b}_0\}$ may consist of $\lambda(\epsilon,\delta)$ and a part of the spectrum in a region with $\mathrm{Im}\,\lambda = \mathcal{O}(\epsilon^{-1})$. If $\tilde{\boldsymbol{v}}(\delta)$ is stable as a solution of (1)–(2), then $\mathrm{sgn}(\lambda(\delta)) = \mathrm{sgn}(\lambda_k\delta^k) = -1$, and hence, by (18), $\mathrm{sgn}(\lambda(\epsilon,\delta)) = -1$ for $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq \delta_1$. Concerning the part of the spectrum in a region with $\mathrm{Im}\,\lambda = \mathcal{O}(\epsilon^{-1})$, we see from [12, Proposition 3.4] and its proof that it lies in the left-half plane strictly away from the imaginary axis uniformly in $0 < \epsilon \leq \epsilon_1$ and $0 < |\delta| \leq \delta_1$ (by taking $\epsilon_1 > 0$ smaller if necessary). As a consequence, there exists a positive constant b_0

such that $\sigma(-L(\epsilon,\delta)) \subset \{\lambda;\, \mathrm{Re}\,\lambda \le -b_0\}$ for $0 < \epsilon \le \epsilon_1$ and $0 < |\delta| \le \delta_1$. This completes the proof. $\square$

It remains to prove Lemma 3.5.

Proof of Lemma 3.5. Let $\Pi(\delta) = \frac{1}{2\pi i}\int_{|\lambda|=\frac{\tilde{b}_0}{4}} (\lambda + \mathbb{L}(\delta))^{-1}\, d\lambda$. Then $\Pi(\delta)$ is the eigenprojection for the eigenvalue $\lambda(\delta)$. Set $\boldsymbol{W}(\delta) = \Pi(\delta)\boldsymbol{w}_0$. Then $\boldsymbol{W}(\delta)$ is an eigenfunction for the eigenvalue $\lambda(\delta)$ and is analytic in δ in $H^2(\Omega)^3$ satisfying $\boldsymbol{W}(0) = \boldsymbol{w}_0$. We write $(\lambda(\delta) + \mathbb{L}(\delta))\boldsymbol{W}(\delta) = \mathbf{0}$ as

$$(\lambda(\delta) + \mathbb{L}_0 + \sum_{k=1}^{\infty} \delta^k \mathbb{P}\mathbb{K}_k)\boldsymbol{W}(\delta) = \mathbf{0}.$$

It follows that

$$\sum_{l,m=0}^{\infty} \delta^{l+m}\lambda_l \boldsymbol{W}_m + \sum_{m=0}^{\infty} \delta^m \mathbb{L}_0 \boldsymbol{W}_m + \sum_{l,m=0}^{\infty} \delta^{l+1+m}\mathbb{P}\mathbb{K}_{l+1}\boldsymbol{W}_m = \mathbf{0}.$$

We equate the coefficient of each power of δ to $\mathbf{0}$. Then we have

$$(\lambda_0 + \mathbb{L}_0)\boldsymbol{W}_m + \sum_{l=1}^{m} \lambda_l \boldsymbol{W}_{m-l} + \sum_{l=0}^{m-1} \mathbb{P}\mathbb{K}_{l+1}\boldsymbol{W}_{m-l-1} = \mathbf{0} \qquad (19)$$

for $m = 0, 1, 2, \cdots$. For $m = 0$, we have $\lambda_0 \boldsymbol{W}_0 + \mathbb{L}_0 \boldsymbol{W}_0 = \mathbf{0}$, and hence

$$\lambda_0 = 0,\ \boldsymbol{W}_0 = \boldsymbol{W}(0) = \boldsymbol{w}_0.$$

We prove Lemma 3.5 by induction on k. Let $k = 1$. We see from (19) with $m = 1$ that

$$(\lambda_0 + \mathbb{L}_0)\boldsymbol{W}_1 + \lambda_1 \boldsymbol{W}_0 + \mathbb{P}\mathbb{K}_1 \boldsymbol{W}_0 = \mathbf{0}. \qquad (20)$$

Taking $\langle\,\cdot\,\rangle$ of this equation, we have $\lambda_1 + \langle \mathbb{K}\boldsymbol{W}_0\rangle = 0$, and so

$$\lambda_1 = -\langle \mathbb{K}_1 \boldsymbol{w}_0 \rangle.$$

Therefore, (20) is solvable for $\boldsymbol{W}_1$, and $\boldsymbol{W}_1$ is given by

$$\boldsymbol{W}_1 = -(\mathbb{L}_0|_{X_1})^{-1}(\lambda_1 \boldsymbol{w}_0 + \mathbb{K}_1 \boldsymbol{w}_0).$$

This shows that Lemma 3.5 holds for $k = 1$. Suppose that Lemma 3.5 holds for $k = m$. We will show that the lemma also holds for $k = m + 1$. Assume that $\lambda_l = 0$ for $l = 0, \cdots, m$. Then, by induction assumption, we have

$$\boldsymbol{W}_k = -(\mathbb{L}_0|_{X_1})^{-1} \sum_{l=0}^{k-1} \mathbb{K}_{l+1} \boldsymbol{W}_{k-l-1}$$

for $k = 1, \cdots, m$. Furthermore, it follows from (19) with m replaced by $m+1$ that $\lambda_{m+1} + \sum_{l=0}^{m} \langle \mathbb{K}_{l+1} \boldsymbol{W}_{m-l} \rangle = 0$, and hence,

$$\lambda_{m+1} = -\sum_{l=0}^{m} \langle \mathbb{K}_{l+1} \boldsymbol{W}_{m-l} \rangle.$$

This implies that

$$\mathbb{L}_0 \boldsymbol{W}_{m+1} = -(\lambda_{m+1} \boldsymbol{w}_0 + \sum_{l=0}^{m} \mathbb{K}_{l+1} \boldsymbol{W}_{m-l})$$

has a unique solution $\boldsymbol{W}_{m+1} = -\mathbb{L}_0^{-1}(\lambda_{m+1}\boldsymbol{w}_0 + \sum_{l=0}^{m} \mathbb{K}_{l+1} \boldsymbol{W}_{m-l})$ with $\langle \boldsymbol{W}_{m+1} \rangle = 0$. This completes the proof. □

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